Topological Defects

18.S995 - L23

Order Parameters, Broken Symmetry, and Topology

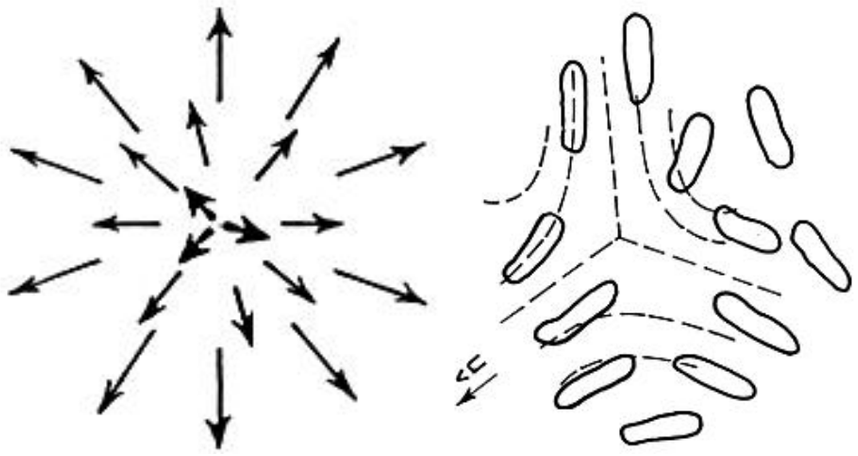
James P. Sethna

Laboratory of Applied Physics, Technical University of Denmark, DK-2800 Lyngby, DENMARK, and NORDITA, DK-2100 Copenhagen Ø, DENMARK and Laboratory of Atomic and Solid State Physics (LASSP), Clark Hall, Cornell University, Ithaca, NY 14853-2501, USA

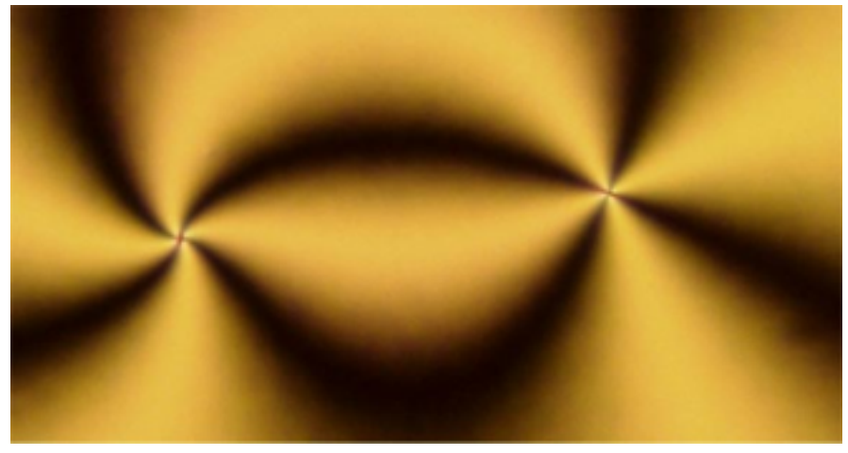
(Dated: May 27, 2003, 10:27 pm)

dunkel@mit.edu

Topological defects are discontinuities in order-parameter fields



- optical effects
- work hardening, etc



"umbilic defects" in a nematic liquid crystal

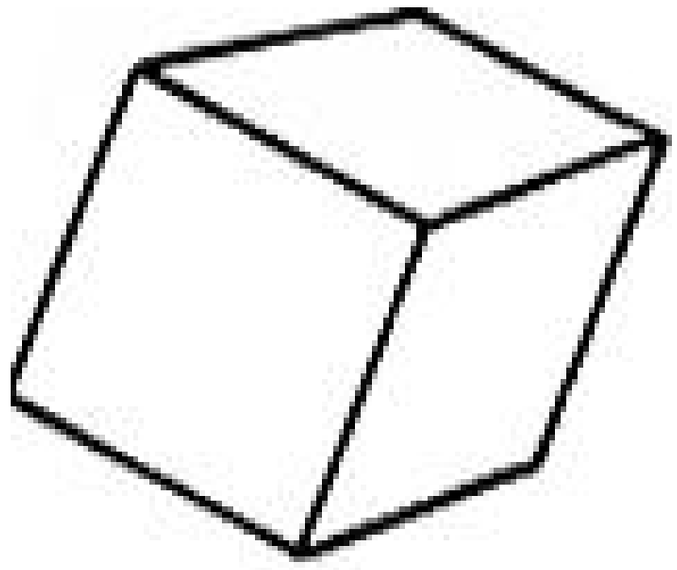
order = symmetry = invariance

(under certain group actions)

symmetry groups can be discrete,
continuous, Lie-groups, ….

More or less symmetric ?

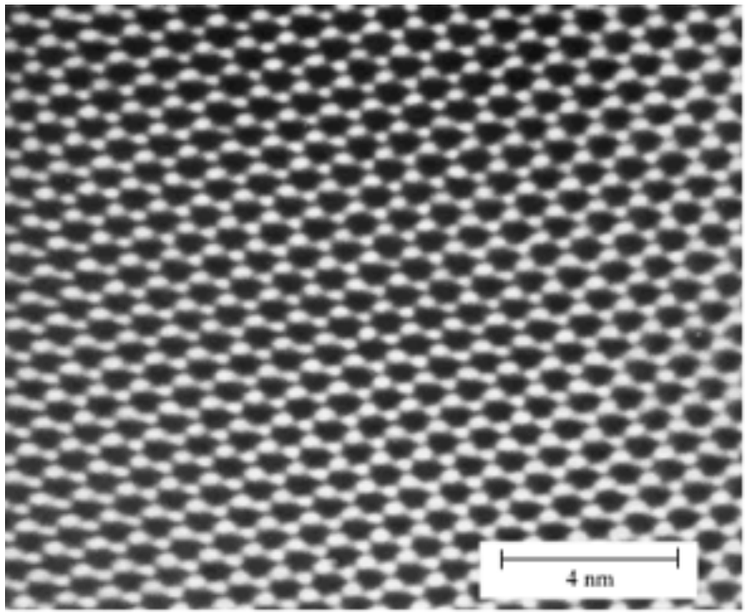
A

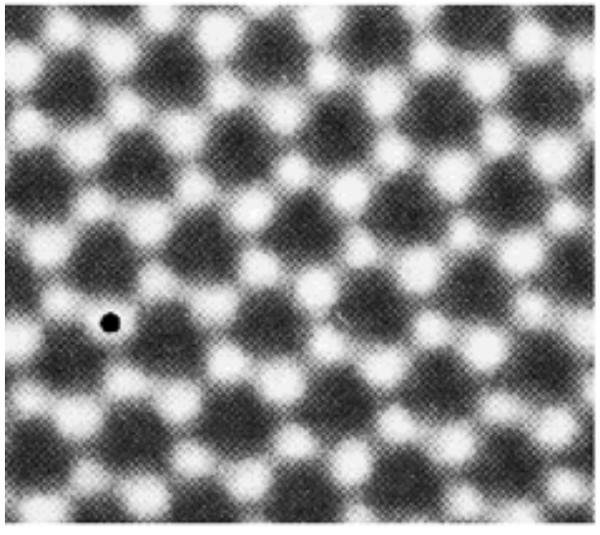


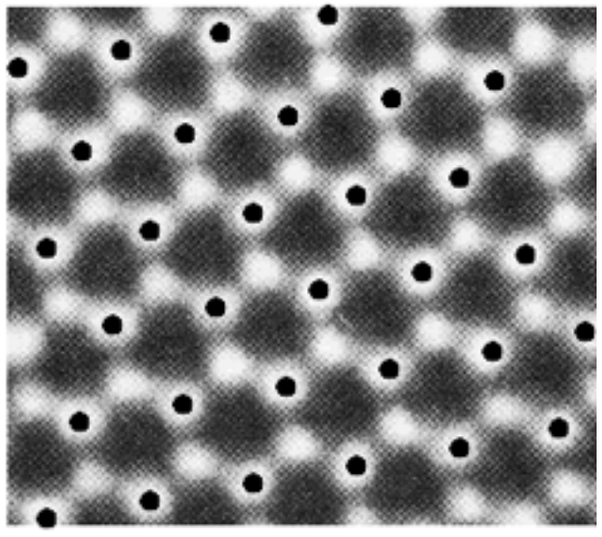
B

More or less symmetric ?

