18.600: Lecture 12

Poisson random variables

Scott Sheffield

MIT

Outline

Outline

Poisson random variable definition

Poisson random variable properties

Poisson random variable problems

Poisson random variables: motivating questions

- How many raindrops hit a given square inch of sidewalk during a ten minute period?

Poisson random variables: motivating questions

- How many raindrops hit a given square inch of sidewalk during a ten minute period?
- How many people fall down the stairs in a major city on a given day?

Poisson random variables: motivating questions

- How many raindrops hit a given square inch of sidewalk during a ten minute period?
- How many people fall down the stairs in a major city on a given day?
- How many plane crashes in a given year?

Poisson random variables: motivating questions

- How many raindrops hit a given square inch of sidewalk during a ten minute period?
- How many people fall down the stairs in a major city on a given day?
- How many plane crashes in a given year?
- How many radioactive particles emitted during a time period in which the expected number emitted is 5?

Poisson random variables: motivating questions

- How many raindrops hit a given square inch of sidewalk during a ten minute period?
- How many people fall down the stairs in a major city on a given day?
- How many plane crashes in a given year?
- How many radioactive particles emitted during a time period in which the expected number emitted is 5?
- How many calls to call center during a given minute?

Poisson random variables: motivating questions

- How many raindrops hit a given square inch of sidewalk during a ten minute period?
- How many people fall down the stairs in a major city on a given day?
- How many plane crashes in a given year?
- How many radioactive particles emitted during a time period in which the expected number emitted is 5?
- How many calls to call center during a given minute?
- How many goals scored during a 90 minute soccer game?

Poisson random variables: motivating questions

- How many raindrops hit a given square inch of sidewalk during a ten minute period?
- How many people fall down the stairs in a major city on a given day?
- How many plane crashes in a given year?
- How many radioactive particles emitted during a time period in which the expected number emitted is 5?
- How many calls to call center during a given minute?
- How many goals scored during a 90 minute soccer game?
- How many notable gaffes during 90 minute debate?

Poisson random variables: motivating questions

- How many raindrops hit a given square inch of sidewalk during a ten minute period?
- How many people fall down the stairs in a major city on a given day?
- How many plane crashes in a given year?
- How many radioactive particles emitted during a time period in which the expected number emitted is 5?
- How many calls to call center during a given minute?
- How many goals scored during a 90 minute soccer game?
- How many notable gaffes during 90 minute debate?
- **Key idea for all these examples:** Divide time into large number of small increments. Assume that during each increment, there is some small probability of thing happening (independently of other increments).

Remember what e is?

- The number e is defined by $e = \lim_{n\to\infty}(1 + 1/n)^n$.

Remember what e is?

- The number e is defined by $e = \lim_{n\to\infty}(1 + 1/n)^n$.
- It's the amount of money that one dollar grows to over a year when you have an interest rate of 100 percent, continuously compounded.

Remember what e is?

- The number e is defined by $e = \lim_{n\to\infty}(1 + 1/n)^n$.
- It's the amount of money that one dollar grows to over a year when you have an interest rate of 100 percent, continuously compounded.
- Similarly, $e^\lambda = \lim_{n\to\infty}(1 + \lambda/n)^n$.

Remember what e is?

- The number e is defined by $e = \lim_{n\to\infty}(1 + 1/n)^n$.
- It's the amount of money that one dollar grows to over a year when you have an interest rate of 100 percent, continuously compounded.
- Similarly, $e^\lambda = \lim_{n\to\infty}(1 + \lambda/n)^n$.
- It's the amount of money that one dollar grows to over a year when you have an interest rate of 100λ percent, continuously compounded.

Remember what e is?

- The number e is defined by $e = \lim_{n\to\infty}(1+1/n)^n$.
- It's the amount of money that one dollar grows to over a year when you have an interest rate of 100 percent, continuously compounded.
- Similarly, $e^\lambda = \lim_{n\to\infty}(1+\lambda/n)^n$.
- It's the amount of money that one dollar grows to over a year when you have an interest rate of 100λ percent, continuously compounded.
- It's also the amount of money that one dollar grows to over λ years when you have an interest rate of 100 percent, continuously compounded.

Remember what e is?

