UTMOST

Undergraduate Teaching in Mathematics with Open Software and Textbooks

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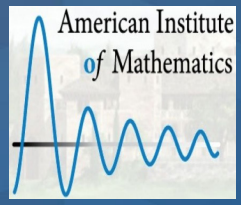
Electronic Seminars in Mathematics Education, March 16, 2021

DOCUMENTS FOR TEACHING A LESSON: LECTURE NOTES AND THEIR PRODUCTION

SCHOOL OF EDUCATION UNIVERSITY OF MICHIGAN



University of Colorado

REFLECTION BEFORE WE BEGIN…

- What kinds of personal documents (e.g., lesson plan, lecture notes) do you create when you are getting ready to teach a lesson?
- What things do you gather? (e.g., textbook, paper, computer…)
- What does the document that you use for teaching look like?

PLAN FOR THE CONVERSATION

- Why I am interested in this work
- Theoretical tools to understand instructors' documentation work
- Larger study, exploratory questions, sources
- Individual work and sharing in small groups
- Preliminary findings
- Questions

DOCUMENTS FOR TEACHING A LESSON

- Teaching is a complex set of activities
  - Planning
  - Implementing the plan in the classroom
  - Assessing student learning
  - Evaluating how things went
- Few studies at the university level
  - What goes on as teachers engage in those activities?
  - What role do resources play in those activities?

THEORETICAL TOOLS

- Human activity is mediated by the artifacts used to achieve a particular goal
- An *artifact*, together with a *scheme of use*, becomes an *instrument*
  - Knife used to spread butter *butter knife*
  - Knife used to tighten a loose screw *screwdriver*
- Resources: A collection of artifacts gathered for a specific purpose
- Documentational genesis: the processes involved in creating documents that support various activities of teaching

(Gueudet & Trouche, 2009; Rabardel & Wearn, 2003; Verillon & Rabardel, 1995)