$Mg_2Al_4Si_5O_{18}$

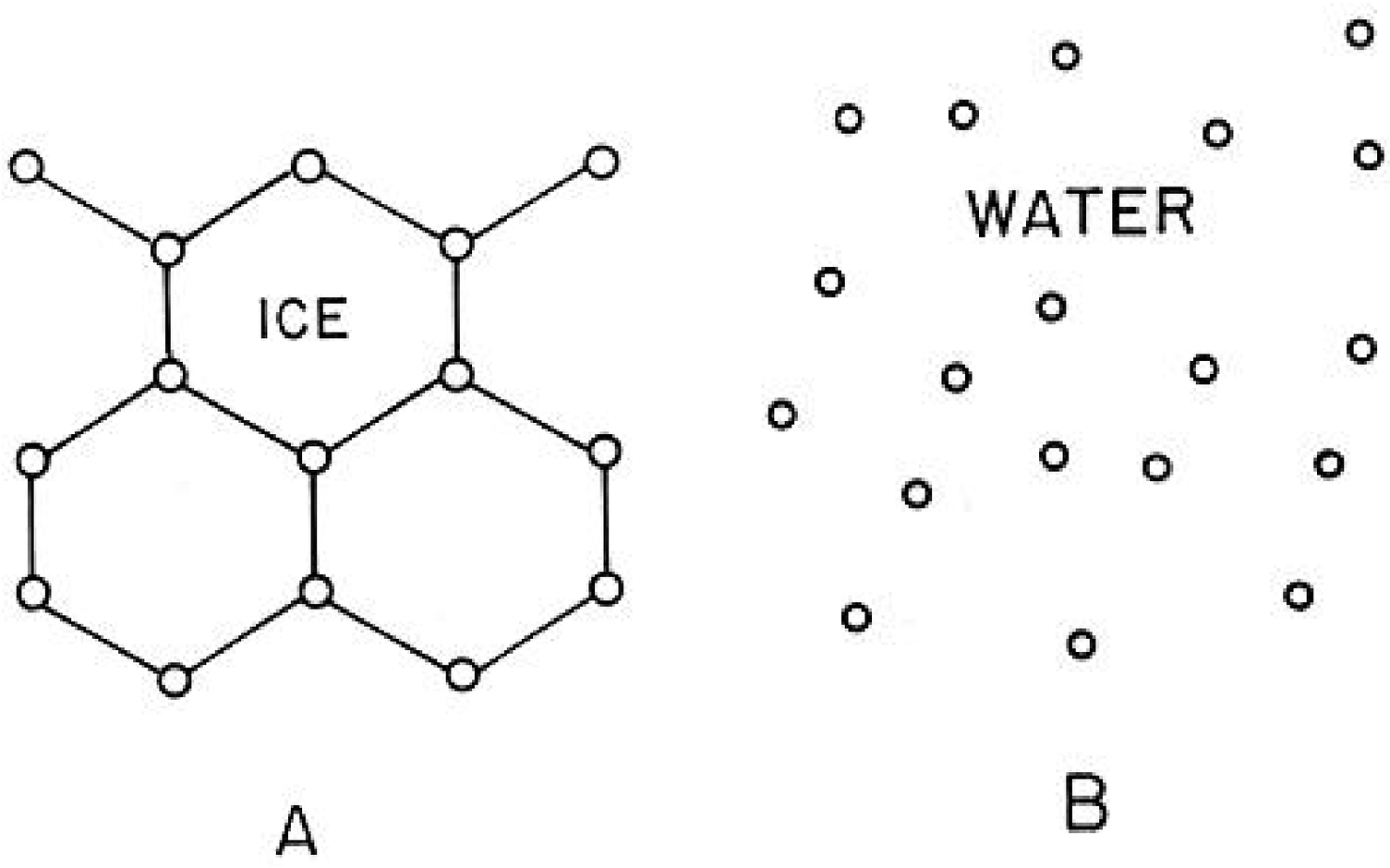






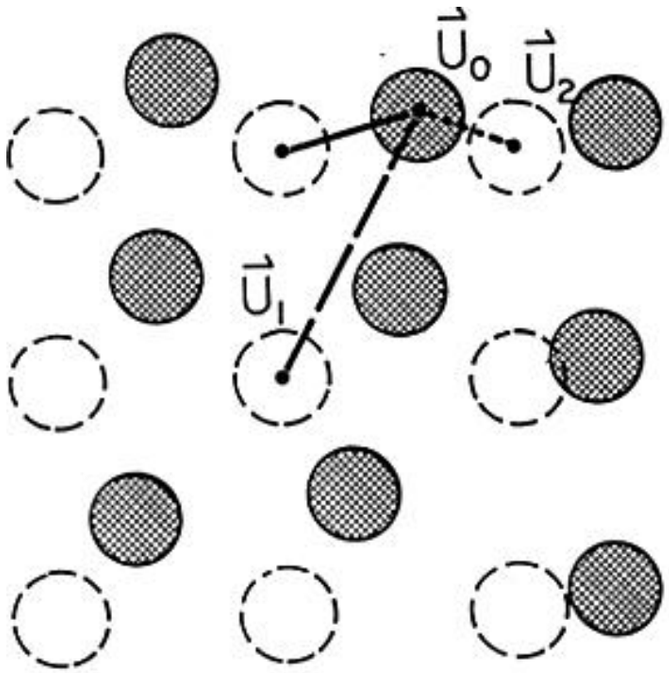
http://www.doitpoms.ac.uk/tlplib/crystallography3/printall.php

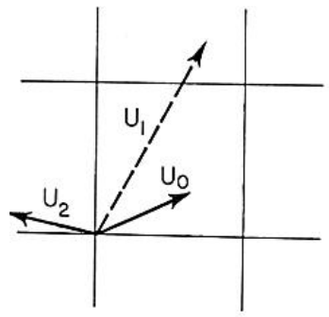
More or less symmetric ?

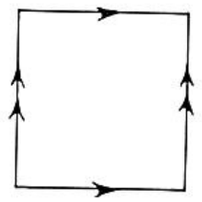


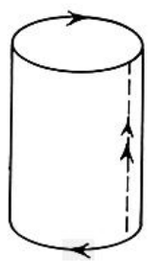
broken continuous
translation/rotation
symmetry (invariance)

