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## 18.354 – Nonlinear Dynamics II: Continuum systems

### Problem Set 1: DIMENSIONAL ANALYSIS, HAMILTONIAN DYNAMICS AND RANDOM WALKS

Due: Thursday, Feb 26 (by 1pm in class)

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#### Problem 1: LEAPING ANIMALS

There is strong evidence (including the Olympic world records in power lifting) that the force which may be sustained or generated by an animal scales as its cross-sectional area. Use this fact to deduce the dependence on body size of the height which may be achieved by a leaping animal. (*Hint: assume that all dimensions are geometrically similar.*)

#### Problem 2: AN EXPERIMENT ON SURFACE TENSION

Get some paper clips of different sizes. Now, fill a glass with water and try placing the paper clip on the surface of the water. If you are very careful, you will find that some of the paper clips will sit on the surface. (check out the picture gallery in the "surface tension" of wikipedia, in case you lose faith). How is this possible? Use dimensional analysis to determine the size of the largest paper clip that can be placed on the surface of water? Following on from this, use a scaling argument to explain why it is that insects, and not elephants, reside on the surface of lakes. (*Hint: The surface tension at an air water interface is  $0.07\text{Nm}^{-1}$ , and you may assume that paper-clips, insects and elephants are isometric.*)

#### Problem 3: CONSERVATION LAWS

Consider a Hamiltonian system consisting of  $N$  particles (of same mass  $m$ ), with standard kinetic energy and identical pair interaction potentials that only depend on particle separation. Show that the total linear momentum and the total angular momentum is conserved.

#### Problem 4: BIASED RANDOM WALKS

The goal of this problem is to derive the continuum approximation for a particle undergoing a biased random walk. Consider a random walker on a one dimensional lattice with:

- (a) probability of  $1/3$  of staying still and probability of  $1/3$  of moving to the left or to the right.
- (b) probability of  $3/4$  of moving to the left and probability of  $1/4$  of moving to the right.

Following the procedure from class, write down the continuum equation for a cloud of random walkers obeying these rules. Use both particle fluxes and probabilities (see class notes) to derive the continuum equation, and check whether they agree. If not, why not? For each case, sketch what happens to an initial distribution of particles at the origin. What is the diffusion constant? Case (b) is an example of an advection-diffusion equation, in which you will find that the cloud of particles drifts to one side as well as diffusing. What is the drift velocity? How does the maximum density of the distribution decrease with time in each case?