A Touch of Calculus: Shaking Up the Pre-Requisite Structure of College Mathematics

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Electronic Seminar on
Math Education
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Thanks!

- To Haynes Miller and Tara Holm for the invitation.
- To TPSE-Math (Transforming Post-Secondary Education) for encouragement
- To my colleagues at Babson, Bentley, Cornell and St. Michael's for years of discussion and innovation

Motivation

- Co-chair of TPSE group on upper-division pathways; with Bill Velez (U. of Arizona). Goal to increase enrollments; remove barriers.
- My own experience teaching in programs both traditional and non-standard.

Outline

I.) Calculus' place in the undergraduate mathematics curriculum.

II.) Examining the pre-requisite structure in the math major.

III.) Alternate pathways to higher level mathematics and STEM employment

Poll #1

-How many calculus courses are math majors at your school required to take? If variable, report minimum. Assume no AP credit.

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-Tradition: That's been the curriculum for a long time.

-Service role: Demand from other disciplines.

-Content and math maturity pre-requisite.

-We all love teaching calculus! (I miss it!)

-Feel free to suggest others in the chat.

Poll #2

-At your school, how many courses that count toward a math major can a student take with no calculus pre-requisite?

Career paths…

Now … math majors are getting jobs in data science and other quantitative fields. High demand for workers in these areas.

-What role should mathematicians play?

-How do we coordinate with other departments; two year colleges; employers; and graduate school in many fields?

Current problems…

- D/F/W rates in traditional pathway courses from college algebra through calculus sequence are high.
- Articulation issues from high schools and two year colleges to four year schools reinforce unequal opportunities.

Two types of response…

- Curricular response: Is there a legitimate mathematics pathway for students interested primarily in careers instead of graduate school in math?

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- Environmental response: How can departments be more welcoming and encouraging to all students?

Poll #3:

For which of the following courses is at least one semester of calculus essential as a pre-requisite? (Choose all that apply)

A.) Linear Algebra B.) Discrete Math

C.) Probability (Level of SOA Exam P)

D.) Financial Math (Option Pricing)

E.) Differential Equations

F.) None of the above

Pre-reqs needed:

My Answer … and Babson college curriculum answer… Calculus sequence is needed for:

E.) Differential Equations … maybe! (we don't have a DE course or we might try it.)

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Can be very liberating for faculty … we can teach what we think is important!

At Bentley University…

- Mathematics major with three tracks … Actuarial science, data analytics, mathematical sciences … with applied courses.
- Calculus required, but not much “pure math.”
- Popular majors with strong job placement … one of biggest majors.

Data Science Sensibility…

- Note that for students interested in data science, linear algebra and discrete mathematics are probably the most appropriate math courses. Teaching them without a calculus pre-req expands the pool of potential students AND eases articulation issues with two year schools.

Electives w/o calculus…

Linear Algebra/Discrete Math:

If you suggested calculus is necessary for Linear Algebra and/or Discrete Math, please use the chat to suggest the particular topics that you believe are needed.

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- To do well on Exam P, students need to know how to do these integrals by hand.
- But to understand the material, the integrals can be done with technology!

Example (SOA practice)

An insurance company insures a large number of drivers. Let *X* be the random variable representing the company's losses under collision insurance, and let *Y* represent the company's losses under liability insurance. *X* and *Y* have joint density function:

$$f(x,y) = .25*(2x + 2 - y) \text{ for } 0 < x < 1; 0 < y < 2$$

Calculate the probability that the total company loss is at least 1.

Example (SOA practice)

An insurance company insures a large number of drivers. Let *X* be the random variable representing the company's losses under collision insurance, and let *Y* represent the company's losses under liability insurance. *X* and *Y* have joint density function:

$$f(x,y) = .25*(2x + 2 - y) \text{ for } 0 < x < 1;\ 0 < y < 2$$

Calculate the probability that the total company loss is at least 1.

To solve: Need idea of joint distribution and algebra skill to define the region where $x + y > 1$ in rectangle. Then call for help!

