

18.354J Nonlinear Dynamics II: Continuum Systems

Spring 2014 – Course Info

Lectures:	MW 3-4:30	in	E17-128
Instructor:	Jörn Dunkel		
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Office Hours:	M 4:30-5:30 (E17-412) & T 3-4 (E18-401r)		
Course website:	math.mit.edu/~dunkel/Teach/18.354/		
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This course introduces the basic ideas for understanding the dynamics of continuum systems, in the context of specific examples taken from a range of different fields. Our goal will be to explain the general principles, and also to illustrate them via important physical effects. A parallel goal of this course is to give you an introduction to mathematical modeling.

The first part of the course will study diffusion, to demonstrate how continuum descriptions arise from averaging microscopic degrees of freedom. The equations of motion that we derive for continuum systems are typically nonlinear partial differential equations, for which it is very difficult to obtain analytical solutions. We shall therefore briefly foray into dimensional analysis to see how it is possible to obtain qualitative information about a system without having to solve the full equations of motion.

We shall then study the ‘calculus of variations’, which is a minimization approach to finding solutions of continuous systems. We are familiar with these techniques for discrete systems and now adapt the ideas to help us with continuous systems. First we examine the classical brachistochrone problem posed by Bernoulli, and then consider an array of problems in many different physical systems (e.g. orientation of domains in a ferromagnet, shapes of soap films, bending of elastic beams, etc.)

In the second half of the course we will examine singular perturbations. We will see how Prandtl’s boundary layer resolved nineteenth century mysteries about dissipation in low viscosity fluids, and led to an understanding of airplane flight. We will also study the Ekman layer and its role in controlling rotating flows, leading to an understanding of the dynamics of the atmosphere.

Having appreciated the subtleties of constructing continuum descriptions of systems with many degrees of freedom, we will proceed to study the stability of solutions. Time permitting, we shall first study neutral stability to develop an understanding of wave motion (water waves, sound waves, gravity waves) before proceeding to study a range of instabilities (Rayleigh-Taylor, Turing, Taylor-Couette etc.).

GRADING:

- 35%: Problem sets
- 30%: Mid-term exam
- 5%: Project proposal (due March 31)
- 30%: Final project presentation (May, Mon 12 and Wed 14) + report (due May 14)

TEXTBOOKS:

Although there are no textbooks which follow the precise spirit of this course, there are literally hundreds of textbooks where the topics we will cover are discussed. For most lectures, typed notes can be downloaded from the course webpage. Additional material will be handed out in class. One book that will be useful frequently is: D. J. Acheson, *Elementary Fluid Dynamics*, Oxford University Press (1990).

HOMEWORK - PROBLEM SETS:

Homework will be assigned roughly every two-three weeks. Each homework set will contain analytical and computational problems, and even the odd experiment. Assignments must be handed in by 3pm (start of class) on the due date. Either hand them in at the end of class, or in an envelope under my office door. *First unexcused late homework score will be multiplied by 3/4. No subsequent unexcused late homework is accepted.* You are welcome to discuss solution strategies and even solutions, but please write up the solution on your own. Be sure to support your answer by listing any relevant Theorems or by explaining important steps. Be as clear and concise as possible. I strongly encourage the computational problems to be written in MATLAB.

MID-TERM EXAM:

There will be a mid-term exam, which will take place about 3/4's of the way through the course. The exam will be a take home, and will be in place of a homework set. There will be no final exam.

FINAL PROJECT:

The ideas we will be discussing have applications to many fields, many of which we will not cover. To give you a chance to explore an area of interest to you, the course will require a final project, in which you explore in depth something of interest to you and within the course's scope. Final projects will be presented in class during the final two classes.

IMPORTANT DATES:

- Mon Feb 24 - Problem Set 1: Random walks. Diffusion. DUE
- Mon Mar 10 - Problem Set 2: Continuum equations. DUE
- Mon Mar 31 - Problem Set 3: Dimensional analysis. Fermat's principle. DUE
- Mon Mar 31 - Proposal (1 page) for final project. DUE
- Mon Apr 7 - Problem Set 4: Surface tension. Singular perturbations. DUE
- Wed Apr 16 - Take-home Midterm exam. POSTED
- Wed Apr 23 - Take-home Midterm exam. DUE
- Mon/Wed May 12/14 - Final project presentations.
- Wed May 14 - Final project report. DUE

Note: The exact due dates for the P-sets may be subject to change