18.952: Theory of Differential Forms
Spring 2025

Instructor Information

Name: Shaoyun Bai, pronouns: he/him/his.
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Class Information

Time: MW 11:00 am – 12:30 pm
Classroom: 2-142
Office Hours: MW 10:00 am – 10:55 am, 2-269, or by appointment

Teaching Assistants

Name:
Email:
Office hours:

Textbook

Differential forms, by Victor Guillemin and Peter Haine: Online access.

About textbook: **It is not mandatory but highly recommended to read the textbook.** The course will closely follow the textbook, and exercises from the textbook will occasionally be left as homework.

Prerequisites

Calculus (18.01) and Linear Algebra (18.700 or 18.701); some familiarity with manifolds is also helpful.

Course Description

This course is concerned with **fundamental aspects of differential forms**.

- Multi-linear algebra, Tensors, Exterior forms.
- Vector fields and differential forms on open subsets of $\mathbb{R}^n$.
- Integral calculus via forms, Sard's Theorem, Degree theory.
- Vector fields and forms on manifolds, Stokes' Theorem, Divergence Theorem, Degree theory for manifolds, Gauss–Bonnet Theorem.
- De Rham theory (cohomology groups of differential manifolds).
- Intersection theory, Thom forms, Applications

[1] I will try to reply within 24 hours. For questions related to mathematics, coming to my office hour is preferred.

Grading

The course grade is determined by the following components:

Homework	40%
Midterm Exam	30%
Final Exam	30%

Depending on the overall feedback, the grades will be curved accordingly.

Two Extra Problem Sets (which contain somewhat more challenging problems) will be posted, and you can earn extra grades (up to 10 points), though the full score is still 100%. These problems may require some extra work.

Attendance: Although attendance is not part of the grade, per University policy, students are expected to attend all classes.

Grade Scale

Final grades will be assigned according to the following scale, with F for scores lower than 60:

A+	97 – 100	B–	80 – 82
A	93 – 96	C+	77 – 79
A–	90 – 92	C	73 – 76
B+	87 – 89	C–	70 – 72
B	83 – 86	D	60 – 69

Homework

Homework is an integral part of the course, which helps you better understand the course material and get prepared for the exams. There will be **six** homework assignments, roughly assigned bi-weekly. The problem sets will be posted **two weeks before its due time**, and you will be required to hand-in your solutions at the beginning of class. Please refer to the weekly schedule for the due dates. Each problem set accounts for $\frac{1}{5}$ of the total score, and **only the** 5 **highest ones will be counted**. You can miss up to one problem set, but still get a full grade on homework.

Except in extraordinary circumstances, late homework will not be accepted.

Exams

There will be an in-class midterm exam (90 minutes) and one final exam (180 minutes). The dates are as follows.

- Midterm: March 19 (Wed), in-class.
- Final exam: awaiting for announcement from registrar

All the exam problems will be picked out from the textbook. You are encouraged to do as many exercises as you can.

If you think you cannot make it to the in-class midterm, **please let me know at least one week ahead**. We will only do make-up exams if it is absolutely necessary. Unlike midterms, the Final exam cannot be moved at the instructor's discretion. If you have foreseeable difficulty in accommodating the schedule of the Finals, please consult your advisor.

If you fail to inform the instructor one week before the mid-term exam, and you have to be absent, the score can be absorbed to the final exam (i.e., if you have to miss the Midterm, the final exam will occupy 60%).

Tentative weekly schedule

Plan for every class			
Date	Material	Sections	Note
2/3	Review of linear algebra, tensors	§1.1, 1.2, 1.3	
2/5	Alternating tensors and wedge product	§1.4, 1.5, 1.6	
2/10	Operations of alternating tensors	§1.7, 1.8, 1.9	
2/12	Vector fields and one-forms	§2.1, 2.2	
2/18	Differential forms and exterior d	§2.3, 2.4	HW1 Due
2/19	Interior product and pullback	§2.5, 2.6	
2/24	Relation with Calculus	§2.7	
2/26	Application: symplectic geometry	§2.8	
3/3	The Poincaré lemma	§3.1, 3.2, 3.3	HW2 Due
3/5	Degree and change of variables	§3.4, 3.5	
3/10	More on degrees	§3.6	
3/12	Sard's Theorem	§3.7	Extra Pset-I Due
3/17	Manifolds and tangent spaces	§4.1, 4.2	HW3 Due
3/19	In-class Midterm	First 3 Chapters	
3/31	Differential forms on manifolds	§4.3	
4/2	Orientations and integrations	§4.4, 4.5	
4/7	Stokes' theorem	§4.6	
4/9	Degree theory and applications	§4.7, 4.8	
4/14	The index of a vector field	§4.9	HW4 Due
4/16	The de Rham cohomology	§5.1	
4/23	Mayer–Vietoris	§5.2	
4/28	(Čech) Cohomology of good covers	§5.3, 5.8	
4/30	Poincaré duality	§5.4	HW5 Due
5/5	Thom classes and intersection theory	§5.5	
5/7	The Lefschetz theorem	§5.6	Extra Pset-II Due
5/12	The Künneth theorem	§5.7	HW6 Due

Accessibility and accommodations

Your success in this class is very important. We all learn differently. If there are aspects of this course that prevent you from learning or exclude you, please let me know as soon as possible. We can develop strategies to meet both your needs and the requirements of the course.

Staying well

Math, and college, can be hard. If you are facing challenges related to your physical or mental health, or any sort of difficulties, you are encouraged to contact your advisor and/or the Student Health Service. If you feel comfortable doing so, please do not hesitate to get in touch with me to discuss ways we can put you in the best possible position to succeed. If you're finding yourself overwhelmed but don't get help, then the tide may very well sweep you away and leave you completely lost!

Inclusivity

We are part of a learning community and must treat one another with respect at all times. This is especially important with regard to race, religion, nationality, sexual orientation, gender, disability, age, size, immigration status, parental status, and any other aspect of identity. I am committed to ensuring that this class is a supportive, inclusive, and safe environment for all students, and that all students are treated with dignity and respect.