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Accretion of Helium and Metal-Rich Gas onto Neutron Stars and Black Holes at High Luminosities*

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Abstract—Ultraluminous X-ray sources fed by Wolf–Rayet star winds and X-ray bursters in ultracompact binaries with He or C white dwarfs have accretion disks whose properties may differ significantly from those of pure Hα-accretion disks. Therefore, we have included the dependence on charge number Z and mean molecular weights $\mu_{e/I}$ in the Shakura and Sunyaev (1973) scaling relations for the key parameters of the disk. Furthermore, we also consider the pseudo-Newtonian potential of Paczyński and Wiita (1980). These scaling relations might become useful, e.g., when estimating the illumination efficiency of the outer parts of the disk. We also address the changes in the structure of the boundary (spreading) layer on the surfaces of neutron stars that occurs in the case of H-depleted accretion disks.

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1. INTRODUCTION

Recent observations of galaxies by the CHANDRA X-ray observatory confirm the existence of compact, ultraluminous X-ray sources (ULXs) emitting at luminosities considerably higher than the Eddington luminosities of neutrons stars or stellar-mass black holes with 1–15 $M_\odot$ (Liu and Mirabel 2005; Roberts et al. 2004a, 2004b). The most dramatic results were obtained by CHANDRA observations of star-forming galaxies and regions (Grimm et al. 2003), where over the wide luminosity range $L = 10^{36}–10^{40}$ erg s^{-1}, the corresponding high-mass X-ray binary (HMXB) luminosity function is well fitted by a single power law with a strong cut-off at $L \sim 10^{40.5}$ erg s^{-1}, which is more than two orders of magnitude higher than the Eddington luminosity of a solar-mass neutron star. Since the HMXB luminosity function does not exhibit a change in slope or other peculiarities in the region just above the Eddington limit of stellar-mass black holes, it is plausible to assume that ULXs represent the high-mass/high-accretion-rate tail of the "ordinary" HMXB population with masses in the range 10–15 $M_\odot$ (King 2002; Grimm et al. 2002; Gilfanov 2004).

These new observational results increased strongly the theoretical interest in the question whether an accreting object can produce super-Eddington luminosities or whether the Eddington luminosity provides a real upper limit for the luminosity of accreting objects. In principle, one can think of several different mechanisms leading to super-Eddington luminosities and, at present, there is no commonly accepted model for the huge energy output of ULXs as yet. For example, young rotation-powered pulsars or jet sources directed toward us could substantially increase the observed X-ray flux. Other possibilities might be slim accretion disks (Abramowicz et al. 1988) or an accretion disk that experiences radiation-driven inhomogeneities (Begelmann 2002). Alternatively, many observers and theorists believe that the high luminosity of ULXs is due to accretion onto an intermediate-mass black hole with $10^2–10^4$ $M_\odot$ (Kubota et al. 2002; Miller and Colbert 2004).

On the other hand, there are also several minor effects (e.g., related to disk inclination and chemical composition), each being, in principle, able to increase the observable luminosity of an accreting object by a factor of 2 or 3 (Grimm et al. 2002). Due to the aforementioned homogeneity of the HMXB luminosity function, it is worthwhile to carefully reconsider the contributions of these minor effects, even though most of them have been known to theorists for more than 30 years. In this paper, we will focus

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on the case where the chemical composition of the accreting matter deviates greatly from the standard cosmic abundances. Obviously, in this case, the mass per electron in the accreting gas can be higher than that for a pure H plasma or standard cosmic abundances. It is well known that for completely ionized He, C, O, N, or Mg plasmas, having a factor of 2 fewer electrons per baryon than H, the Eddington luminosity is a factor of 2 higher than that for a pure H plasma.

From an observational point of view, there is evidence that, under certain conditions, the accreting gas in binary systems can consist only of elements heavier than H (Hammer et al. 2005). For example, several observations of *X-ray sources in extremely dense binary systems* revealed a neutron star accreting matter supplied by a He, C, O, Ne, or even Mg white dwarf (Juett and Chakrabarty 2003; Nelemans et al. 2004; Schulz et al. 2001). The unusual chemical abundance affects the power release due to nuclear explosions on the surface of the neutron star, which are observable as X-ray bursts. The duration of bursts and their recurrence rate are also affected strongly.

