Yulia's Dream 2022: concrete mathematics

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Syllabus

Meetings take place Thursdays at 11am Boston time via Zoom (link).

Topics	**Sections**	**Chapter exercises**	**Date**
Intro & program logistics	-	-	May 26
Recurrent problems	1.1 - 1.3	1, 2, 5, 6, 8, 9, 11a, 14, 15, 23	Jun 2
Sums I	2.1 - 2.4	1, 2, 3, 4, 13, 22, 23, 25, 26, 31	Jun 9
Sums II	2.5 - 2.7	7, 9, 10, 11, 17, 18, 27, 28	Jun 16
Integer functions	3.1 - 3.2, 3.4	2, 5, 10, 12, 13, 14, 16, 20, 23, 31	Jun 23
Number theory	4.1 - 4.2, 4.5 - 4.6	1, 2, 3, 6, 9, 13, 17, 18, 31, 36, 39	Jun 30
Binomial coefficients	5.1, 5.4 - 5.5	1, 2, 5, 9, 10, 19, 35, 74, 80	Jul 7
Special numbers	6.1, 6.3 - 6.6	1, 2, 6, 8, 9, 13, 22, 23, 35	Jul 14
Generating functions I	7.2 - 7.4	2, 3, 4, 5, 6, 17, 32, 55	Jul 21
Discrete probability	8.1 - 8.4	1, 2, 4, 7, 8, 9, 11, 12, 18, 19	Jul 28
Project feedback	-	-	Aug 4
O notation and manipulation	9.1 - 9.3	1, 2a, 2c, 3, 7, 8, 9, 11, 24, 51	Aug 11

Exercises. Solutions are due the week after the corresponding chapter is covered in lecture; solutions to the *last five* weekly exercises should be typed up in LaTeX and sent to `juliusbl@mit.edu` Start the homeworks early; if you are unfamiliar with LaTeX, it can take longer than expected to write up a solution.

Final paper. All Yulia's Dream students are asked to write a paper for the completion of the program. Guidelines for the paper:

- It should expand on one of the reading topics or exercises; I will suggest a few topics in July, based on our progress until then. You are encouraged to propose your own topic and run it by me.

*Supported in part by the National Science Foundation. *E-mail*: `juliusbl@mit.edu`

- It should be written in Overleaf, with the correct formatting (inline and displayed formulas, correct use of definition, lemma, proposition, theorem and proof environments, references, etc.).
- It should be *at most 5 pages* long.
- A draft is due on **July 31**. I will provide some feedback on this draft.
- The final paper is due on **August 15**.
- I recommend you start working on the paper at least 3 weeks before the final deadline.

Example paper topics.

- Finite and infinite calculus: which mathematical objects and theorems from standard calculus have discrete analogues? Write a paper extending the results of Section 2.6 of the textbook.
- Read up on the p-adic number system. Write a paper defining this number system and prove some of its elementary properties. Contrast these properties with those of the real number system.
- Read up on the notion of the cardinality of a (potentially infinite) set. Write a paper defining this concept and determine the cardinality of some important sets, including the set of natural numbers $\mathbb{N}$, the set of rational number $\mathbb{Q}$, the set of real numbers $\mathbb{R}$, and the set of infinite sequences of real numbers. Show that there are different sizes of infinity. How can arithmetic be performed with cardinal numbers?

References

[GKP] Graham, R. L., Knuth, D. E., & Patashnik, O. (1994). *Concrete mathematics: a foundation for computer science*. 2nd edition. Addison-Wesley.