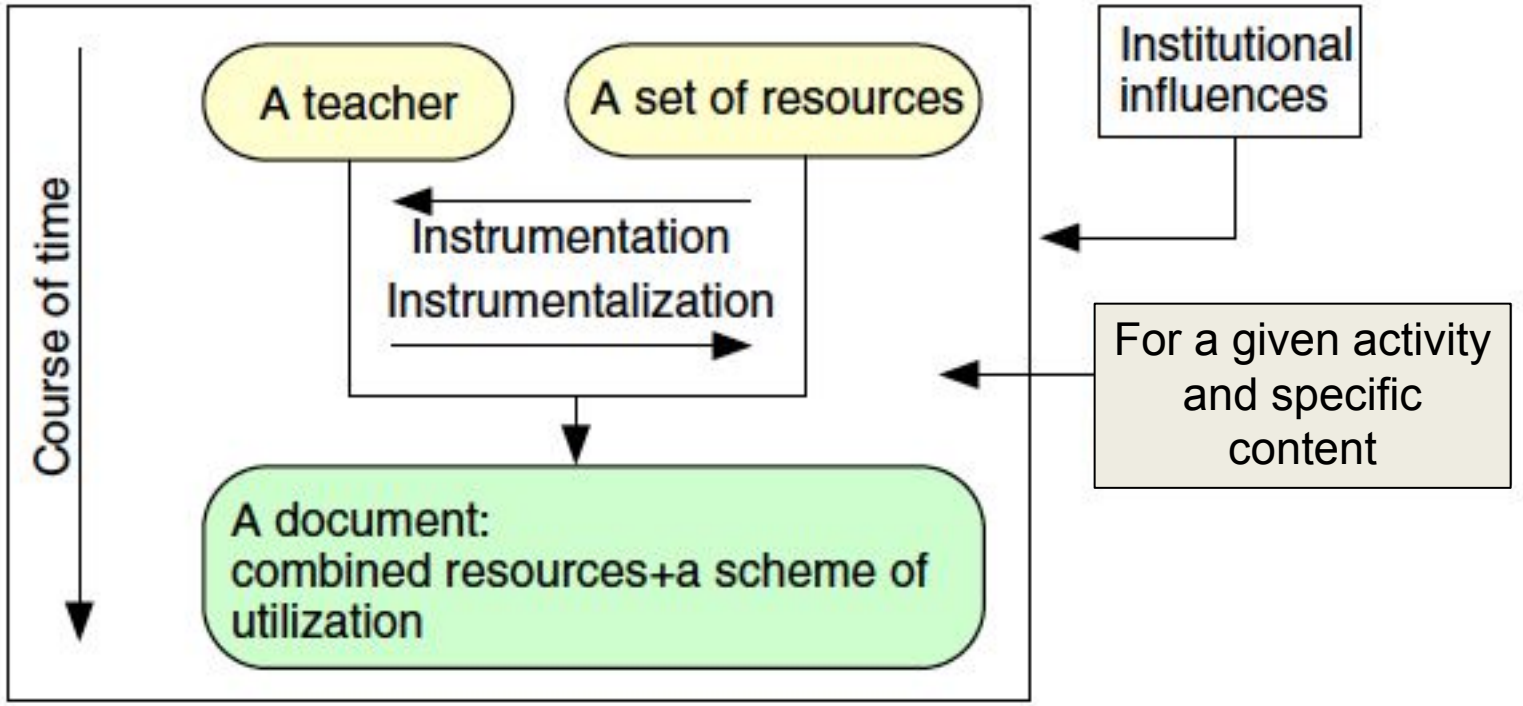
TEACHERS' DOCUMENTATION WORK

- **looking for resources**: textbooks, instructional materials, time (for planning, discussing ideas with colleagues, attending seminars)
- **making sense and use of them**: planning instructional tasks, aligning instruction with the objectives to which teachers are held accountable

"The products of this work at a given point in time are characterized as **documents** [… and they] can in turn become resources in subsequent documentation work. Documentational genesis foregrounds interactions of teachers and resources, and highlights how both are transformed through these interactions"

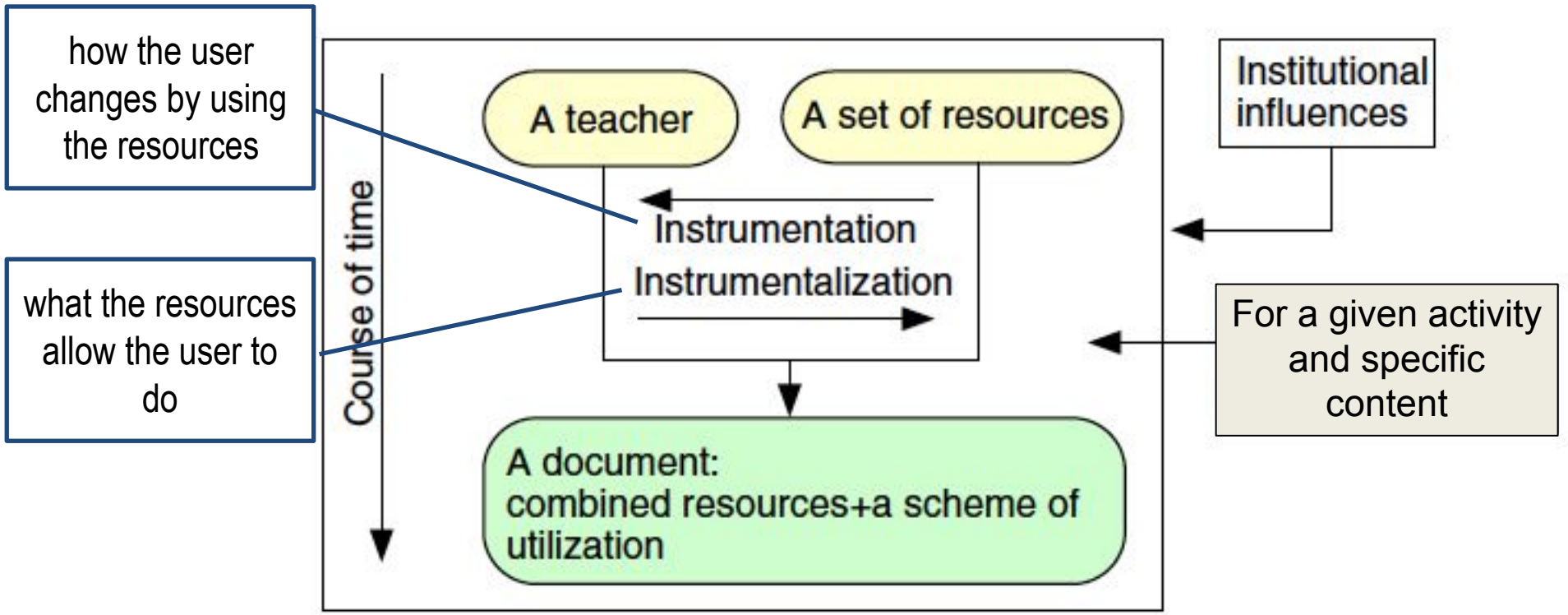
(Vinovska, Cobb, & Dean, 2012)