The problem of nonstandard abundances might also be of relevance if the accretion onto the HMXB black hole takes place in a *star-forming region or galaxy*. Then, many of the HMXBs are fed by stellar-wind accretion. A particularly interesting scenario in this regard corresponds to the case where the donor star is of the Wolf-Rayet type. During their evolution, these stars may have lost a significant amount of their H-rich envelope, leading to stellar winds dominated by He or, in some cases, even by C and N (Abbott et al. 2004; Crowther et al. 2002; Hamann and Koesterke 2000; van der Hucht 2001). For instance, one of the brightest, most luminous X-ray sources in the Milky Way, Cygnus X-3, is fed by the dense, high-velocity wind from its He Wolf-Rayet companion (van Kerkwijk et al. 1992; Lommen et al. 2005).

In the case of accretion onto a black hole, the plasma density in the region of the main energy release of a standard H-dominated accretion disk is not high enough to produce the photons necessary for creating a blackbody radiation spectrum inside the disk (Shakura and Sunyaev 1973). Moreover, in the limit $\dot{M}/\dot{M}_{\rm E} \to 1$ and $\alpha \to 1$, the optical depth becomes rather small (here, as usual, $\dot{M}_{\rm E}$ denotes the accretion rate corresponding to the Eddington luminosity and α is the viscosity parameter). Therefore, the purpose of this paper is to study how the disk density, temperature, and optical depth depend on the chemical abundance in the extreme case of H-poor accretion. By simply repeating the approach of Shakura and Sunyaev (1973), we will show that under such peculiar conditions, the density of electrons and nuclei increases, thereby also increasing the effective optical depth. This might stabilize the disk against the transition to the two-temperature hot flow regime (Shapiro et al. 1976).

Recently, Hammer et al. (2005) have performed numerical simulations to determine the structure in the outer regions of C/O/Ne-dominated accretion disks in ultracompact binary systems (for details of their NLTE-model, see Nagel et al. 2004). Their primary objective was to explain the observed peculiarities in the optical spectra of these systems by taking into account the metal-line blanketting and irradiation by the central object. However, despite the considerable theoretical interest in the properties of H-poor accretion disks (Menou et al. 2002), we failed to find in the literature simple analytical results describing the power-law scaling relations for the key parameters of the standard accretion disk with respect to its chemical composition in the extreme case of H- or even He-depleted accretion flows. Therefore, the formulas given below, which were originally derived as part of a more complex study, will possibly be useful when interpreting observational data and obtaining simple analytical estimates. Finally, it is also worthwhile to mention that a high abundance of He or heavier elements not only decreases the disk (scale) height by a factor of ~ 2, but also changes the width and surface brightness of the spreading layer on the surfaces of neutron stars. Therefore, in the periods between X-ray bursts, the accretion of heavier elements also affects the heating of the outer parts of the disk by X-ray radiation emitted from the central disk regions (this also applies to the case of accretion onto black holes) and from the neutron-star surface (Lyutyi and Sunyaev 1976).

2. THE STRUCTURE OF HELIUM- OR METAL-DOMINATED ACCRETION DISKS

The hydrodynamic equations considered by Shakura and Sunyaev (1973) do not explicitly depend on the chemical composition of the disk. More precisely, in their model, the chemical composition enters only via the thermodynamic equations for the pressure and the energy density of the gas:

$$P_{\rm g} = \frac{\rho k T}{\mu m_p}, \qquad \epsilon_{\rm g} = \frac{3}{2}\frac{\rho k T}{\mu m_p}, \qquad (1a)$$

and via the Rosseland mean opacities for Thomson and free–free processes:

$$\kappa_{\rm T} = \frac{\sigma_{\rm T}}{\mu_e m_p} = \frac{0.398}{\mu_e}\ {\rm cm^2\ g^{-1}}, \qquad (1b)$$

$$\kappa_{\rm ff} = 0.12\frac{Z^2}{\mu_e \mu_{\rm I}}\left(\frac{\rho\,{\rm cm^3}}{m_p}\right)\left(\frac{T}{\rm K}\right)^{-7/2}\ {\rm cm^2\ g^{-1}}. \quad (1c)$$

