

18.781 Problem Set 9: Due Friday, May 5

1. Let α be a quadratic integer of discriminant d . Its *isotropy group* is the group

$$G_\alpha = \{\gamma \in \mathrm{GL}_2(\mathbb{Z}) : \gamma\alpha = \alpha\}.$$

Define $\psi : G_\alpha \rightarrow A(d)$ by sending the matrix $\begin{bmatrix} p & q \\ r & s \end{bmatrix}$ to $r\alpha + s$.

(a) Show that ψ is an *antihomomorphism* from G_α to the group of units $A(d)^*$. (That is, it satisfies $\psi(1) = 1$, $\psi(\gamma\delta) = \psi(\delta)\psi(\gamma)$.)

(b) Show that $\psi : G_\alpha \rightarrow A(d)^*$ is bijective.

2. Use the continued fraction algorithm to find, for the two discriminants listed, (i) the smallest positive solution to $t^2 - du^2 = \pm 4$; (ii) the fundamental unit of $A(d)$; (iii) a generator for G_α , where α is the unique reduced quadratic integer of discriminant d ; (iv) a generator for the group of signed automorphisms of the quadratic form f corresponding to α .

(a) $d = 41$; **(b)** $d = 52$.

3. What is the group-structure of $Cl(-164)$? (In principal, you have to determine the ideals corresponding to quadratic irrationals in $R(-164)$, work out what their products are, and find out which ideals the products are equivalent to. But with care you should not have to do this with many elements.)