Calculus in Context:

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Electronic Seminar on Mathematics Education

April 14, 2020

Calculus in Context: Introducing Calculus Ideas through Epidemiology Models

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ALTERNATIVE COURSE TITLE:

CALCULUS, SYSTEMS, AND MODELING

CALCULUS:

The mathematics of change

SYSTEM:

A set of things working together as parts of a mechanism or an interconnecting network

MODEL:

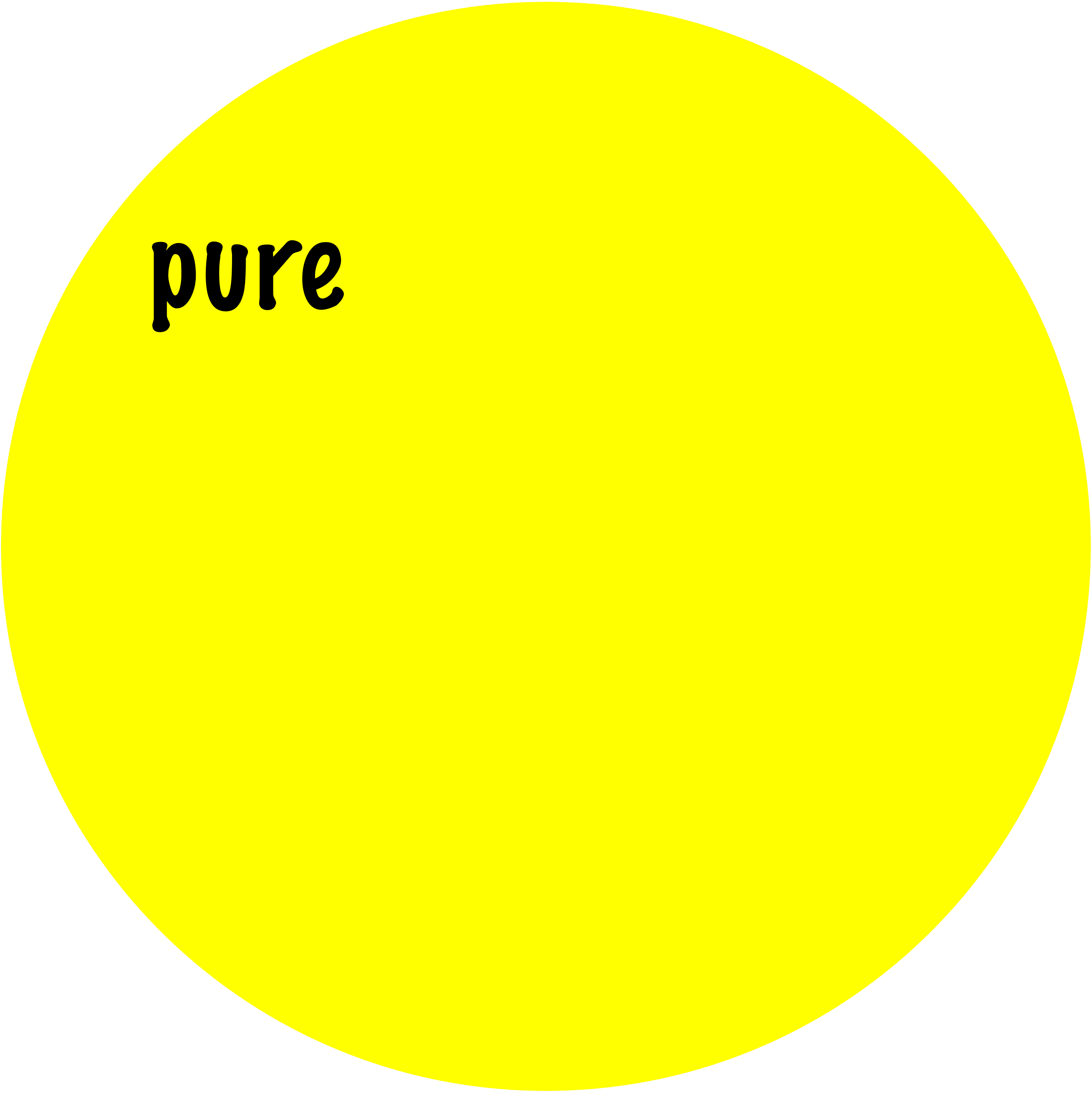
To devise a representation, esp. a mathematical one, of a phenomenon or system