Here, Z is the ion charge number, ρ is the mass density of the matter, k is the Boltzmann constant, $\sigma_{\rm T}$ is the Thomson scattering cross section, T is the temperature, m_p is the proton mass, $\mu_{e,I}$ is the mean molecular weight of the electrons and ions, respectively, and $\mu^{-1} = \mu_e^{-1} + \mu_I^{-1}$. The results for the disk scale height, surface density, etc. given below in Eqs. (4)–(6) were obtained by using Eqs. (1) and following the approach of Shakura and Sunyaev (1973). In particular, Eqs. (4)–(6) refer to a Newtonian potential $\Phi = -GM/(R^2+z^2)^{1/2}$, where R is the cylindrical radius. It is however easy to see that the scaling relations with respect to the chemical parameters (see also Table 1) remain also valid, when including relativistic corrections in the form of the pseudo-Newtonian potential of Paczyńsky and Witta (1980), as is evident from the results in Appendix A. It is convenient to express the results in terms of nondimensional quantities $m = M/M_\odot$, $\dot{m} = \dot{M}/\dot{M}_{\rm E}$, $r = R/r_0$, and $s = 1 - r^{-1/2}$, where $r_0 = 3r_G = 6\,GM/c^2 = m8.86$ km. The total luminosity of an infinite α-disk ranging from r_0 to ∞ reads $L_\infty = \zeta \dot{M} c^2$ with an efficiency $\zeta = 1/12$ in the case of a Newtonian potential and $\zeta \approx 0.06$ for a Schwarzschild black hole. The critical Eddington luminosity that provides an upper limit for spherically symmetric accretion can then be written as

$$L_{\rm E} = \mu_e L_{\rm E,H} = \mu_e \frac{2\pi r_{\rm G} m_p c^3}{\sigma_{\rm T}} = \mu_e m (1.26 \times 10^{38}\ {\rm erg\ s^{-1}}), \quad (2)$$

where, in what follows, the quantities labeled by H refer to the case of a fully ionized, pure H plasma. The corresponding critical accretion rate obtained from the condition $L_\infty = L_{\rm E}$ reads

$$\dot{M}_{\rm E} = \mu_e \dot{M}_{\rm E,H} = \mu_e \zeta^{-1} m (2.22 \times 10^{-9} M_\odot\ {\rm yr}^{-1}). \quad (3)$$

For the inner disk zone (A) characterized by dominating radiation pressure $P_g \ll P_\gamma$ and $\kappa_{\rm T} \gg \kappa_{\rm ff}$, we find

$$H = \zeta^{-1} m \dot{m} s (2.2\ {\rm km}), \quad (4)$$

$$\Sigma = \zeta \mu_e \alpha^{-1} \dot{m}^{-1} r^{3/2} s^{-1} (9.9 \times 10^2\ {\rm g\ cm^{-2}}),$$

$$\rho_c = \zeta^2 \mu_e \alpha^{-1} m^{-1} \dot{m}^{-2} r^{3/2} s^{-2} (2.2 \times 10^{-4}\ {\rm g\ cm^{-3}}),$$

$$n_{ec} = \zeta^2 \alpha^{-1} m^{-1} \dot{m}^{-2} r^{3/2} s^{-2} (1.3 \times 10^{20}\ {\rm cm^{-3}}),$$

$$\bar{v}_R = \zeta^{-2} \alpha \dot{m}^2 r^{-5/2} s (-2.5 \times 10^3\ {\rm km\ s^{-1}}),$$

$$\epsilon_{\gamma c} = \mu_e \alpha^{-1} m^{-1} r^{-3/2} (3.1 \times 10^{15}\ {\rm erg\ cm^{-3}}),$$

$$P_c = \mu_e \alpha^{-1} m^{-1} r^{-3/2} (1.0 \times 10^{15}\ {\rm erg\ cm^{-3}}),$$

$$T_c = \mu_e^{1/4} \alpha^{-1/4} m^{-1/4} r^{-3/8} (2.5 \times 10^7\ {\rm K}),$$

Relative quantitative changes compared to a pure, fully ionized H accretion disk with $\mu_I = \mu_e = Z = 1$ and the same parameters $(\alpha, M, \dot{M})$.