- The number e is defined by $e = \lim_{n\to\infty}(1+1/n)^n$.
- It's the amount of money that one dollar grows to over a year when you have an interest rate of 100 percent, continuously compounded.
- Similarly, $e^\lambda = \lim_{n\to\infty}(1+\lambda/n)^n$.
- It's the amount of money that one dollar grows to over a year when you have an interest rate of 100λ percent, continuously compounded.
- It's also the amount of money that one dollar grows to over λ years when you have an interest rate of 100 percent, continuously compounded.
- Can also change sign: $e^{-\lambda} = \lim_{n\to\infty}(1-\lambda/n)^n$.

Bernoulli random variable with n large and $np = \lambda$

- Let λ be some moderate-sized number. Say $\lambda = 2$ or $\lambda = 3$. Let n be a huge number, say $n = 10^6$.

Bernoulli random variable with n large and $np = \lambda$

- Let λ be some moderate-sized number. Say $\lambda = 2$ or $\lambda = 3$. Let n be a huge number, say $n = 10^6$.
- Suppose I have a coin that comes up heads with probability λ/n and I toss it n times.

Bernoulli random variable with n large and $np = \lambda$

- Let λ be some moderate-sized number. Say $\lambda = 2$ or $\lambda = 3$. Let n be a huge number, say $n = 10^6$.
- Suppose I have a coin that comes up heads with probability λ/n and I toss it n times.
- How many heads do I expect to see?

Bernoulli random variable with n large and $np = \lambda$

- Let λ be some moderate-sized number. Say $\lambda = 2$ or $\lambda = 3$. Let n be a huge number, say $n = 10^6$.
- Suppose I have a coin that comes up heads with probability λ/n and I toss it n times.
- How many heads do I expect to see?
- Answer: $np = \lambda$.

Bernoulli random variable with n large and $np = \lambda$

- Let λ be some moderate-sized number. Say $\lambda = 2$ or $\lambda = 3$. Let n be a huge number, say $n = 10^6$.
- Suppose I have a coin that comes up heads with probability λ/n and I toss it n times.
- How many heads do I expect to see?
- Answer: $np = \lambda$.
- Let k be some moderate sized number (say $k = 4$). What is the probability that I see exactly k heads?

Bernoulli random variable with n large and $np = \lambda$

- Let λ be some moderate-sized number. Say $\lambda = 2$ or $\lambda = 3$. Let n be a huge number, say $n = 10^6$.
- Suppose I have a coin that comes up heads with probability λ/n and I toss it n times.
- How many heads do I expect to see?
- Answer: $np = \lambda$.
- Let k be some moderate sized number (say $k = 4$). What is the probability that I see exactly k heads?
- Binomial formula: $\binom{n}{k}p^k(1-p)^{n-k} = \frac{n(n-1)(n-2)\dots(n-k+1)}{k!}p^k(1-p)^{n-k}$.

Bernoulli random variable with n large and $np = \lambda$

- Let λ be some moderate-sized number. Say $\lambda = 2$ or $\lambda = 3$. Let n be a huge number, say $n = 10^6$.
- Suppose I have a coin that comes up heads with probability λ/n and I toss it n times.
- How many heads do I expect to see?
- Answer: $np = \lambda$.
- Let k be some moderate sized number (say $k = 4$). What is the probability that I see exactly k heads?
- Binomial formula: $\binom{n}{k} p^k (1-p)^{n-k} = \frac{n(n-1)(n-2)\ldots(n-k+1)}{k!} p^k (1-p)^{n-k}$.
- This is approximately $\frac{\lambda^k}{k!}(1-p)^{n-k} \approx \frac{\lambda^k}{k!} e^{-\lambda}$.

Bernoulli random variable with n large and $np = \lambda$

- Let λ be some moderate-sized number. Say $\lambda = 2$ or $\lambda = 3$. Let n be a huge number, say $n = 10^6$.
- Suppose I have a coin that comes up heads with probability λ/n and I toss it n times.
- How many heads do I expect to see?
- Answer: $np = \lambda$.
- Let k be some moderate sized number (say $k = 4$). What is the probability that I see exactly k heads?
- Binomial formula: $\binom{n}{k} p^k (1-p)^{n-k} = \frac{n(n-1)(n-2)\dots(n-k+1)}{k!} p^k (1-p)^{n-k}$.
- This is approximately $\frac{\lambda^k}{k!}(1-p)^{n-k} \approx \frac{\lambda^k}{k!} e^{-\lambda}$.
- A **Poisson random variable** X with parameter λ satisfies $P\{X = k\} = \frac{\lambda^k}{k!} e^{-\lambda}$ for integer $k \geq 0$.