TEACHERS' DOCUMENTATION WORK



(Gueudet & Trouche, 2009)

TEACHERS' DOCUMENTATION WORK



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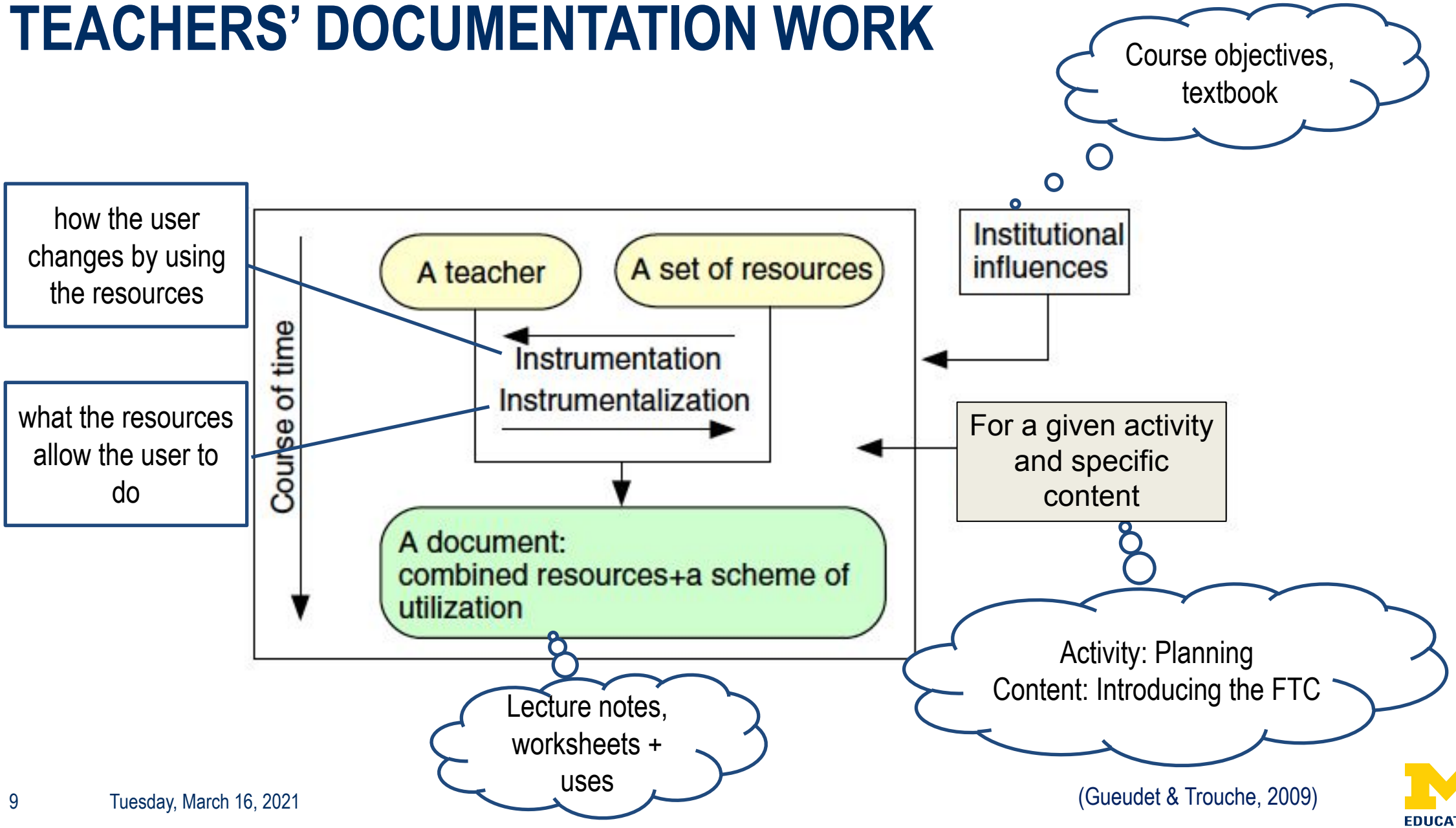
TEACHERS' DOCUMENTATION WORK