I^{Z+}	He^{2+}	C^{6+}	N^{7+}	O^{8+}	Mg^{12+}
Z	2	6	7	8	12
μ	4/3	12/7	14/8	16/9	24/13
μ_e	2	2	2	2	2
μ_I	4	12	14	16	24
$\dot{M}_{\rm E}/\dot{M}_{\rm E,H}$	2	2	2	2	2
Zone A					
$H/H_{\rm H}$	0.5	0.5	0.5	0.5	0.5
$\Sigma/\Sigma_{\rm H}$	4	4	4	4	4
$\rho_c/\rho_{c,\rm H}$	8	8	8	8	8
$n_{ec}/n_{ec,\rm H}$	4	4	4	4	4
$\bar{v}_R/\bar{v}_{R,\rm H}$	0.25	0.25	0.25	0.25	0.25
$\epsilon_{\gamma c}/\epsilon_{\gamma c,\rm H}$	2	2	2	2	2
$P_c/P_{c,\rm H}$	2	2	2	2	2
$T_c/T_{c,\rm H}$	1.19	1.19	1.19	1.19	1.19
$\tau_c/\tau_{c,\rm H}$	4.18	7.23	7.81	8.35	10.23
$t_{\rm co}/t_{\rm co,H}$	0.5	0.5	0.5	0.5	0.5
Zone B					
$H/H_{\rm H}$	0.63	0.57	0.57	0.56	0.55
$\Sigma/\Sigma_{\rm H}$	2.51	3.08	3.13	3.17	3.27
$\rho_c/\rho_{c,\rm H}$	3.99	5.40	5.54	5.64	5.90
$n_{ec}/n_{ec,\rm H}$	2.00	2.70	2.77	2.82	2.95
$\bar{v}_R/\bar{v}_{R,\rm H}$	0.40	0.32	0.32	0.32	0.31
$\epsilon_{\gamma c}/\epsilon_{\gamma c,\rm H}$	1.26	1.54	1.56	1.58	1.63
$P_c/P_{c,\rm H}$	1.59	1.75	1.77	1.78	1.81
$T_c/T_{c,\rm H}$	1.06	1.11	1.12	1.12	1.13
$\tau_c/\tau_{c,\rm H}$	2.27	5.13	5.66	6.15	7.84
$t_{\rm co}/t_{\rm co,H}$	0.79	0.65	0.64	0.63	0.61
Zone C					
$H/H_{\rm H}$	0.64	0.64	0.64	0.64	0.65
$\Sigma/\Sigma_{\rm H}$	2.24	2.42	2.42	2.42	2.39
$\rho_c/\rho_{c,\rm H}$	3.34	3.76	3.76	3.76	3.69
$n_{ec}/n_{ec,\rm H}$	1.67	1.88	1.88	1.88	1.84
$\bar{v}_R/\bar{v}_{R,\rm H}$	0.45	0.41	0.41	0.41	0.42
$\epsilon_{\gamma c}/\epsilon_{\gamma c,\rm H}$	2.02	4.03	4.38	4.69	5.73
$P_c/P_{c,\rm H}$	1.50	1.56	1.56	1.55	1.54
$T_c/T_{c,\rm H}$	1.19	1.41	1.45	1.47	1.55
$\tau_c/\tau_{c,\rm H}$	1.50	2.21	2.30	2.38	2.61
$t_{\rm co}/t_{\rm co,H}$	0.50	0.25	0.23	0.21	0.17

Note. Inner zone A: $P_g \ll P_\gamma$ and $\kappa_{\rm T} \gg \kappa_{\rm ff}$. Intermediate zone B: $P_g \gg P_\gamma$ and $\kappa_{\rm T} \gg \kappa_{\rm ff}$. Outer zone C: $P_g \gg P_\gamma$ and $\kappa_{\rm T} \ll \kappa_{\rm ff}$. All values refer to pure disk plasmas containing only a single ion species. The same results are obtained for the pseudo-Newtonian potential of Paczyńsky and Witta (1980), see Appendix A.