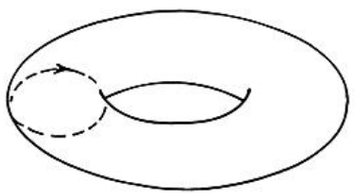
Order parameters: 2D crystal







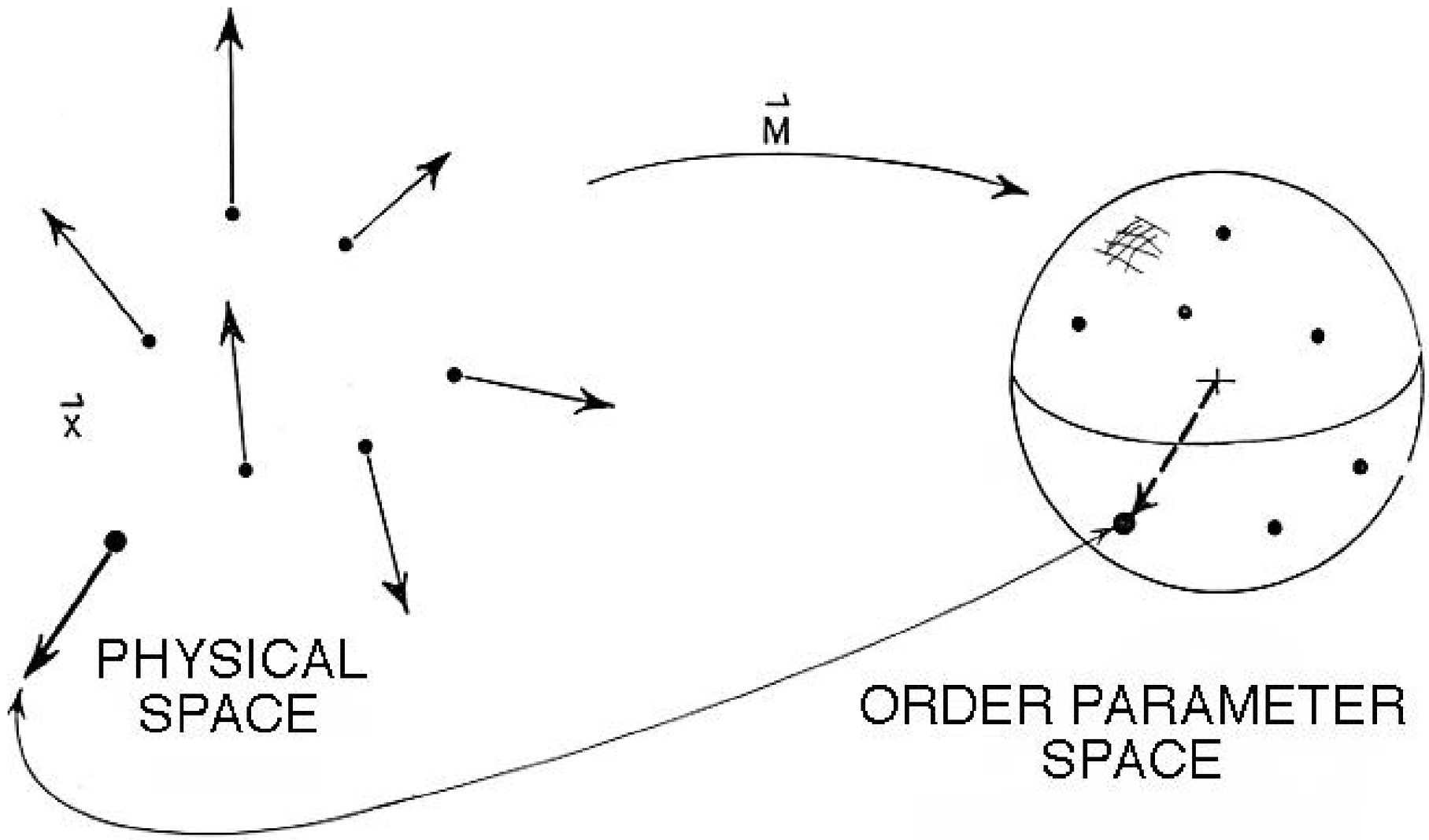




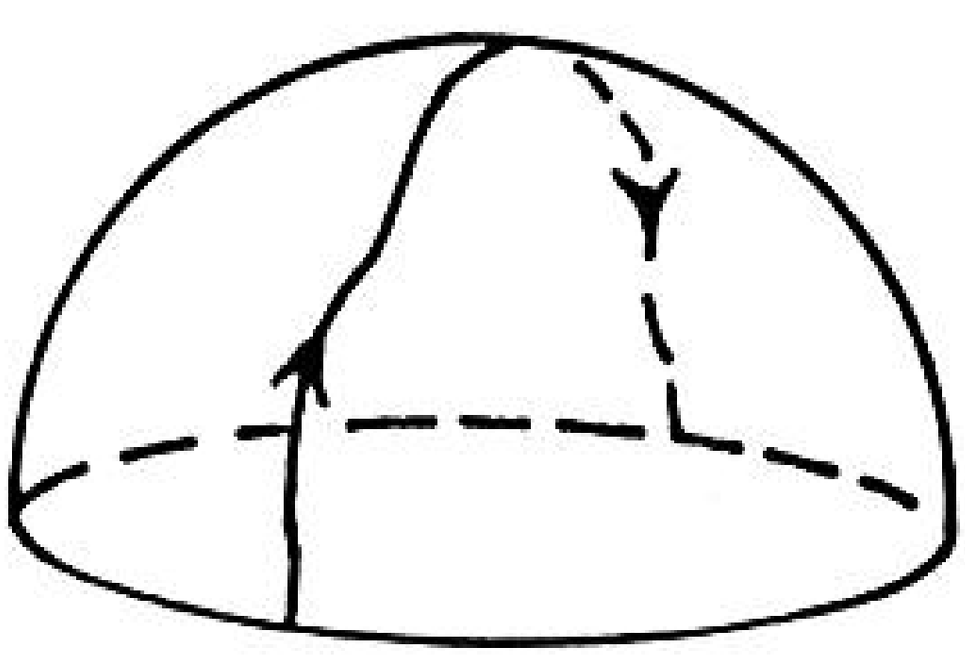
$$\vec{u} \equiv \vec{u} + a\hat{x} = \vec{u} + ma\hat{x} + na\hat{y}.$$

$$\mathcal{E} = \int dx\,(\kappa/2)(du/dx)^2.$$

Order parameters: magnets

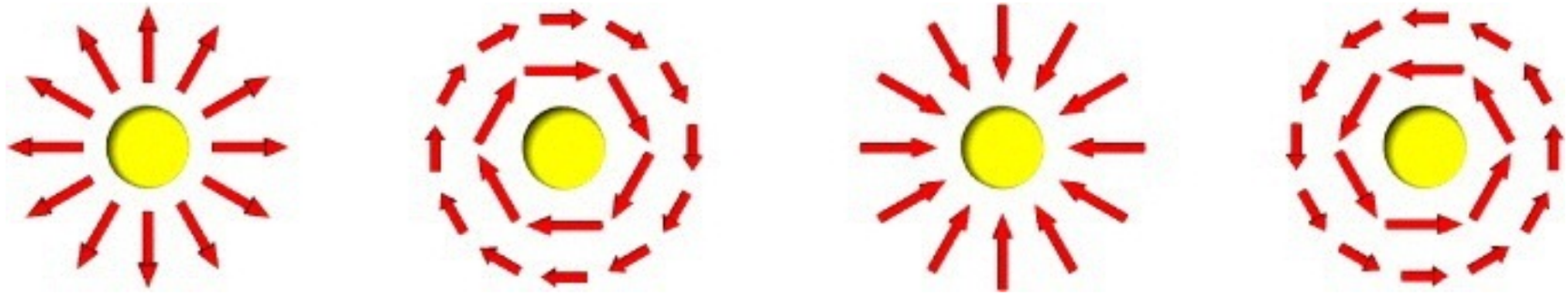


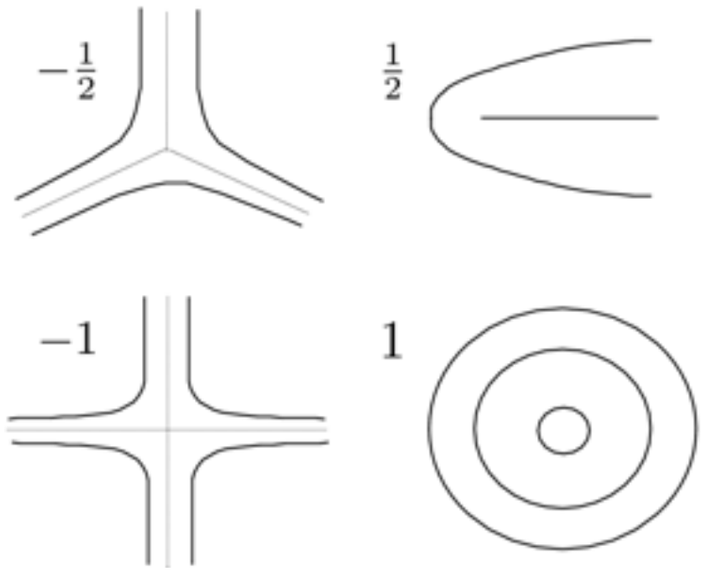
Order parameters: nematic liquid crystals