Outline

Poisson random variable definition

Poisson random variable properties

Poisson random variable problems

Outline

Probabilities sum to one

- A **Poisson random variable** X with parameter λ satisfies $p(k) = P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$.

Probabilities sum to one

- A **Poisson random variable** X with parameter λ satisfies $p(k) = P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$.
- How can we show that $\sum_{k=0}^{\infty} p(k) = 1$?

Probabilities sum to one

- A **Poisson random variable** X with parameter λ satisfies $p(k) = P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$.
- How can we show that $\sum_{k=0}^{\infty} p(k) = 1$?
- Use Taylor expansion $e^{\lambda} = \sum_{k=0}^{\infty} \frac{\lambda^k}{k!}$.

Probabilities sum to one

- A **Poisson random variable** X with parameter λ satisfies $p(k) = P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$.
- How can we show that $\sum_{k=0}^{\infty} p(k) = 1$?
- Use Taylor expansion $e^{\lambda} = \sum_{k=0}^{\infty} \frac{\lambda^k}{k!}$.
- Multiply both sides by 1 to get $1 = \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!}$.

Probabilities sum to one

- A **Poisson random variable** X with parameter λ satisfies $p(k) = P\{X = k\} = \frac{\lambda^k}{k!} e^{-\lambda}$ for integer $k \geq 0$.
- How can we show that $\sum_{k=0}^{\infty} p(k) = 1$?
- Use Taylor expansion $e^{\lambda} = \sum_{k=0}^{\infty} \frac{\lambda^k}{k!}$.
- Multiply both sides by 1 to get $1 = \sum_{k=0}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!}$.
- This is one way to *remember* the Poisson probability mass function. Just remember that it comes from Taylor expansion of e^{λ}.

Remembering/understanding the formula

- Is there a kind of more motivated term-by-term way to remember where $e^{-\lambda}\lambda^k/k!$ comes from? Look at, say, $k = 3$.

Remembering/understanding the formula

- Is there a kind of more motivated term-by-term way to remember where $e^{-\lambda}\lambda^k/k!$ comes from? Look at, say, $k = 3$.
- Say you toss $n = 50$ coins, each heads with probability $\lambda/50$.

Remembering/understanding the formula

- Is there a kind of more motivated term-by-term way to remember where $e^{-\lambda}\lambda^k/k!$ comes from? Look at, say, $k = 3$.
- Say you toss $n = 50$ coins, each heads with probability $\lambda/50$.
- How many "3-head sequences" like $TTTTTTTTTTTTTTHT$ $TTTTTTTTTTTTTHTTTTTTTTTTTTTTTTHTTTTTT$?

Remembering/understanding the formula

- Is there a kind of more motivated term-by-term way to remember where $e^{-\lambda}\lambda^k/k!$ comes from? Look at, say, $k = 3$.
- Say you toss $n = 50$ coins, each heads with probability $\lambda/50$.
- How many "3-head sequences" like $TTTTTTTTTTTTTHT$ $TTTTTTTTTTTTTHTTTTTTTTTTTTTTHTTTTTT$?
- *Number is about* $n^3/3!$. Because have about n^3 ways to pick ordered triple $(5, 35, 24)$, and *nearly all* such triples are distinct, and 3! such triples correspond to each sequence (since each sequence corresponds to an unordered triple).

Remembering/understanding the formula

- Is there a kind of more motivated term-by-term way to remember where $e^{-\lambda}\lambda^k/k!$ comes from? Look at, say, $k = 3$.
- Say you toss $n = 50$ coins, each heads with probability $\lambda/50$.
- How many "3-head sequences" like *TTTTTTTTTTTTTHT TTTTTTTTTTTTTHTTTTTTTTTTTTTTHTTTTTT*?
- *Number is about* $n^3/3!$. Because have about n^3 ways to pick ordered triple $(5, 35, 24)$, and *nearly all* such triples are distinct, and 3! such triples correspond to each sequence (since each sequence corresponds to an unordered triple).
- *Each sequence* has probability about $(\lambda/n)^3 e^{-\lambda}$. Multiplying number by probability gives about $e^{-\lambda}\lambda^k/k!$.