(Gueudet & Trouche, 2009)

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LARGER STUDY

Undergraduate Teaching and learning in Mathematics with Open Source Textbooks (UTMOST) project (https://utmost.aimath.org/)

How are open-source mathematics textbooks used for teaching and learning in post-secondary settings? What do we gain?

Three textbooks:

- Boelkins' *Active Calculus* (AC)
- Beezer's *First Course in Linear Algebra* (FCLA)
- Judson's *Abstract Algebra Theory and Applications* (AATA)

EXPLORATORY QUESTION, SOURCES

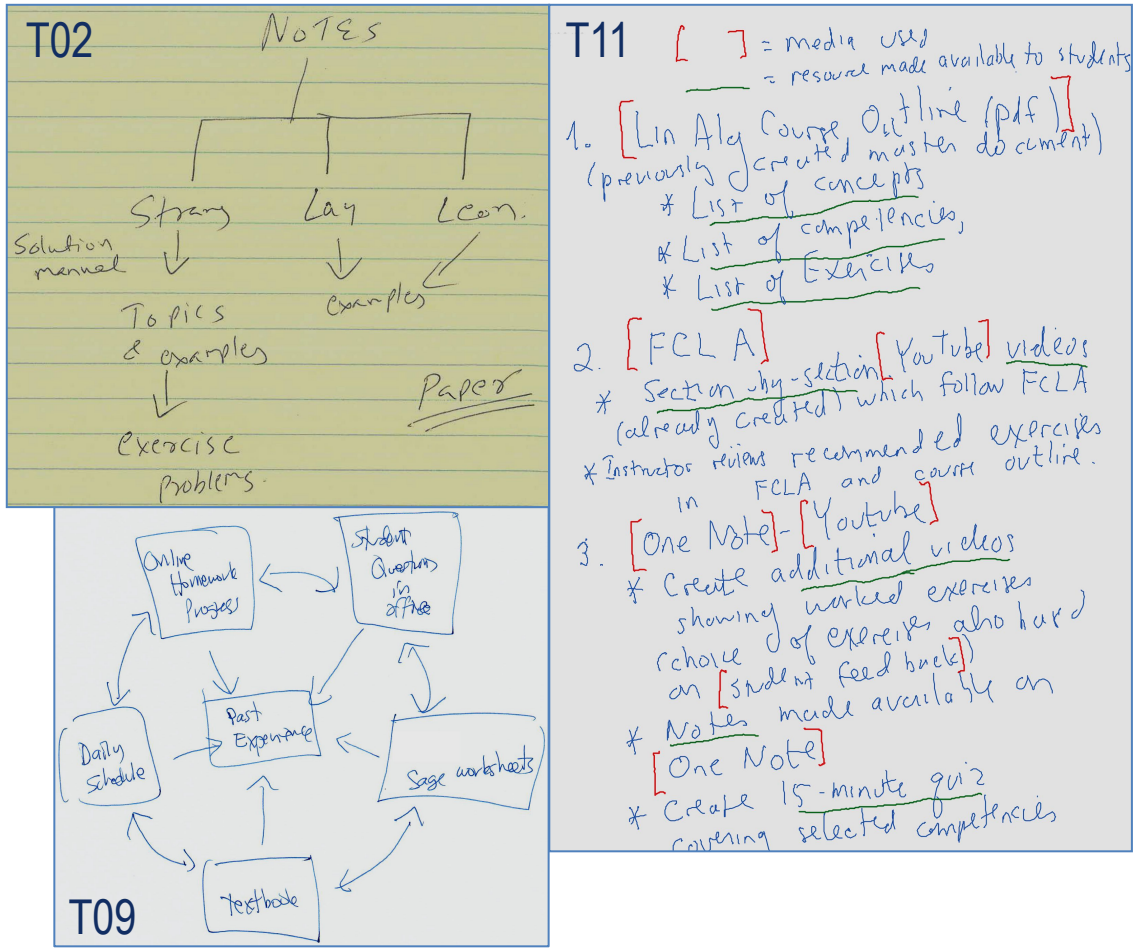
- What resources are used to generate documents to teach a lesson?
- How are the resources instrumented to generate the lecture notes?