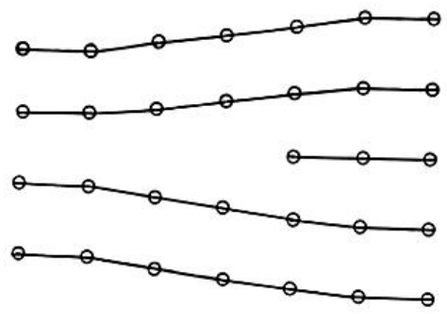


"projective plane" = half-sphere with opposite points on equator identified

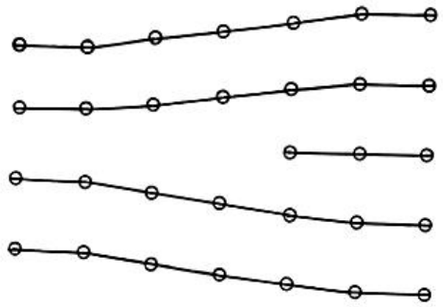
Topological defects



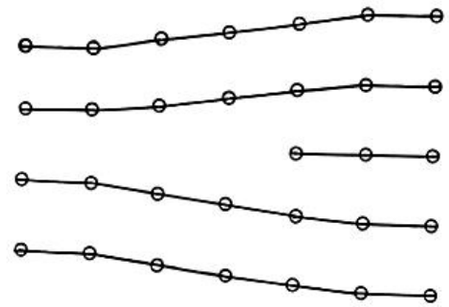




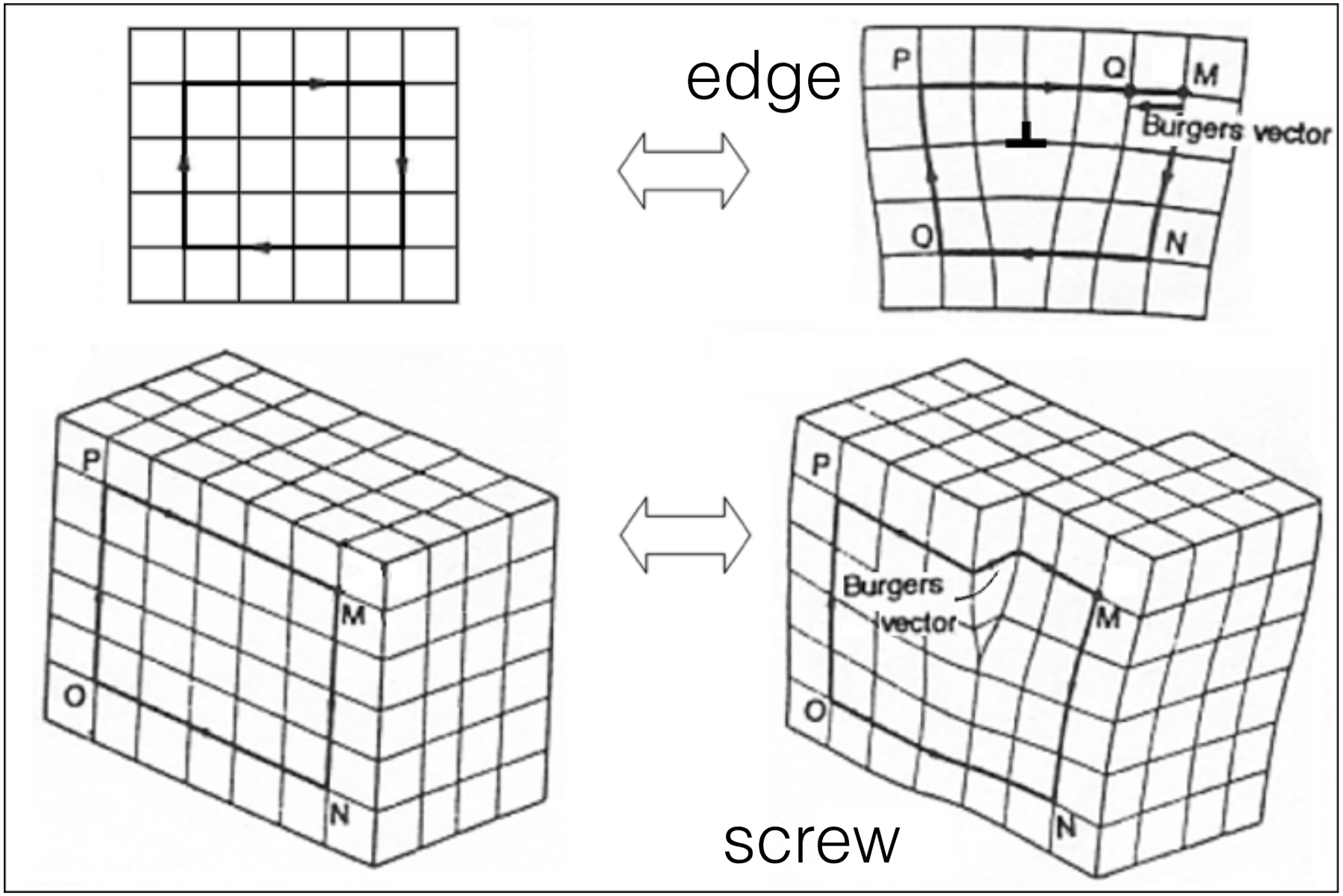
Work hardening

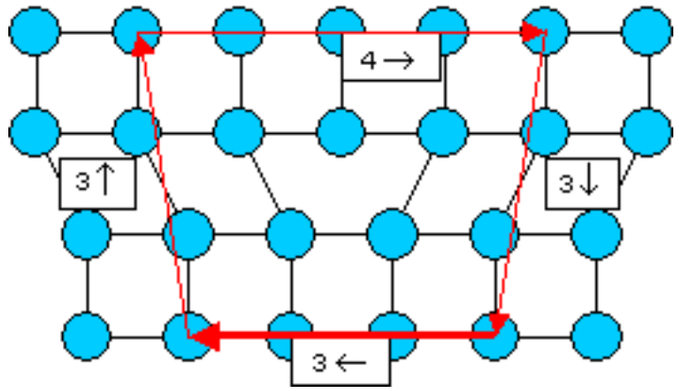


Disclinations



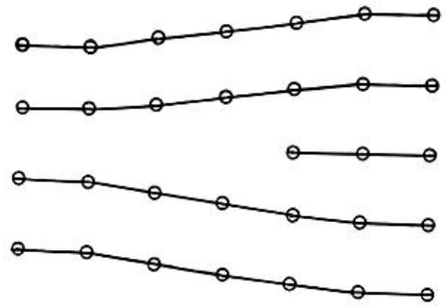
edge

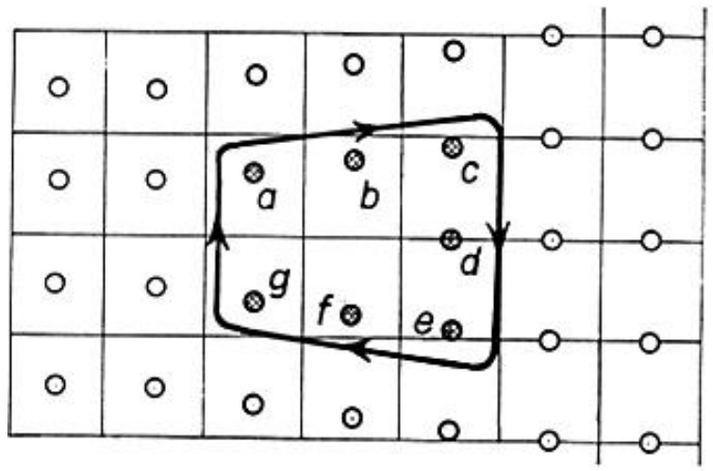


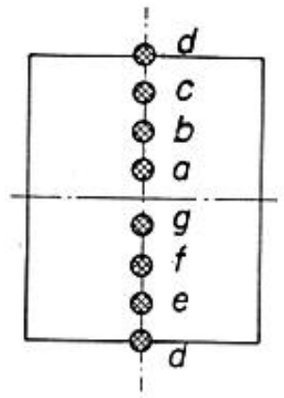


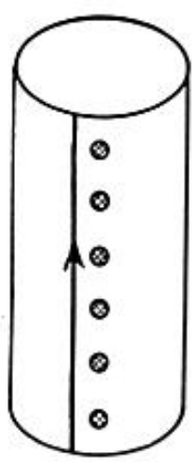
$$\|\mathbf{b}\| = (a/2)\sqrt{h^2 + k^2 + l^2}$$