Remembering/understanding the formula

- Is there a kind of more motivated term-by-term way to remember where $e^{-\lambda}\lambda^k/k!$ comes from? Look at, say, $k = 3$.
- Say you toss $n = 50$ coins, each heads with probability $\lambda/50$.
- How many "3-head sequences" like $TTTTTTTTTTTTTTTHT$ $TTTTTTTTTTTTTHTTTTTTTTTTTTTTTTHTTTTTTT$?
- *Number is about* $n^3/3!$. Because have about n^3 ways to pick ordered triple $(5, 35, 24)$, and *nearly all* such triples are distinct, and 3! such triples correspond to each sequence (since each sequence corresponds to an unordered triple).
- *Each sequence* has probability about $(\lambda/n)^3 e^{-\lambda}$. Multiplying number by probability gives about $e^{-\lambda}\lambda^k/k!$.
-
  - $e^{-\lambda}$ is approximate probability of all tails sequence.
  - λ^k comes from fact that *given* sequence with k heads is $(\lambda/n)^k$ times more probable than *given* sequence with zero heads.
  - $k!$ is "ordered vs. unordered overcount factor."

Expectation and variance

- **Recall:** Last lecture we showed that a binomial random variable with parameters (n, p) has expectation np and variance npq (where $q = 1 - p$).

Expectation and variance

- **Recall:** Last lecture we showed that a binomial random variable with parameters (n, p) has expectation np and variance npq (where $q = 1 - p$).
- **Recall:** We had two proof approaches: easier one using additivity of expectation and trickier one using the identify $i\binom{n}{i} = n\binom{n-1}{i-1}$.

Expectation and variance

- **Recall:** Last lecture we showed that a binomial random variable with parameters (n, p) has expectation np and variance npq (where $q = 1 - p$).
- **Recall:** We had two proof approaches: easier one using additivity of expectation and trickier one using the identify $i\binom{n}{i} = n\binom{n-1}{i-1}$.
- Now consider **Poisson random variable** X with parameter λ, which satisfies $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$.

Expectation and variance

- **Recall:** Last lecture we showed that a binomial random variable with parameters (n, p) has expectation np and variance npq (where $q = 1 - p$).
- **Recall:** We had two proof approaches: easier one using additivity of expectation and trickier one using the identify $i\binom{n}{i} = n\binom{n-1}{i-1}$.
- Now consider **Poisson random variable** X with parameter λ, which satisfies $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$.
- What are the expectation and variance of X?

Expectation and variance

- **Recall:** Last lecture we showed that a binomial random variable with parameters (n, p) has expectation np and variance npq (where $q = 1 - p$).
- **Recall:** We had two proof approaches: easier one using additivity of expectation and trickier one using the identify $i\binom{n}{i} = n\binom{n-1}{i-1}$.
- Now consider **Poisson random variable** X with parameter λ, which satisfies $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$.
- What are the expectation and variance of X?
- Before seeking *easiest way to rigorously derive* what is *easiest way to guess (and subsquently remember)*?

Expectation and variance

- **Recall:** Last lecture we showed that a binomial random variable with parameters (n, p) has expectation np and variance npq (where $q = 1 - p$).
- **Recall:** We had two proof approaches: easier one using additivity of expectation and trickier one using the identify $i\binom{n}{i} = n\binom{n-1}{i-1}$.
- Now consider **Poisson random variable** X with parameter λ, which satisfies $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$.
- What are the expectation and variance of X?
- Before seeking *easiest way to rigorously derive* what is *easiest way to guess (and subsquently remember)*?
- **Guess:** $E[X] = \mathrm{Var}[X] = \lambda$. **Reason:** If Y is binomial with parameter (n, p), where $np = \lambda$ with n very large so that $p \approx 0$ and $q \approx 1$, then $E[Y] = \lambda$ and $\mathrm{Var}[Y] = npq \approx \lambda$.