$$\tau_c = Z\zeta^2\mu_e^{1/16}\mu_{\rm I}^{-1/2}\alpha^{-17/16} \times m^{-1/16}\dot{m}^{-2}r^{93/32}s^{-2}(1.4\times10^{-2}),$$

where we note that $\dot{m} = \dot{m}_{\rm H}/\mu_e$. Here, the index c is used to label the midplane values, H is the disk scale height, Σ is the surface mass density, $\bar{v}_R$ is the (vertically averaged) radial velocity, $\epsilon_{\gamma c}$ is the radiation energy density, $n_{ec} = \rho_c/(\mu_e m_p)$ is the electron number density, and $\tau = (\kappa_{\rm T}\kappa_{\rm ff})^{1/2}\,\Sigma/2$ is the effective optical depth (Shakura and Sunyaev 1973).

Similarly, for the intermediate zone (B) with $P_g \gg P_\gamma$ and $\kappa_{\rm T} \gg \kappa_{\rm ff}$,

$$H = \zeta^{-1/5}\mu^{-2/5}\mu_e^{1/10}\alpha^{-1/10} \times m^{9/10}\dot{m}^{1/5}r^{21/20}s^{1/5}(1.0\times10^{-1}\ {\rm km}), \quad (5)$$

$$\Sigma = \zeta^{-3/5}\mu^{4/5}\mu_e^{4/5}\alpha^{-4/5}m^{1/5} \times \dot{m}^{3/5}r^{-3/5}s^{3/5}(4.7\times10^4\ {\rm g\ cm^{-2}}),$$

$$\rho_c = \zeta^{-2/5}\mu^{6/5}\mu_e^{7/10}\alpha^{-7/10} \times m^{-7/10}\dot{m}^{2/5}r^{-33/20}s^{2/5}(2.3\ {\rm g\ cm^{-3}}),$$

$$n_{ec} = \zeta^{-2/5}\mu^{6/5}\mu_e^{-3/10}\alpha^{-7/10} \times m^{-7/10}\dot{m}^{2/5}r^{-33/20}s^{2/5}(1.4\times10^{24}\ {\rm cm^{-3}}),$$

$$\bar{v}_R = \zeta^{-2/5}\mu^{-4/5}\mu_e^{1/5}\alpha^{4/5} \times m^{-1/5}\dot{m}^{2/5}r^{-2/5}s^{-3/5}(-5.3\ {\rm km\ s^{-1}}),$$

$$\epsilon_{\gamma c} = \zeta^{-8/5}\mu^{4/5}\mu_e^{4/5}\alpha^{-4/5} \times m^{-4/5}\dot{m}^{8/5}r^{-18/5}s^{8/5}(1.5\times10^{18}\ {\rm erg\ cm^{-3}}),$$

$$P_c = \zeta^{-4/5}\mu^{2/5}\mu_e^{9/10}\alpha^{-9/10} \times m^{-9/10}\dot{m}^{4/5}r^{-51/20}s^{4/5}(2.3\times10^{16}\ {\rm erg\ cm^{-3}}),$$

$$T_c = \zeta^{-2/5}\mu^{1/5}\mu_e^{1/5}\alpha^{-1/5} \times m^{-1/5}\dot{m}^{2/5}r^{-9/10}s^{2/5}(1.2\times10^8\ {\rm K}),$$

$$\tau_c = Z\zeta^{-1/10}\mu^{21/10}\mu_e^{-1/5}\mu_I^{-1/2}\alpha^{-4/5} \times m^{1/5}\dot{m}^{1/10}r^{3/20}s^{1/10}(4.5\times10^1),$$

and for the outer zone (C) with $P_g \gg P_\gamma$ and $\kappa_{\rm T} \ll \kappa_{\rm ff}$,

$$H = Z^{1/10}\zeta^{-3/20}\mu^{-3/8}\mu_e^{1/10}\mu_{\rm I}^{-1/20}\alpha^{-1/10} \times m^{9/10}\dot{m}^{3/20}r^{9/8}s^{3/20}(6.1\times10^{-2}\ {\rm km}), \quad (6)$$

$$\Sigma = Z^{-1/5}\zeta^{-7/10}\mu^{3/4}\mu_e^{4/5}\mu_{\rm I}^{1/10}\alpha^{-4/5} \times m^{1/5}\dot{m}^{7/10}r^{-3/4}s^{7/10}(1.3\times10^5\ {\rm g\ cm^{-2}}),$$