- A sample of 21 instructors
- Responses to survey questions…
  - How do you create your lecture notes for a class session?
  - What resources are you using to create your lecture notes? (e.g., course textbook, CoCalc, lecture notes from previous years….)
- Drawings illustrating the process of creating the lecture notes
- Lecture notes/Lesson notes
- Course syllabus

INDIVIDUAL WORK AND SHARING IN SMALL GROUPS

- How do you create lecture notes/lesson plans for a class session?
- What resources do you use to create them?
- Create on paper a diagram that showcases the resources you use when planning the lesson. How are they related?

Be ready to share in a small group…



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WHAT RESOURCES ARE USED TO GENERATE DOCUMENTS TO TEACH A LESSON?

"Material"			"Non-Material"
Print available	Electronic only	Physical	
• textbook • past lecture notes • graduate school notes • other textbooks • course syllabus • college/department competencies • past exams • documents provided by authors (prep assignments, solutions to problems, worksheets) • publications (MAA, research)	• YouTube • Wolfram alpha • software (Sage, Desmos, GeoGebra, Mathlab, Mathematica, LaTeX, PreTeXt, Beamer, Remind, OneNote, Word, PPT, Google docs…) • Wikipedia • course management systems • repositories (GitHub, MS OneDrive, Google drive…) • Zoom	• manipulatives (Rubik's cube, D4 models) • computer • printer • scanner • document projector • tablet	• experience • personal knowledge • own thinking • student questions • discussions with others (students, colleagues, partners, children, IBL/NExT) • "divine" inspiration • time

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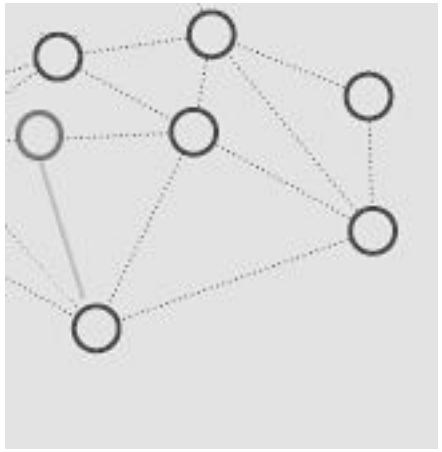
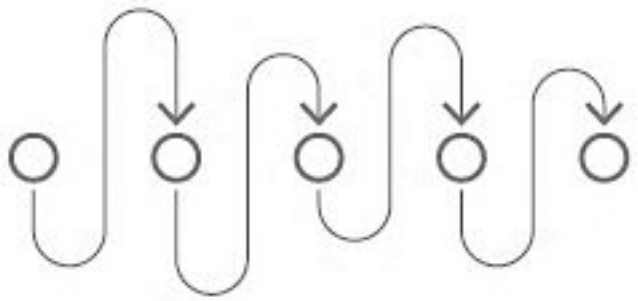
HOW ARE RESOURCES INSTRUMENTED?