Expectation and variance

- **Recall:** Last lecture we showed that a binomial random variable with parameters (n, p) has expectation np and variance npq (where $q = 1 - p$).
- **Recall:** We had two proof approaches: easier one using additivity of expectation and trickier one using the identify $i\binom{n}{i} = n\binom{n-1}{i-1}$.
- Now consider **Poisson random variable** X with parameter λ, which satisfies $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$.
- What are the expectation and variance of X?
- Before seeking *easiest way to rigorously derive* what is *easiest way to guess (and subsquently remember)*?
- **Guess:** $E[X] = \mathrm{Var}[X] = \lambda$. **Reason:** If Y is binomial with parameter (n, p), where $np = \lambda$ with n very large so that $p \approx 0$ and $q \approx 1$, then $E[Y] = \lambda$ and $\mathrm{Var}[Y] = npq \approx \lambda$.
- **Mnemonic:** binomial has variance npq, and Poisson is obtained by fixing $\lambda = np$ and taking $q \to 1$, so Poisson has variance $\lambda = np$. It's like npq without the q.

Expectation: formal derivation

- Let us formally derive the expectation of a **Poisson random variable** X with parameter λ, which satisfies $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$.

Expectation: formal derivation

- Let us formally derive the expectation of a **Poisson random variable** X with parameter λ, which satisfies $P\{X = k\} = \frac{\lambda^k}{k!} e^{-\lambda}$ for integer $k \geq 0$.
- We use a variant of the "trickier" derivation of binomial expectation. It's not too complicated.

Expectation: formal derivation

- Let us formally derive the expectation of a **Poisson random variable** X with parameter λ, which satisfies $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$.
- We use a variant of the "trickier" derivation of binomial expectation. It's not too complicated.
- By definition of expectation

$$E[X] = \sum_{k=0}^{\infty} P\{X = k\}k = \sum_{k=0}^{\infty} k\frac{\lambda^k}{k!}e^{-\lambda} = \sum_{k=1}^{\infty} \frac{\lambda^k}{(k-1)!}e^{-\lambda}.$$

Expectation: formal derivation

- Let us formally derive the expectation of a **Poisson random variable** X with parameter λ, which satisfies $P\{X = k\} = \frac{\lambda^k}{k!} e^{-\lambda}$ for integer $k \geq 0$.
- We use a variant of the "trickier" derivation of binomial expectation. It's not too complicated.
- By definition of expectation

$$E[X] = \sum_{k=0}^{\infty} P\{X = k\} k = \sum_{k=0}^{\infty} k \frac{\lambda^k}{k!} e^{-\lambda} = \sum_{k=1}^{\infty} \frac{\lambda^k}{(k-1)!} e^{-\lambda}.$$

- Setting $j = k - 1$, this is $\lambda \sum_{j=0}^{\infty} \frac{\lambda^j}{j!} e^{-\lambda} = \lambda$.

Variance: formal derivation

- Given $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$, what is $\mathrm{Var}[X]$?

Variance: formal derivation

- Given $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$, what is $\mathrm{Var}[X]$?
- Use a variant of the "trickier" derivation of binomial variance.

Variance: formal derivation

- Given $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$, what is $\mathrm{Var}[X]$?
- Use a variant of the "trickier" derivation of binomial variance.
- Compute

$$E[X^2] = \sum_{k=0}^{\infty} P\{X = k\}k^2 = \sum_{k=0}^{\infty} k^2 \frac{\lambda^k}{k!} e^{-\lambda} = \lambda \sum_{k=1}^{\infty} k \frac{\lambda^{k-1}}{(k-1)!} e^{-\lambda}.$$

Variance: formal derivation

- Given $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$, what is $\mathrm{Var}[X]$?
- Use a variant of the "trickier" derivation of binomial variance.
- Compute

$$E[X^2] = \sum_{k=0}^{\infty} P\{X = k\}k^2 = \sum_{k=0}^{\infty} k^2 \frac{\lambda^k}{k!} e^{-\lambda} = \lambda \sum_{k=1}^{\infty} k \frac{\lambda^{k-1}}{(k-1)!} e^{-\lambda}.$$

- Setting $j = k - 1$, this is

$$\lambda \left(\sum_{j=0}^{\infty} (j+1) \frac{\lambda^j}{j!} e^{-\lambda} \right) = \lambda E[X+1] = \lambda(\lambda+1).$$

Variance: formal derivation

- Given $P\{X = k\} = \frac{\lambda^k}{k!}e^{-\lambda}$ for integer $k \geq 0$, what is $\mathrm{Var}[X]$?
- Use a variant of the "trickier" derivation of binomial variance.
- Compute

$$E[X^2] = \sum_{k=0}^{\infty} P\{X = k\}k^2 = \sum_{k=0}^{\infty} k^2 \frac{\lambda^k}{k!} e^{-\lambda} = \lambda \sum_{k=1}^{\infty} k \frac{\lambda^{k-1}}{(k-1)!} e^{-\lambda}.$$

- Setting $j = k - 1$, this is

$$\lambda \left(\sum_{j=0}^{\infty} (j+1) \frac{\lambda^j}{j!} e^{-\lambda} \right) = \lambda E[X+1] = \lambda(\lambda+1).$$

- Then $\mathrm{Var}[X] = E[X^2] - E[X]^2 = \lambda(\lambda+1) - \lambda^2 = \lambda$.