$$\rho_c = Z^{-3/10}\zeta^{-11/20}\mu^{9/8}\mu_e^{7/10}\mu_{\rm I}^{3/20}\alpha^{-7/10} \times m^{-7/10}\dot{m}^{11/20}r^{-15/8}s^{11/20}(1.0\times10^1\ {\rm g\ cm^{-3}}),$$

$$n_{ec} = Z^{-3/10}\zeta^{-11/20}\mu^{9/8}\mu_e^{-3/10}\mu_{\rm I}^{3/20}\alpha^{-7/10} \times m^{-7/10}\dot{m}^{11/20}r^{-15/8}s^{11/20}(6.3\times10^{24}\ {\rm cm^{-3}}),$$

$$\bar{v}_R = Z^{1/5}\zeta^{-3/10}\mu^{-3/4}\mu_e^{1/5}\mu_{\rm I}^{-1/10}\alpha^{4/5} \times m^{-1/5}\dot{m}^{3/10}r^{-1/4}s^{-7/10}(-2.0\ {\rm km\ s^{-1}}),$$

$$\epsilon_{\gamma c} = Z^{4/5}\zeta^{-6/5}\mu\mu_e^{4/5}\mu_{\rm I}^{-2/5}\alpha^{-4/5} \times m^{-4/5}\dot{m}^{6/5}r^{-3}s^{6/5}(2.7\times10^{16}\ {\rm erg\ cm^{-3}}),$$

$$P_c = Z^{-1/10}\zeta^{-17/20}\mu^{3/8}\mu_e^{9/10}\mu_{\rm I}^{1/20}\alpha^{-9/10} \times m^{-9/10}\dot{m}^{17/20}r^{-21/8}s^{17/20}(3.8\times10^{16}\ {\rm erg\ cm^{-3}}),$$

$$T_c = Z^{1/5}\zeta^{-3/10}\mu^{1/4}\mu_e^{1/5}\mu_{\rm I}^{-1/10}\alpha^{-1/5} \times m^{-1/5}\dot{m}^{3/10}r^{-3/4}s^{3/10}(4.3\times10^7\ {\rm K}),$$

$$\tau_c = Z^{3/10}\zeta^{-9/20}\mu^{7/8}\mu_e^{-1/5}\mu_{\rm I}^{-3/20}\alpha^{-4/5} \times m^{1/5}\dot{m}^{9/20}r^{-3/8}s^{9/20}(1.5\times10^3).$$

The disk zones are labeled by A, B, and C as in Shakura and Sunyaev (1973). Numerical values based on the above results for different types of pure, fully ionized plasmas are given in Table 1. For example, this table indicates that, compared to a pure H disk with the same parameters $(\alpha, M, \dot{M})$, the disk becomes typically thinner by a factor of 2 if it is dominated by heavier elements. As one can check using Saha's equation (Landau and Lifshitz 1980), considering fully ionized He, C, Mg, etc. is a good approximation in the case of ultracompact binaries possessing hot accretion disks with small radial diameters.

In addition to the quantities from Eqs. (4)–(6), Table 1 lists the relative changes in the characteristic time scale for Comptonization (Pozdnyakov et al. 1983)

$$t_{\rm co} = \frac{3}{8}\frac{m_e c}{\sigma_{\rm T}\epsilon_{\gamma c}}. \quad (7)$$

Another important time scale is given by the equipartition time for the energy exchange between fast electrons and slow ions (Spitzer 1962),

$$t_{\rm eq} = \frac{A}{Z^2\ln\Lambda}\left(\frac{\rm cm^{-3}}{n_{Ic}}\right) \times \left(\frac{kT_c}{m_e c^2}\right)^{3/2}(1.1\times10^{17}\ {\rm s}) = \frac{A\mu_{\rm I}}{Z^2\ln\Lambda}\left(\frac{\rm g\ cm^{-3}}{\rho_c}\right)\left(\frac{kT_c}{m_e c^2}\right)^{3/2}(1.9\times10^{-7}\ {\rm s}), \quad (8)$$

where $n_{Ic} = \rho_c/(\mu_I m_p)$ is the ion number density in the disk mid-plane. The quantities $t_{\rm eq}$ and $t_{\rm co}$ can be compared with, e.g., the time scale of radial motions $t_R = R/\bar{v}_R$, or, alternatively, with the revolution time scale $t_\Omega = 2\pi R/\Omega$. It is worthwhile to note that, in contrast to $t_{\rm eq/co/R}$, the revolution time scale t_Ω is independent of the chemical composition. A two-temperature flow regime can develop in the hot, inner disk region if the latter becomes optically thin and if $t_{\rm eq} \gg t_\Omega$ (Shapiro et al. 1976).