Various resources contribute to the document Resources are (re)used at different times





INSTRUMENTED ACTIVITY, TWO PROCESSES

My lecture notes tend to follow the text as much as possible. With this course, I find that the vocabulary is very important, so following the definitions in the text helps the students follow the development of the new ideas. At times, I find that there is an example that I prefer to the text, and I slip that in instead. This gives a little more variety to the students, too. The textbook does a good job of highlighting the various definitions, theorem, and examples, and my previous lecture notes help me remember the points that I like to emphasize. (T15, LA)

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Instrumentalization User → Resources

The user (T15) takes the textbook content (definitions, theorems, examples) for creating the lecture notes.

INSTRUMENTED ACTIVITY, TWO PROCESSES

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Instrumentation: Resources → User

Something missing in the textbook makes the user (T15) search for other examples. Previous lecture notes act as reminder.

WHAT DO THE PRODUCTS LOOK LIKE?

Type

- Personal notes
- Notes to share ahead of class
- Notes to project during class
- Notes to share after class

Style

- Fully/partially written out text
- A table with approximate times for various activities
- A template to be filled out live
- Bulleted list

Content

- Definitions, examples
- Theorems, proofs
- List of homework problems
- Reminders to self/narrative
- Administrative reminders

Means of production

- Handwritten
- Word processor
- Presentation programs
- LaTeX
- PreTeXt

Means of presentation

- White/black board
- Computer
- Tablet
- Projector/document camera
- Video
- Zoom

TWO EXAMPLES

Type	Style	Content	Means of production
• Personal notes • Notes to share ahead of class • Notes to project during class • Notes to share after class	• Fully/partially written out text • A table with approximate times for various activities • A template that will be filled out live • Bulleted list	• Definitions, examples • Theorems, proofs • List of homework problems • Reminders to self/narrative • Administrative reminders	• Handwritten • Word processor • Presentation programs • LaTeX • PreTeXt

T23

More Subgroups

Last time we introduced the concept of a *subgroup* of a group. This is defined as a subset that is also a group under the same operation. We decided that to check whether a subset was a group, we need to check three properties: (1) $e \in H$ (H contains the identity of G), (2) $\forall a, b, \in H$ we have $ab \in H$ (H is closed under the operation), and (3) $\forall a \in H$ we have $a^{-1} \in H$ (H is closed under inverses).

Note though that we still need the operation to be the same. In particular, $\mathbb{Z}_4$ is not a subgroup of $\mathbb{Z}_8$.

A few examples of subgroups:

- Let $G = \mathbb{Z}$, the group of integers under addition. What are the subgroups? Is $3\mathbb{Z}$ a subgroup? These are all the multiples of 3. Check the 3 things.
- Let $G = \mathscr{F}(\mathbb{R})$, the group of all real-valued functions under addition. One subgroup is the set of all continuous functions. Also the set of all differentiable functions, or linear functions, or polynomials.

We would also like to say some things in general. For example, let's prove that if G is *any* abelian group, then $H = \{g^2 : g \in G\}$ is a subgroup of G.

- Another way to write the subgroup: $H = \{g \in G : g = a^2 \text{ for some } a \in G\}$.
- First, H contains the identity, since $e = e^2$ and $e \in G$.
- For closure: assume $a, b \in H$. That is, $a = x^2$ and $b = y^2$ for $x, y \in G$. What is ab? Well, $ab = x^2y^2 = (xy)^2$ because G is abelian. But $xy \in G$, so $ab \in H$.
- For inverses: assume $a \in H$. This means, $a = x^2$ for some $x \in G$. What about a^{-1}? Well, $a^{-1} = (x^2)^{-1} = (x^{-1})^2$, and since $x^{-1} \in G$ we see that $a \in H$.

By the way, what is H for $\mathbb{Z}$ here?

Try two more: Finish the proof that $Z(G) = \{c \in G : cx = xc \text{ for every } x \in G\}$ is a subgroup of G. This is called the **center** of G (the set of elements that commute with everything).

Then prove that the **centralizer** of H in G is a subgroup: $C(H) = \{g \in G : ghg^{-1} = h \text{ for all } h \in H\}$.

If there is time, consider $\langle a \rangle = \{a^n : n \in \mathbb{Z}\}$. That is, the set containing all the positive and negative powers of a.