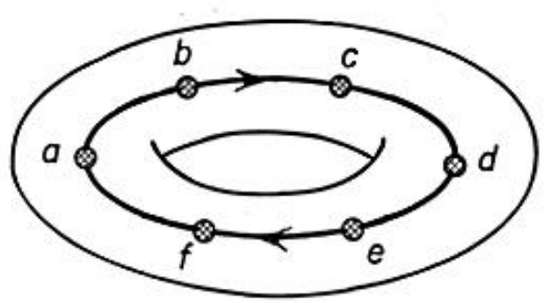
Disclineations





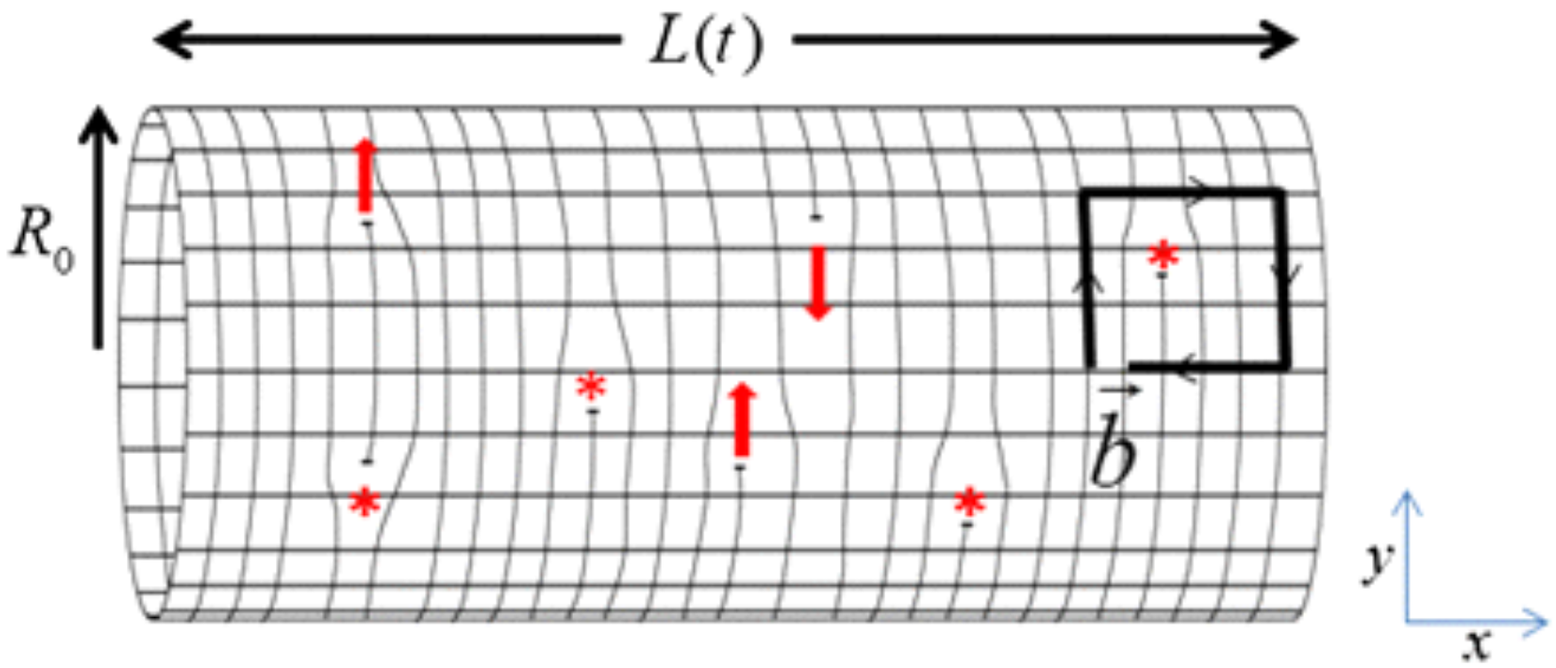






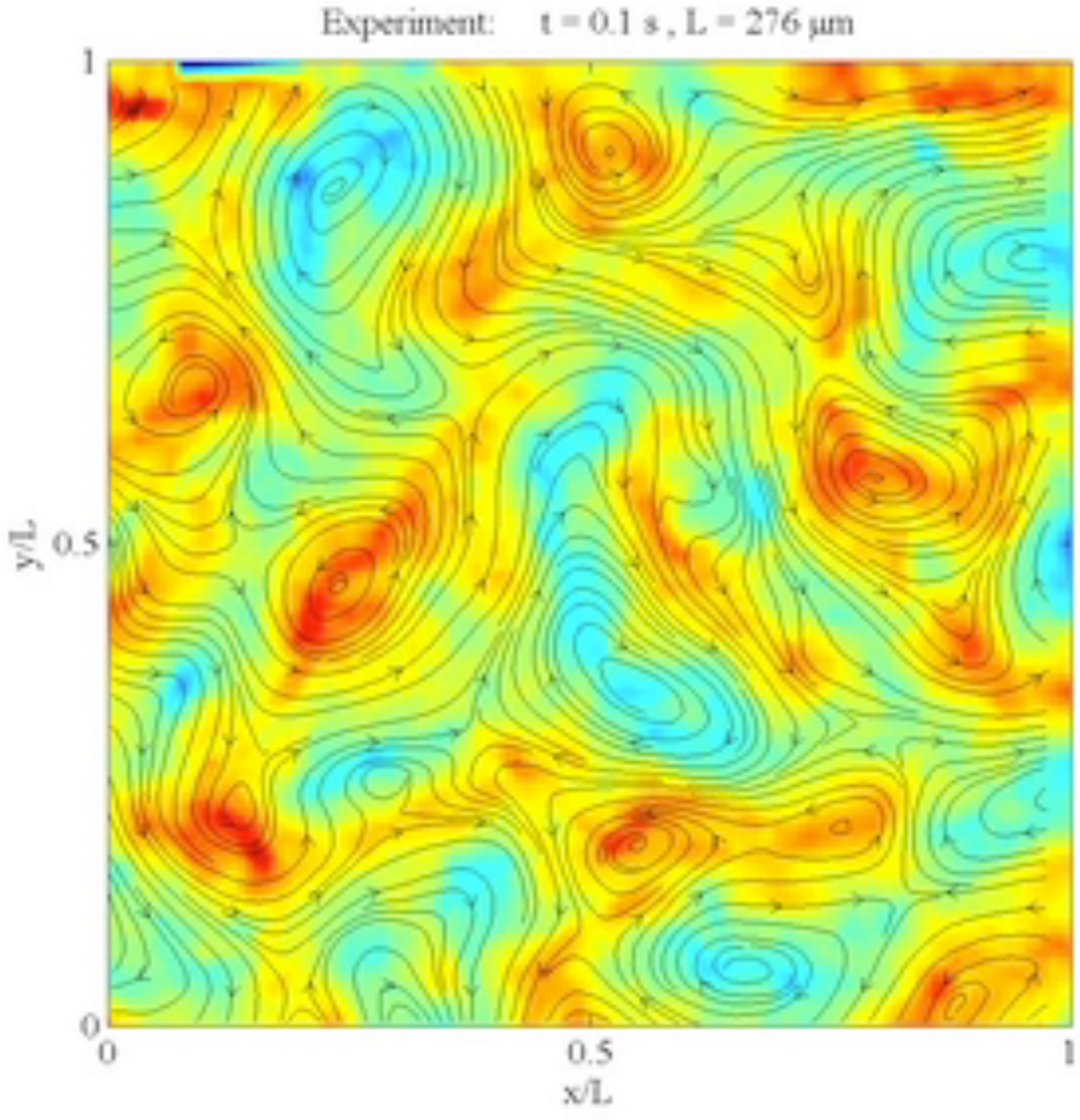
Dislocation-mediated growth of bacterial cell walls

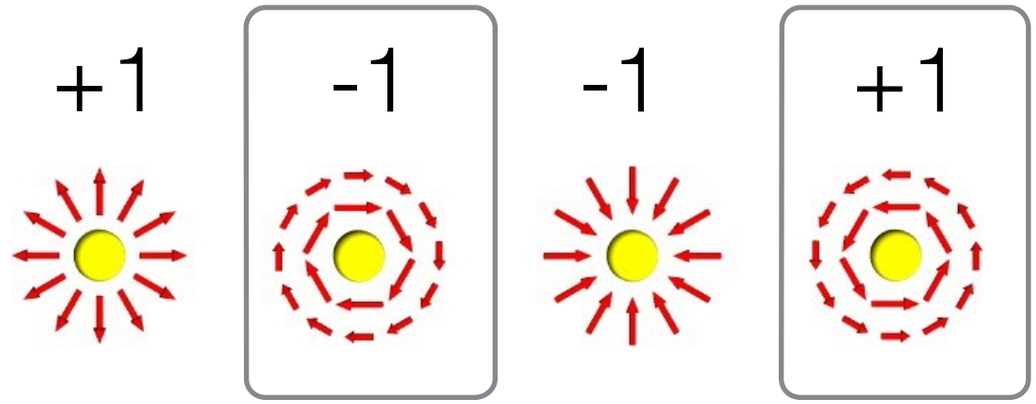
Ariel Amir and David R. Nelson[1]