3. THE BOUNDARY (SPREADING) LAYER OF NEUTRON STARS

The main difference between the accretion onto black holes and that onto neutron stars is that the kinetic energy of the accreting matter on the neutron-star surface transforms into radiation, whereas in the former case this energy is absorbed by the black hole. There are several different models for the boundary layers of neutron stars (Sunyaev and Shakura 1986; Popham and Sunyaev 2001). In the spreading-layer model of Inogamov and Sunyaev (1999), two bright belts located equidistant from the equator on the surface of the (slowly rotating) star emit radiation corresponding to the local Eddington flux

$$q_{\rm E}(\theta) = \frac{L_{\rm E}}{4\pi R_*^2}\left[1-\left(\frac{v_\phi(\theta)}{v_{\rm K}}\right)^2\right] \quad (9)$$
$$= \mu_e \frac{GM}{R_*^2}\frac{m_p c}{\sigma_{\rm T}}\left[1-\left(\frac{v_\phi(\theta)}{v_{\rm K}}\right)^2\right],$$

where R_* denotes the radius of the neutron star, θ the latitude angle ($\theta = 0$ corresponds to the "equatorial" disk midplane), $v_{\rm K} = (GM/R_*)^{1/2}$ is Keplerian velocity, and v_ϕ is the rotation velocity on the surface of the star (to be obtained by solving the boundary-layer equations of Inogamov and Sunyaev (1999)). The local Eddington flux $q_{\rm E}(\theta)$ is determined by the difference between the gravitational force and the centrifugal force due to the rotation of the accreting matter on the stellar surface. Hence, because of $L_{\rm E} = \mu_e L_{\rm E,H}$, formula (9) implies that the flux that may be emitted from the surface of a neutron star increases with μ_e (small deviations from this simple proportionality may occur due to the weak dependence of v_ϕ on the chemical parameters μ_e and μ). Therefore, in the case of He- or metal-rich accretion flows, this flux increases by a factor of 2 (at the same values of $\dot M$) compared to a pure H disk. Moreover, similarly to the scale height H of the disk, the meridional height $H_{\rm SL}$ of the boundary (spreading) layer, which can be estimated from the energy balance (Inogamov and Sunyaev 1999)

$$\frac{\dot M v_{\rm K}^2}{2} = L_{\rm SL} = L_{\rm E}\frac{H_{\rm SL}}{R_*} = \mu_e L_{\rm E,H}\frac{H_{\rm SL}}{R_*}, \quad (10)$$

decreases at the same values of $\dot M$ if the disk is dominated by elements heavier than H.

APPENDIX

THE PASZYŃSKY-WIITA POTENTIAL

As discussed by Paczyńsky and Witta (1980), the general relativity effects can be modeled by replacing the Newtonian potential $\Phi = -GM/r$ with the modified potential

$$\hat\Phi(R,z) = -\frac{GM}{r-r_{\rm G}}, \quad (11)$$

corresponding to an efficiency factor $\zeta = 1/16$, in contrast to the Newtonian value of $\zeta = 1/12$. To simplify the subsequent formulas, it is convenient to define three dimensionless correction factors by

$$\chi_0 \equiv \frac{R}{R-r_{\rm G}} = \frac{r}{r-1/3}, \quad (12a)$$
$$\chi_1 \equiv \frac{3}{2} - \frac{R+\sqrt{3Rr_{\rm G}}}{2(R-r_{\rm G})} = \frac{3}{2} - \frac{r+\sqrt{r}}{2(r-1/3)}, \quad (12b)$$
$$\chi_2 \equiv \frac{R(3R-r_{\rm G})}{6(R-r_{\rm G})^3}\left(2R-\sqrt{3Rr_{\rm G}}-3r_{\rm G}\right) \quad (12c)$$
$$= \frac{r(3r-1/3)}{6(r-1/3)^3}\left(2r-\sqrt{r}-1\right).$$

As can be seen from these factors, the relativistic corrections are important only in the innermost parts of the disk. Therefore, we will restrict ourselves here to giving the results for zones A and B.