For next time, read section 4.1 and do the subgroup proofs on Canvas.

T20

§ 3.2 Groups: Definitions & Examples

Binary operation on a set G: A function $*: G \times G \to G$, $(a, b) \mapsto a*b$ (*) must be well-def

A group $(G, *)$ is a set G together w/ a binary op. $*$ that satisfies the following axioms:

(i) associativity $(a*b)*c = a*(b*c)$ $\forall a, b, c \in G$

(iii) ~~identity~~ inverse element For each $a \in G$ $\exists$ $a^{-1} \in G$ s.t. $a*a^{-1} = a^{-1}*a = e$.

(ii) identity element: $\exists$ $e \in G$ s.t. $\forall a \in G$ $e*a = a*e = a$.

A group G with the property that $a*b = b*a$ $\forall a, b \in G$ is called Abelian or commutative.

Activity 2: Do Ex 3 ← reading assignment

(*) check well-def

Ex: $(\mathbb{Z}, +)$ ✓ $(\mathbb{Z}, \times)$ × $(\mathbb{Z}_n, \oplus_n)$ ✓ check conditions use equiv classes $(\mathbb{Z}_n, \odot_n)$ depends on n.

$(D_2, \circ)$ Activity 1: Do Ex. 2

Prop. 3.4 Let $a \in \mathbb{Z}$ $a \neq 0$. Then $\gcd(a, n) = 1 \iff \exists\ b \in \mathbb{Z}$ s.t. §3.1 $ab \equiv 1 \pmod n$

$\Rightarrow$ $1 = ar + ns \Rightarrow ns = 1 - ar \therefore 1 \equiv ar \pmod n$

$\Leftarrow$ Supp $\exists b$ st. $ab \equiv 1 \pmod n$. $\Rightarrow n \mid ab - 1$

$\Rightarrow nk = ab - 1 \Rightarrow 1 = ab - nk$.

Let $d = \gcd(a, n)$. Since $d \mid a$ & $d \mid n \Rightarrow$

$d \mid ab - nk \Rightarrow d \mid 1 \therefore d = 1$. □

$U(n) = \{0 \leq a < n \mid \gcd(a, n) = 1\}$

$U(n)$ = group of units of $\mathbb{Z}_n$.

Activity 3: Cayley table for $U(6)$.

WHAT DO THE DOCUMENTS REVEAL ABOUT INSTRUCTORS' WORK? INSTRUCTORS AS DESIGNERS

DOCUMENTS: Products of documentation + schemes of use

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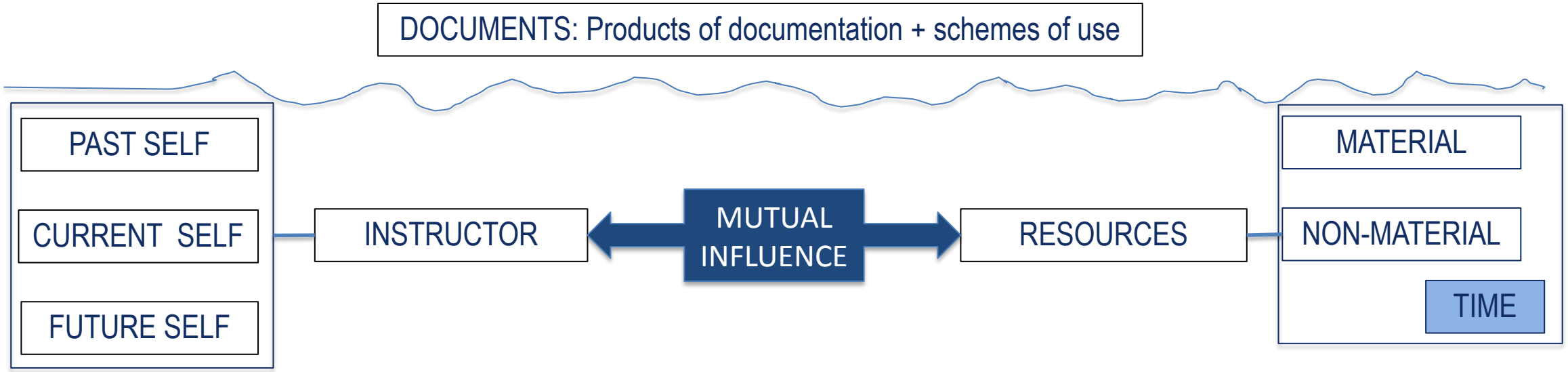
WHAT DO THE DOCUMENTS REVEAL ABOUT INSTRUCTORS' WORK? INSTRUCTORS AS DESIGNERS