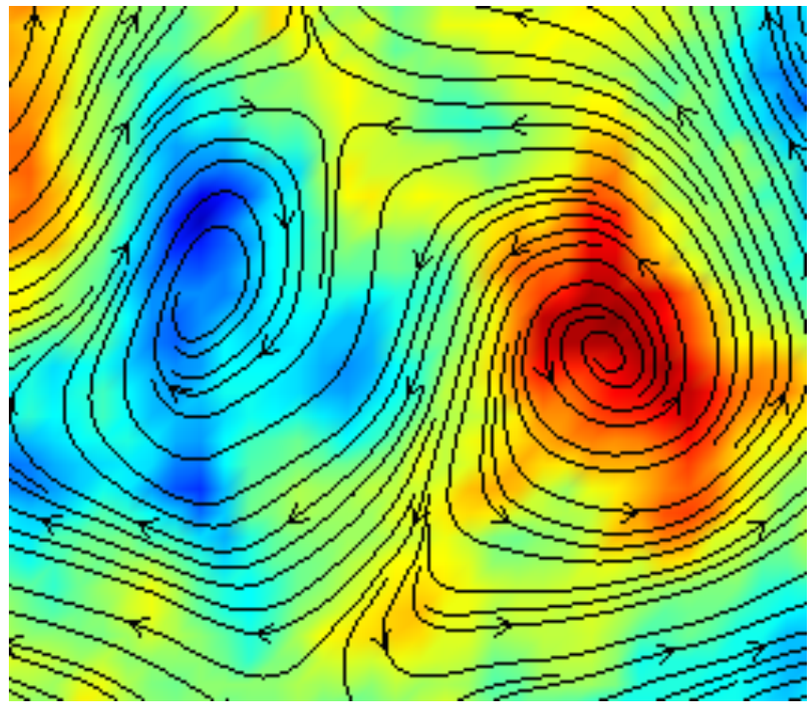


Bacterial vortices

PIV



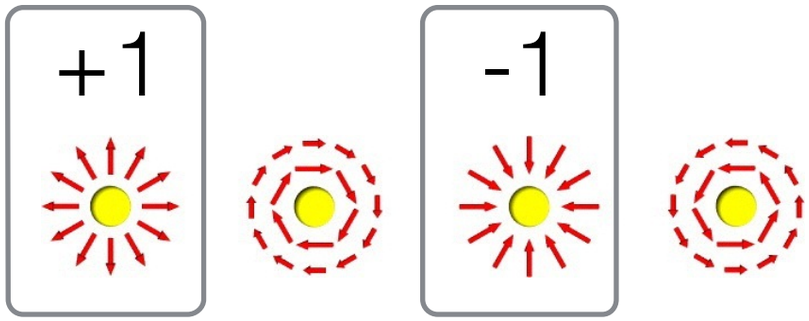




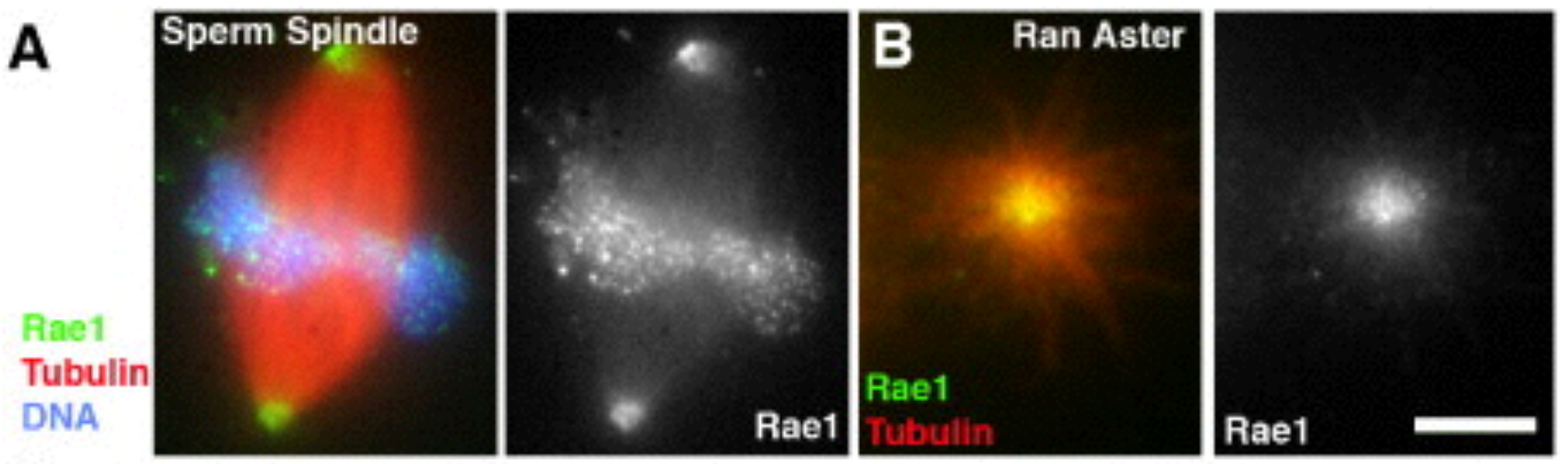
Dunkel et al PRL 2013

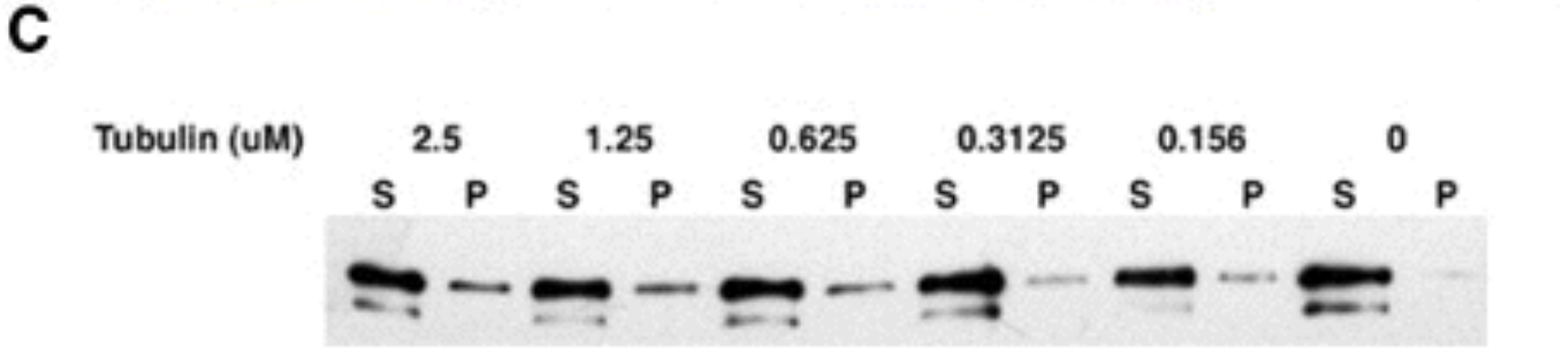
Microtubule asters

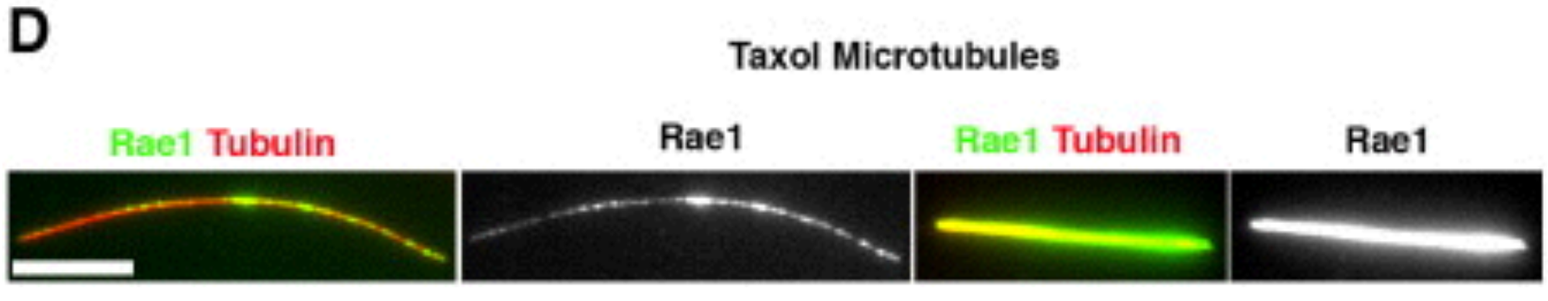
mitotic spindle organization



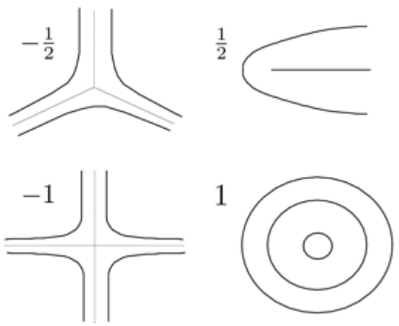
Blower et al (2005) Cell

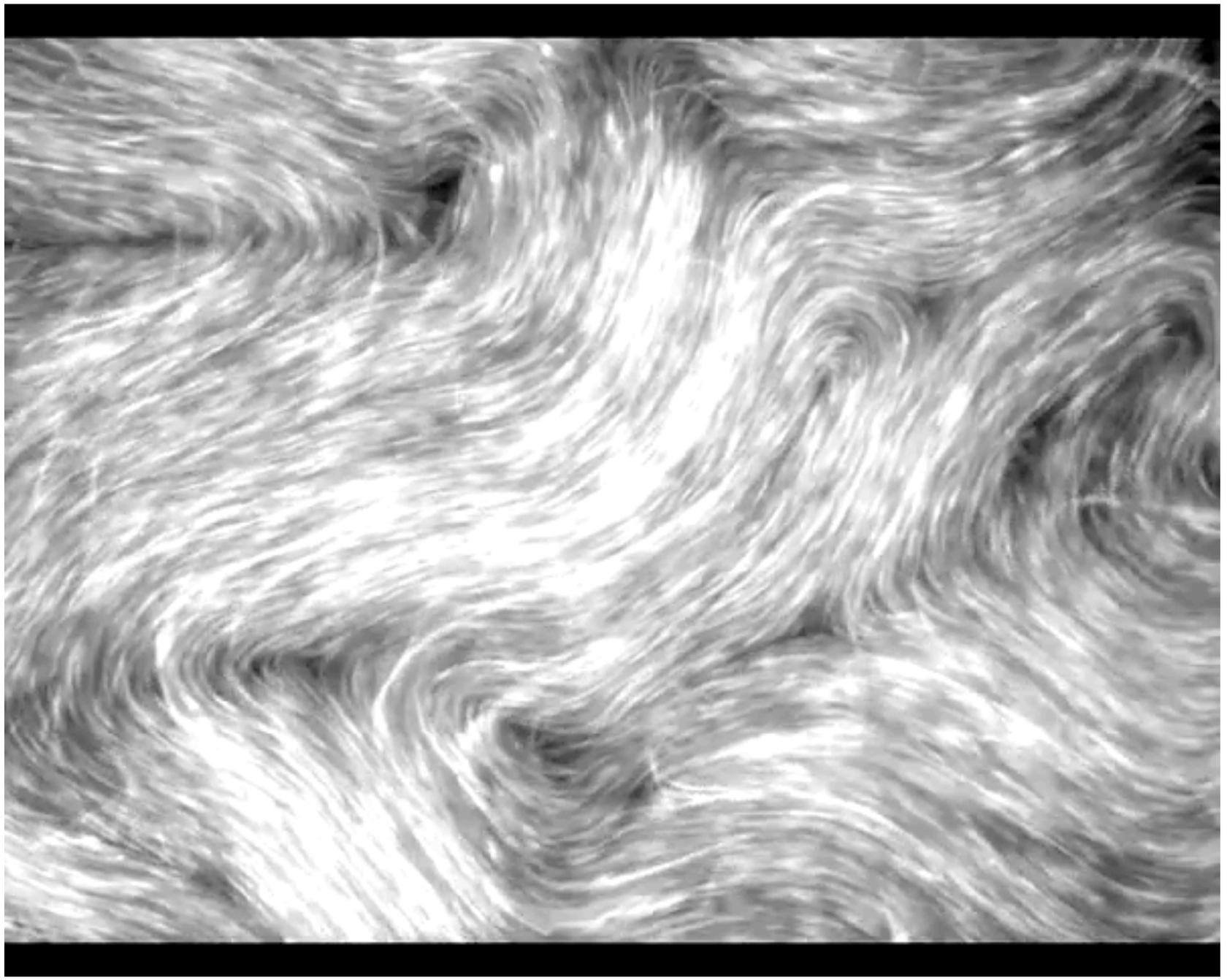






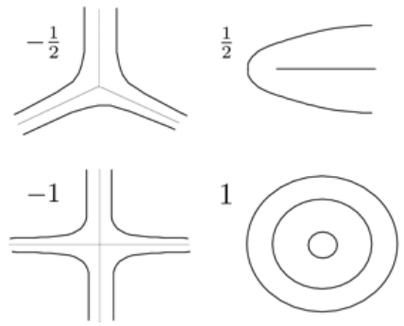
Active nematics