Outline

Poisson random variable definition

Poisson random variable properties

Poisson random variable problems

Outline

Poisson random variable problems

- A country has an average of 2 plane crashes per year.

Poisson random variable problems

- A country has an average of 2 plane crashes per year.
- How reasonable is it to assume the number of crashes is Poisson with parameter 2?

Poisson random variable problems

- A country has an average of 2 plane crashes per year.
- How reasonable is it to assume the number of crashes is Poisson with parameter 2?
- Assuming this, what is the probability of exactly 2 crashes? Of zero crashes? Of four crashes?

Poisson random variable problems

- A country has an average of 2 plane crashes per year.
- How reasonable is it to assume the number of crashes is Poisson with parameter 2?
- Assuming this, what is the probability of exactly 2 crashes? Of zero crashes? Of four crashes?
- $e^{-\lambda}\lambda^k/k!$ with $\lambda = 2$ and k set to 2 or 0 or 4

Poisson random variable problems

- A country has an average of 2 plane crashes per year.
- How reasonable is it to assume the number of crashes is Poisson with parameter 2?
- Assuming this, what is the probability of exactly 2 crashes? Of zero crashes? Of four crashes?
- $e^{-\lambda}\lambda^k/k!$ with $\lambda = 2$ and k set to 2 or 0 or 4
- A city has an average of five major earthquakes a century. What is the probability that there is at least one major earthquake in a given decade (assuming the number of earthquakes per decade is Poisson)?

Poisson random variable problems

- A country has an average of 2 plane crashes per year.
- How reasonable is it to assume the number of crashes is Poisson with parameter 2?
- Assuming this, what is the probability of exactly 2 crashes? Of zero crashes? Of four crashes?
- $e^{-\lambda}\lambda^k/k!$ with $\lambda = 2$ and k set to 2 or 0 or 4
- A city has an average of five major earthquakes a century. What is the probability that there is at least one major earthquake in a given decade (assuming the number of earthquakes per decade is Poisson)?
- $1 - e^{-\lambda}\lambda^k/k!$ with $\lambda = .5$ and $k = 0$

Poisson random variable problems

- A country has an average of 2 plane crashes per year.
- How reasonable is it to assume the number of crashes is Poisson with parameter 2?
- Assuming this, what is the probability of exactly 2 crashes? Of zero crashes? Of four crashes?
- $e^{-\lambda}\lambda^k/k!$ with $\lambda = 2$ and k set to 2 or 0 or 4
- A city has an average of five major earthquakes a century. What is the probability that there is at least one major earthquake in a given decade (assuming the number of earthquakes per decade is Poisson)?
- $1 - e^{-\lambda}\lambda^k/k!$ with $\lambda = .5$ and $k = 0$
- A casino deals one million five-card poker hands per year. Approximate the probability that there are exactly 2 royal flush hands during a given year.

Poisson random variable problems

- A country has an average of 2 plane crashes per year.
- How reasonable is it to assume the number of crashes is Poisson with parameter 2?
- Assuming this, what is the probability of exactly 2 crashes? Of zero crashes? Of four crashes?
- $e^{-\lambda}\lambda^k/k!$ with $\lambda = 2$ and k set to 2 or 0 or 4
- A city has an average of five major earthquakes a century. What is the probability that there is at least one major earthquake in a given decade (assuming the number of earthquakes per decade is Poisson)?
- $1 - e^{-\lambda}\lambda^k/k!$ with $\lambda = .5$ and $k = 0$
- A casino deals one million five-card poker hands per year. Approximate the probability that there are exactly 2 royal flush hands during a given year.
- Expected number of royal flushes is $\lambda = 10^6 \cdot 4/\binom{52}{5} \approx 1.54$. Answer is $e^{-\lambda}\lambda^k/k!$ with $k = 2$.