18.440 Midterm 2, Spring 2011: 50 minutes, 100 points

1. Carefully and clearly *show your work* on each problem (without writing anything that is technically not true). In particular, if you use any known fact (or fact proved in lecture) you should state clearly what fact you are using and why it applies.

2. No calculators, books, or notes may be used.

3. Simplify your answers as much as possible.

4. Point total is 90 (10 points are free)

1. (20 points) Jill polishes her resume and sends it to 900 companies she finds on monster.com. Each company responds with probability .1 (independently of what all the other companies do). Let R be the number of companies that respond.

(a) Compute the expectation of R (give an exact number).

(b) Compute the standard deviation of R (given an exact number).

(c) Use a normal random variable approximation to estimate the probability $P\{R > 113\} = P\{R \geq 114\}$. You may use the function $\Phi(a) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{a} e^{-x^2/2} dx$ in your answer.

2. (20 points) Let X_1, X_2, and X_3 be independent uniform random variables on $[0, 1]$.

    (a) Write $X = \max\{X_1, X_2, X_3\}$. Compute $P\{X \le a\}$ for $a \in [0, 1]$.

    (b) Compute the probability density function for X on the interval $[0, 1]$.

    (c) Compute the variance of the first variable X_1.

    (d) Compute the following covariance: $\mathrm{Cov}(X_1 + X_2, X_2 + X_3)$.

3. (10 points) Toss 3 fair coins independently.

   (a) What is the *conditional* expected number of heads given that the first coin comes up heads?

   (b) What is the *conditional* expected number of heads given that there are at least two heads among the three tosses.

4. (10 points) Suppose that the amount of time until a certain radioactive particle decays is exponential with parameter λ. If there are three such particles, and their decay times are independent of each other, what is the expected amount of time until all three particles have decayed?

5. (10 points) Let X be the number on a standard die roll (so X is chosen uniformly from the set $\{1, 2, 3, 4, 5, 6\}$).

(a) What is the moment generating function $M_X(t)$?

(b) Suppose that ten dice are rolled independently and Y is the sum of the numbers on all the dice. What is the moment generating function $M_Y(t)$?

6. (20 points) On a certain hiking trail, it is well known that the lion, tiger, and bear attacks are independent Poisson point processes with respective λ values of .1/hour, .2/hour, and .3/hour. Let T be the number of hours until the first animal of any kind attacks.

(a) What is the probability that there are no lion attacks during the first hour?

(b) What is the probability density function for T?

(c) What is the expected amount of time until the first tiger attack?

(d) What is the distribution of the time until the fifth attack by any animal? (Give both the name of the distribution and an explicit formula for the probability density function.)