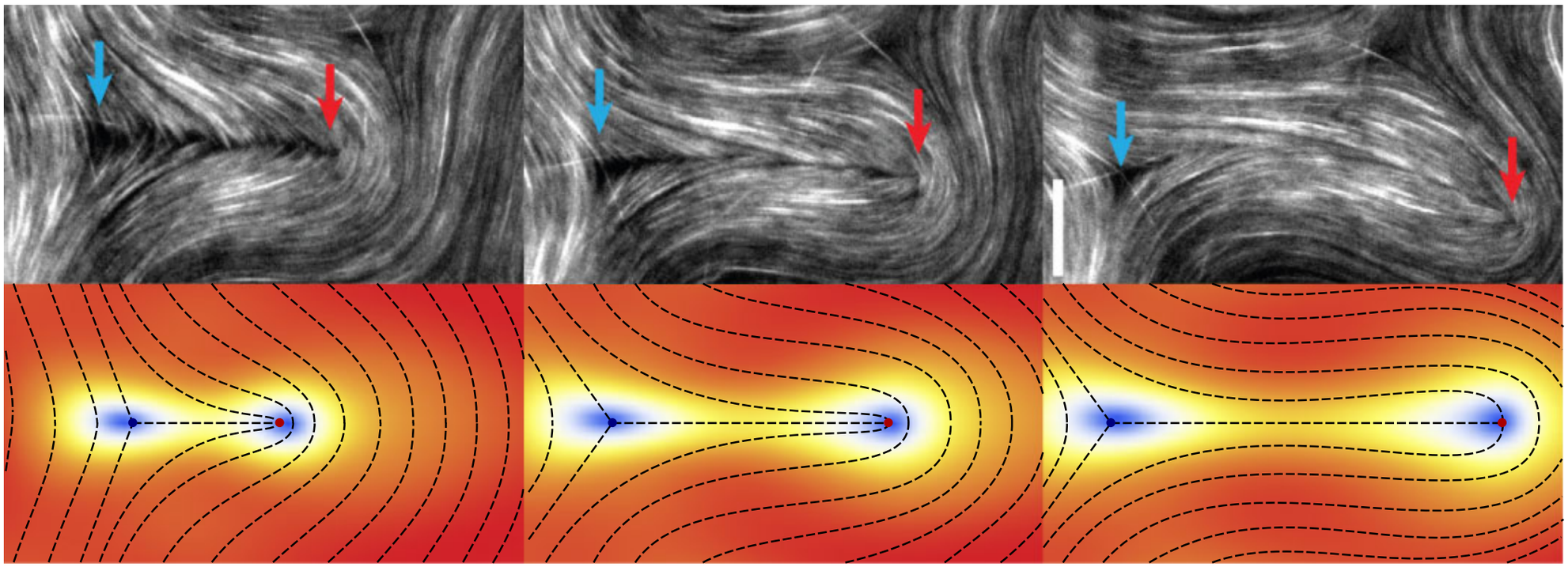




Dogic lab (Brandeis) Nature 2012

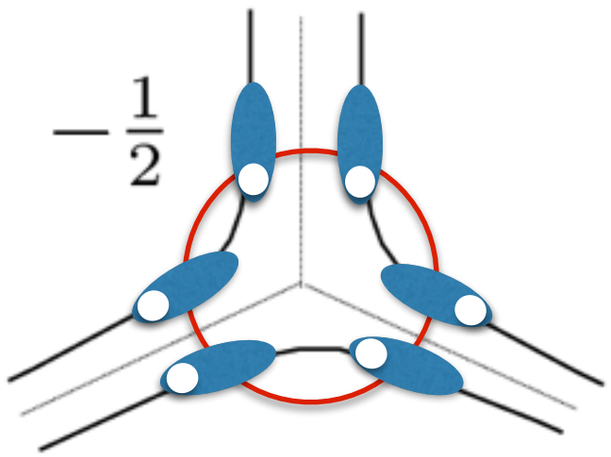
Active nematics

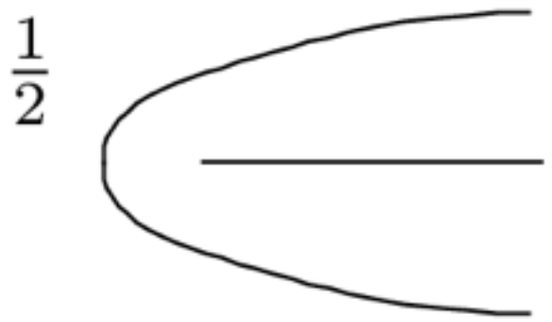




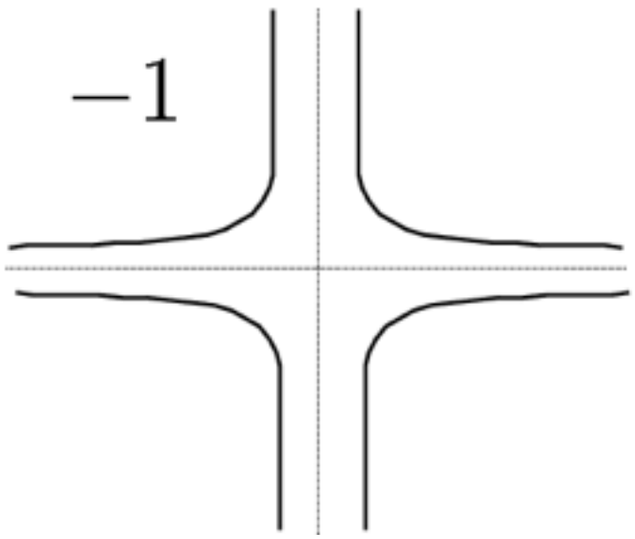
Giomi et al PRL 2012

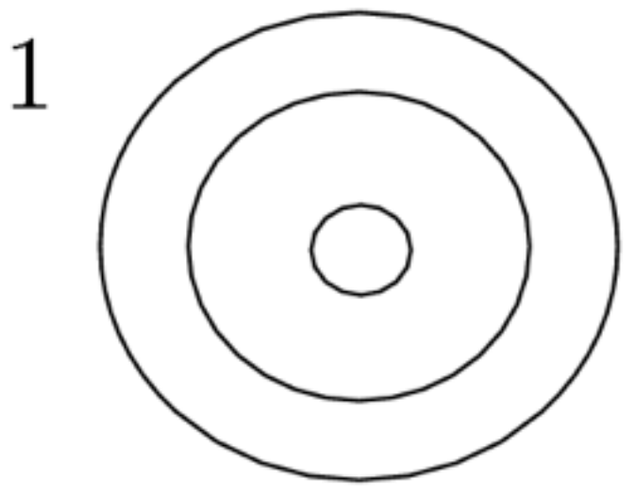
Defects in nematics



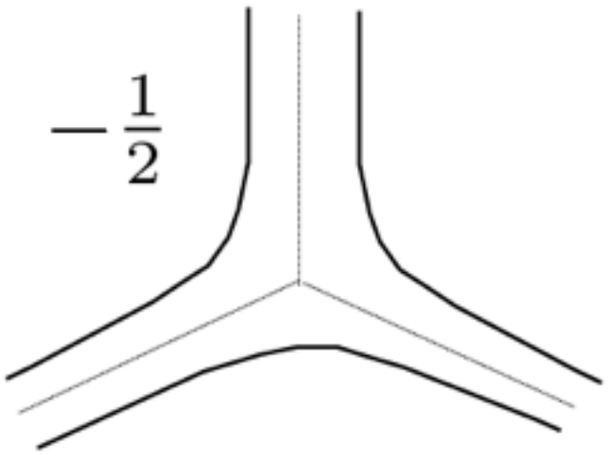


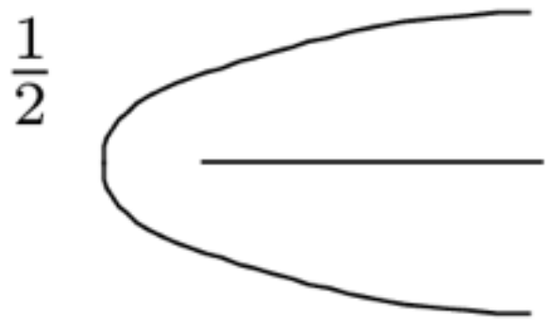
winding number



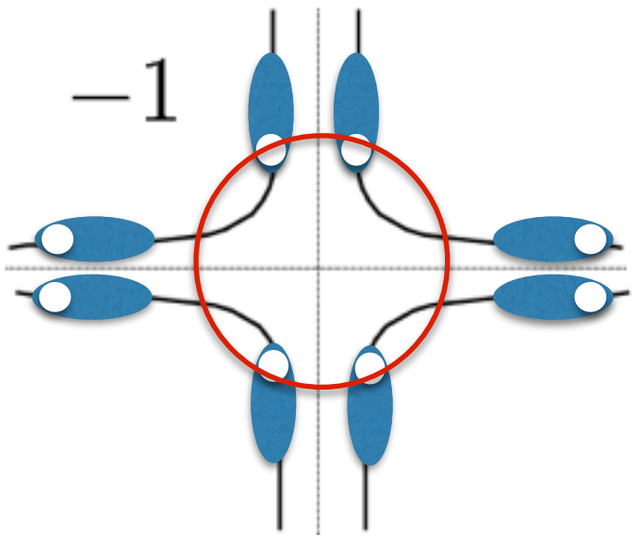


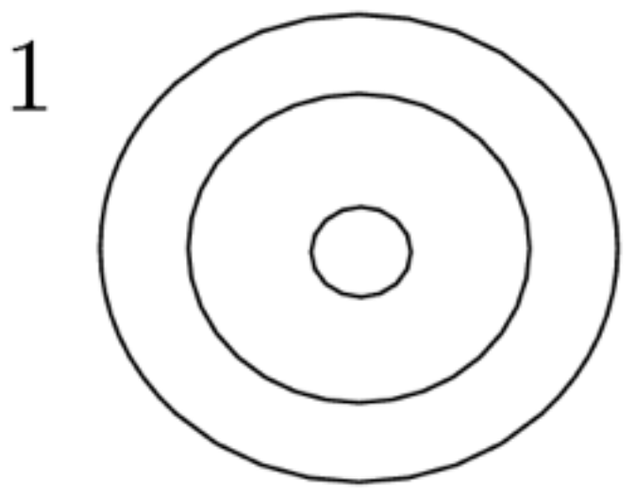