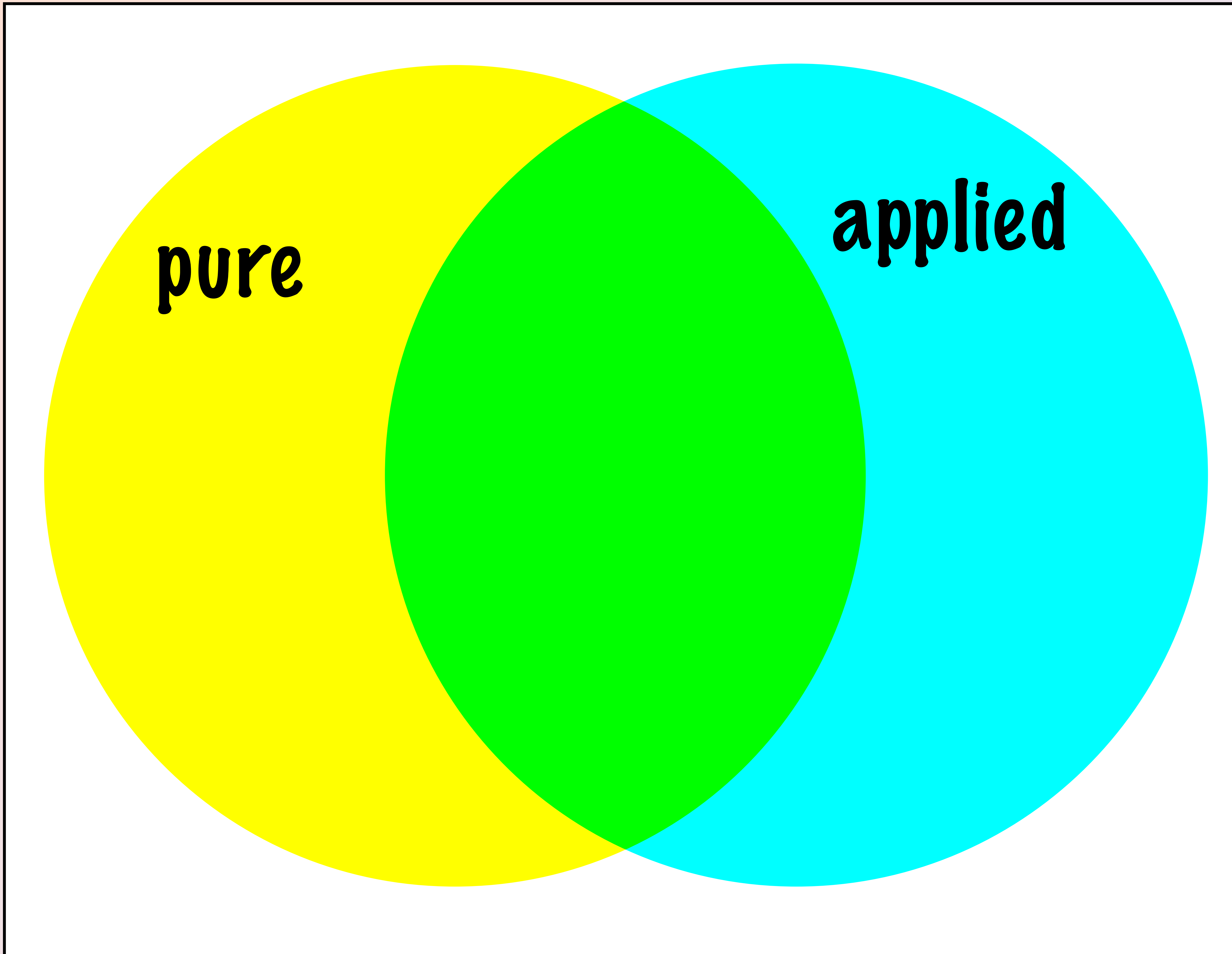
“Applications are not just something to stick at the end of the chapter.”