Zone A: In the case of dominating radiation pressure, $P_g \ll P_\gamma$, and dominating Thomson scattering, $\kappa_{\rm T} \gg \kappa_{\rm ff}$, we find

$$\hat H(R) = \chi_0^{-2}\chi_2 H, \quad (13)$$
$$\hat\Sigma(R) = \chi_0^2\chi_1\chi_2^{-2}\Sigma,$$
$$\hat\rho_c(R) = \chi_0^4\chi_1\chi_2^{-3}\rho_c,$$
$$\hat n_{ec}(R) = \chi_0^4\chi_1\chi_2^{-3}n_{ec},$$
$$\hat{\bar v}_R(R) = \chi_0^{-2}\chi_1^{-1}\chi_2^2\bar v_R,$$
$$\hat\epsilon_{\gamma c}(R) = \chi_0^2\chi_1\chi_2^{-1}\epsilon_{\gamma c},$$
$$\hat P_c(R) = \chi_0^2\chi_1\chi_2^{-1}P_c$$
$$\hat T_c(R) = \chi_0^{1/2}\chi_1^{1/4}\chi_2^{-1/4}T_c,$$
$$\hat\tau_c(R) = \chi_0^{25/8}\chi_1^{17/16}\chi_2^{-49/16}\tau_c,$$

where on the right-hand sides the results from Eqs. (4) are to be inserted (in what follows, the quantities without hats refer to the expressions derived for the Newtonian potential).

Zone B: In the case of dominating gas pressure, $P_g \gg P_\gamma$, and dominating Thomson scattering, $\kappa_{\rm T} \gg \kappa_{\rm ff}$, we find

$$\hat H(R) = \chi_0^{-1}\chi_1^{1/10}\chi_2^{1/10}H, \quad (14)$$

$$\hat{\Sigma}(R) = \chi_1^{4/5}\chi_2^{-1/5}\Sigma,$$
$$\hat{\rho}_c(R) = \chi_0\chi_1^{7/10}\chi_2^{-3/10}\rho_c,$$
$$\hat{n}_{ec}(R) = \chi_0\chi_1^{7/10}\chi_2^{-3/10}n_{ec},$$
$$\hat{\bar{v}}_R(R) = \chi_1^{-4/5}\chi_2^{1/5}\bar{v}_R,$$
$$\hat{\epsilon}_{\gamma c}(R) = \chi_1^{4/5}\chi_2^{4/5}\epsilon_{\gamma c},$$
$$\hat{P}_c(R) = \chi_0\chi_1^{9/10}\chi_2^{-1/10}P_c,$$
$$\hat{T}_c(R) = \chi_1^{1/5}\chi_2^{1/5}T_c,$$
$$\hat{\tau}_c(R) = \chi_0^{1/2}\chi_1^{4/5}\chi_2^{-7/10}\tau_c,$$

where on the right-hand sides the results from Eqs. (5) are to be inserted.

The dimensionless transition radius r_{AB} separating zones A and B can be determined from the condition $\hat{T}^A(r_{AB}) = \hat{T}^B(r_{AB})$; i.e., r_{AB} is given by the solution of the equation

$$r^{21/16}s^{-1}\chi_0^{5/4}\chi_1^{1/8}\chi_2^{-9/8} = 47.3\zeta^{-1}\mu^{1/2}\mu_e^{-1/8}\alpha^{1/8}m^{1/8}\dot{m}. \quad (15)$$

The function on the left-hand side attains its *minimum* value of 14.8 at $r = 2.54$. Consequently, zone A exists only if

$$0.02 < \mu^{1/2}\mu_e^{-1/8}\alpha^{1/8}m^{1/8}\dot{m}, \quad (16)$$

where we have already inserted the efficiency factor $\zeta = 1/16$.

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