WolframAlpha computational intelligence

.25*(integral (2x+2-y) for x=0 to x=1, y=1-x to y=2)

Extended Keyboard | Upload | Examples | Random

Input:

$$0.25 \int_0^1 \int_{1-x}^2 (2x + 2 - y)\, dy\, dx$$

Result: More digits

0.708333333333333333333333333333333333333333333333333333333333333333333333333333
...

Indefinite integral assuming all variables are real:

$0.708333\, x$ + constant

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POWERED BY THE WOLFRAM LANGUAGE

Benefits…

-Reduces need for notation and encourages algebra/geometry connection.

-Learning calculus concepts in the context of applications may encourage understanding.

-Students likely to use technology tools in career.

A ‘calc on demand’ curriculum…

-Statistics I
-Stat II (Lin. Models)
-Discrete Math
-Linear Algebra
-Probability
-Financial Math
-Cryptography
-Sports Applications
-Coding/Algorithms
-Dynamical Systems
-’Depth’ Experience (internship/research/independent study)

Poll #4:

-Statistics I -Stat II (Lin. Models)

-Discrete Math -Linear Algebra

-Probability -Financial Math

-Cryptography -Sports Applications

-Coding/Algorithms -Dynamical Systems

-'Depth' Experience (internship/research/independent study)

Poll #4: Do you think the list of courses shown is a mathematics majors? Do you think your department would ever approve it as a mathematics major?

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- What about the logic, beauty, structure of mathematics?
- Perhaps it's there, and more accessible, when it's colloquial rather than formal!
- Written and oral expression are the career oriented equivalents of formal proofs.

Oral exam questions…

Applications and Explanations – Imagine you are at a job interview and potential employer looking at your transcript sees that you took Linear Algebra and asks:

I.) Tell me something interesting you learned about eigenvalues and eigenvectors.

2.) So linear algebra can be taught in a theoretical way or in an applied way. Where do you think your course fell along this spectrum?

3.) You took Linear Algebra? I took that, it was challenging at times! What was a concept you found difficult, and what did you do to help you understand it?

Oral exam questions…

Applications and Explanations

4.) Did your course cover any ways Linear Algebra is used to solve applied problems? What's one you remember, and what was one of the linear algebra tools or techniques needed to solve it?

5.) I know matrix multiplication is a big deal in a linear algebra course. What are some examples of how it came up, either in theory or in an application?

6.) As a title for a course, "Linear Algebra" has meaning to people who have already taken it. However it does not provide much explanation to those without experience. If you could rename the course, what would you call it, and how would you write a brief (two sentence) description for the course catalog?

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Do you know how many of these there are? Are you succeeding?

-Chances for collaboration across institutions to “co-teach” theory.

-Expanding bridge programs and making them sustainable.

But what about (… #3)

- Can we get faculty to buy into a different curriculum?

Caveat!

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Avoid:

“Oh ,we used to have a REAL math major but not the one you’re doing…”

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- Can we get faculty to buy into a different curriculum?

We should try! We claim to be problem solvers; and to have learned a subject that allows us to solve problems. “That’s not how we’ve always done it” is not a solution.

Continuing discussion…

-Not every school can be as flexible as Babson or Bentley; but we can all think creatively about curriculum and attitude!

-Changes in curriculum and attitude can lead to larger enrollments, and be more inclusive by lowering barriers to entry.

-Changes can be relatively low cost and encourage interdisciplinary cooperation.

TPSE Next Steps

-Find, promote and publicize innovative curricula that encourage upper division mathematics enrollments.

This work is well underway with TPSE commissioned study from Rutgers Education and Employment research center and other sources.

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Reach out to departments to initiate these conversations, perhaps with data science as a motivator.

Follow Up/Resources

Use the many resources available from professional societies. For example:

-MAA suggestions for program review:

MAAProgramReviewPage

-AMS Committee on Education

AMSCoE (Mini-conference coming up 10/20/20).

Thanks for listening!

- Please contact me (rcleary@Babson.edu) with responses and suggestions.
- Follow the work of TPSE at

https://www.tpsemath.org/