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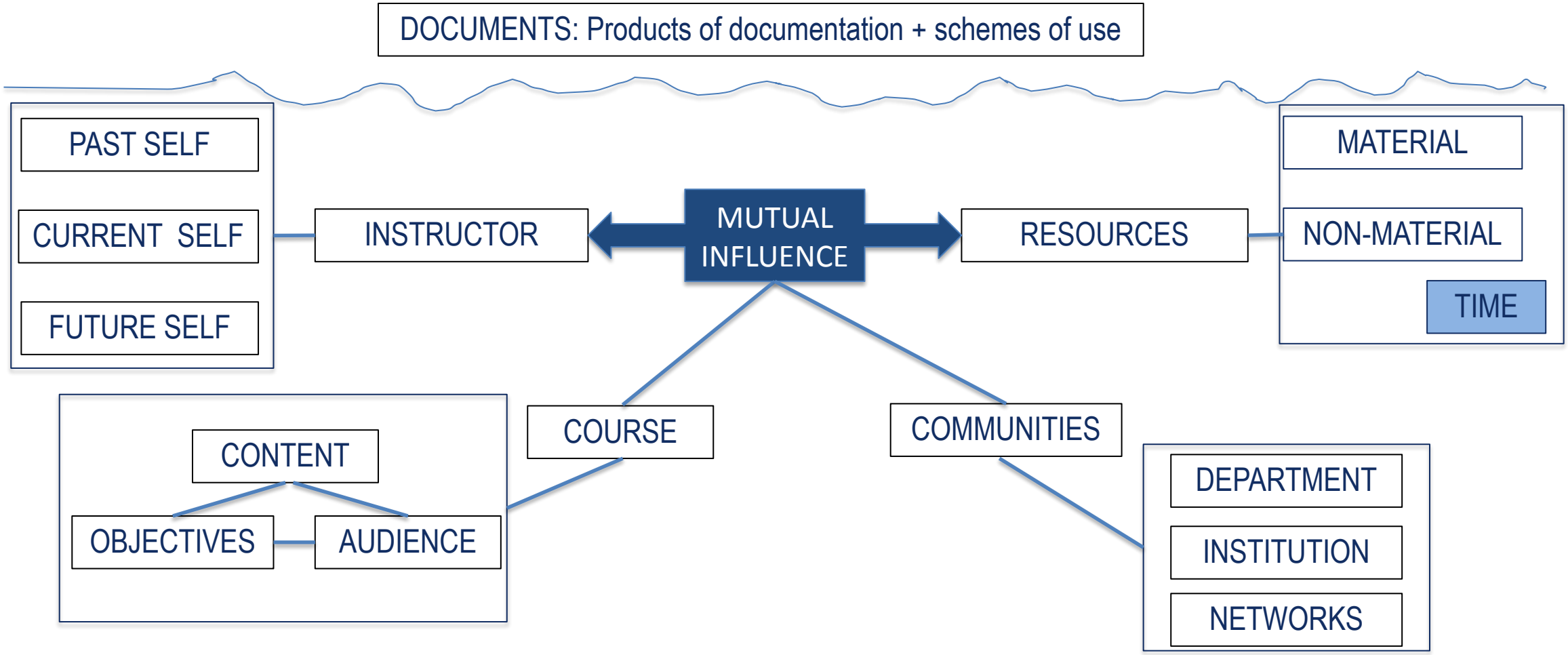

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THANK YOU! UTMOST 3.0

Collaborators:

- Rob Beezer — University of Puget Sound
- Tom Judson — Stephen F Austin State U
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* Former members of the UTMOST project

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