Defects in nematics





winding number

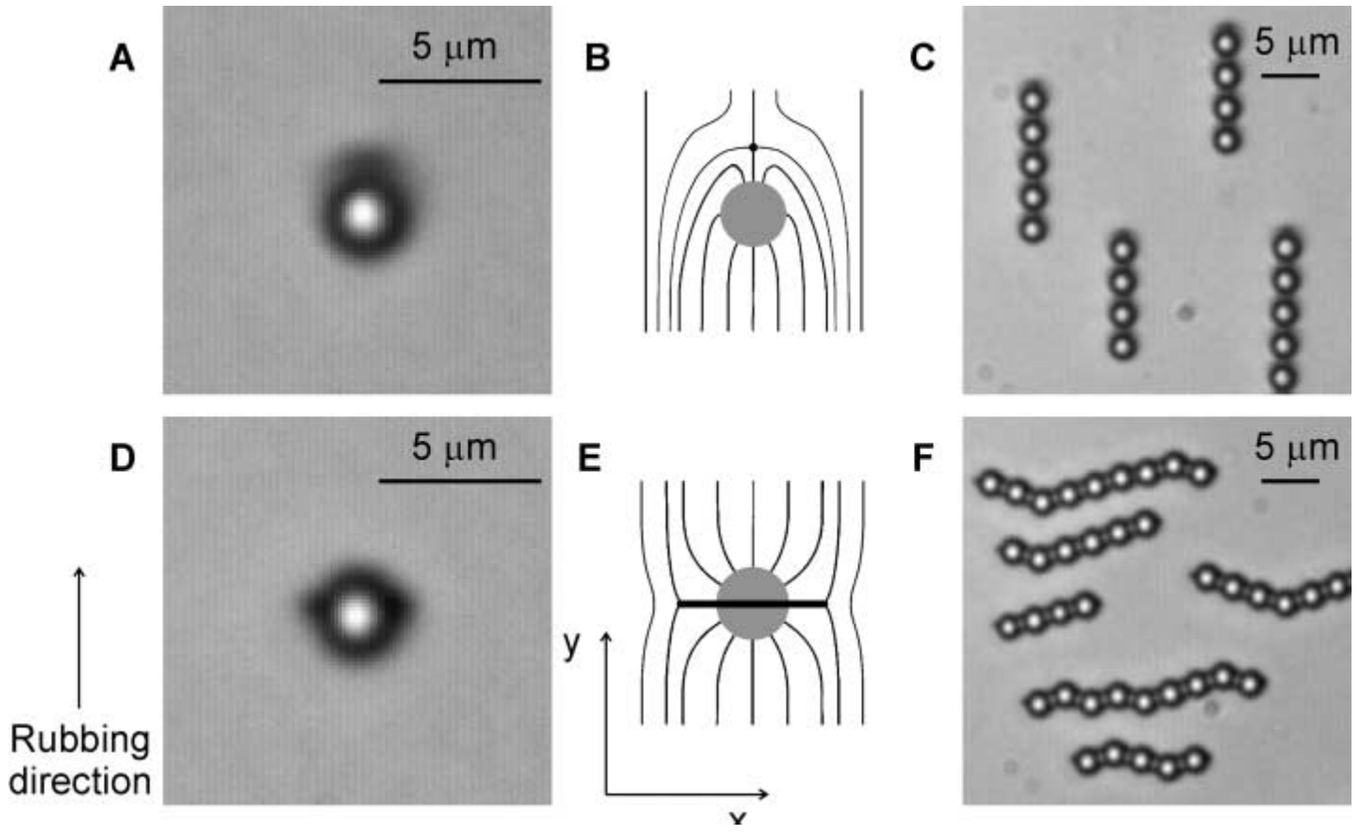




Two-Dimensional Nematic Colloidal Crystals Self-Assembled by Topological Defects

Igor Musevic *et al.*

Science **313**, 954 (2006);
DOI: 10.1126/science.1129660



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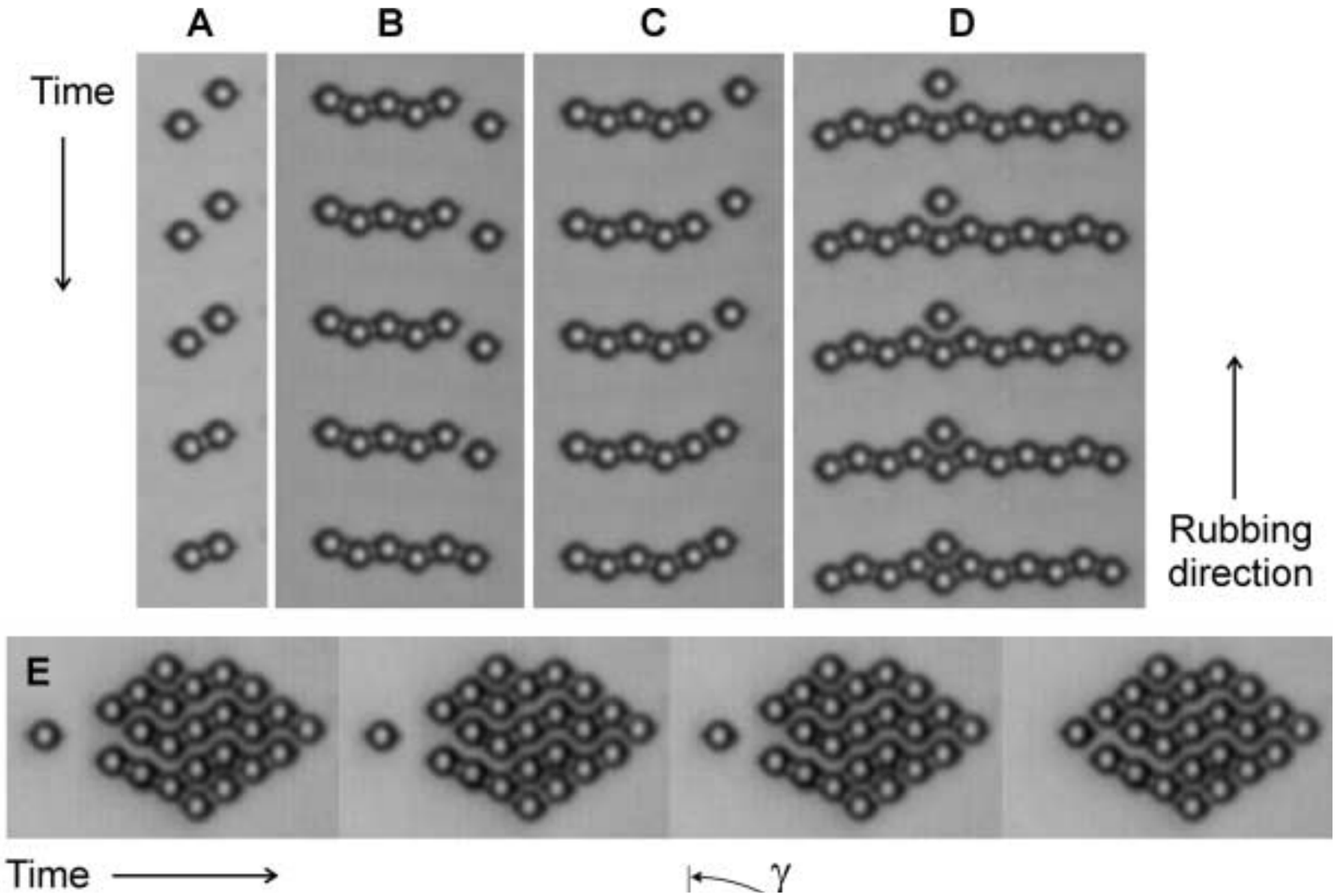

Reconfigurable Knots and Links in Chiral Nematic Colloids
Uros Tkalec *et al.*
Science **333**, 62 (2011);
DOI: 10.1126/science.1205705