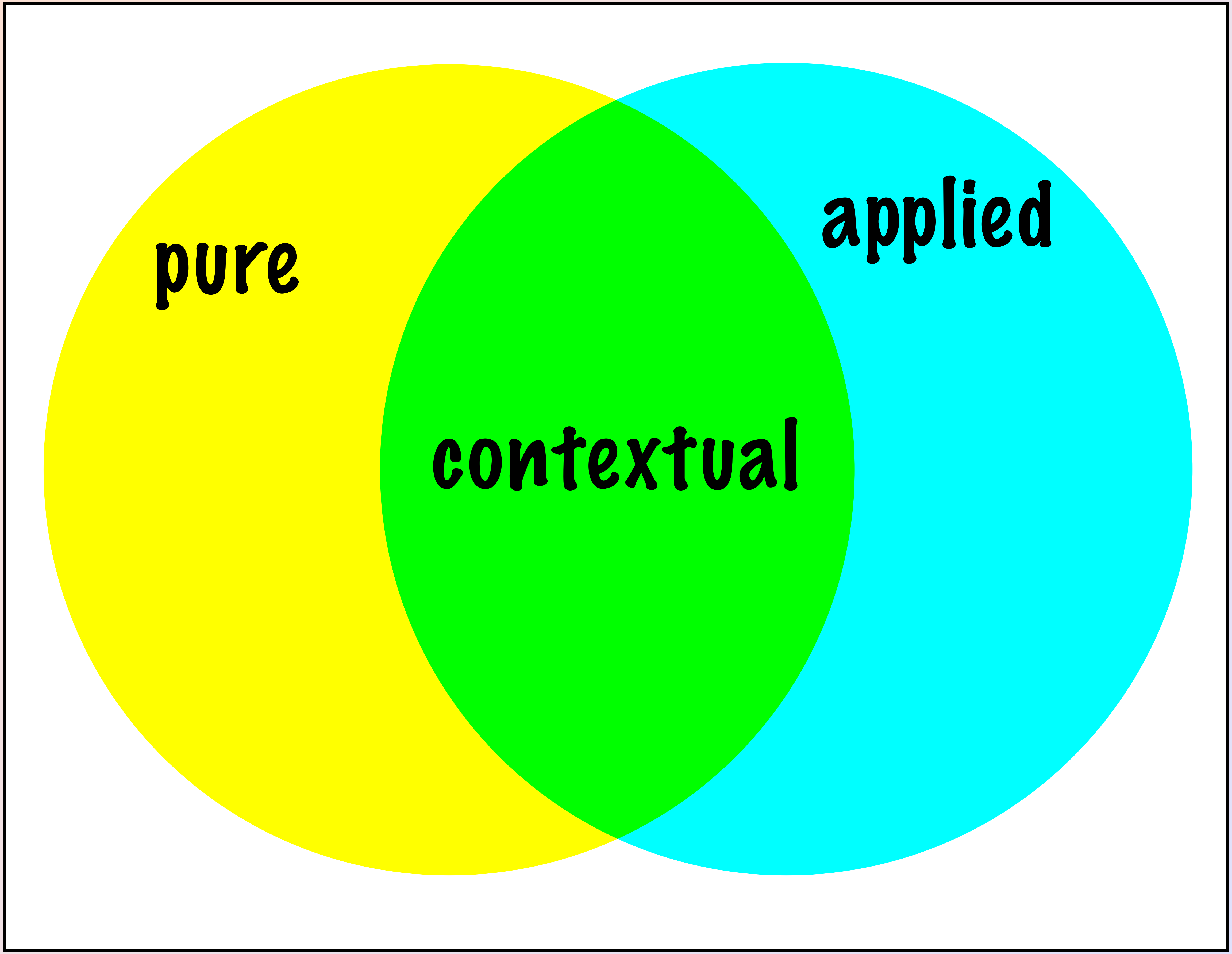
–David Webb
School of Education
University of Colorado Boulder

"Mathematicians love mathematics and want other people to love it too.... But our problem is to get a hook into students who aren't already 'into' mathematics."

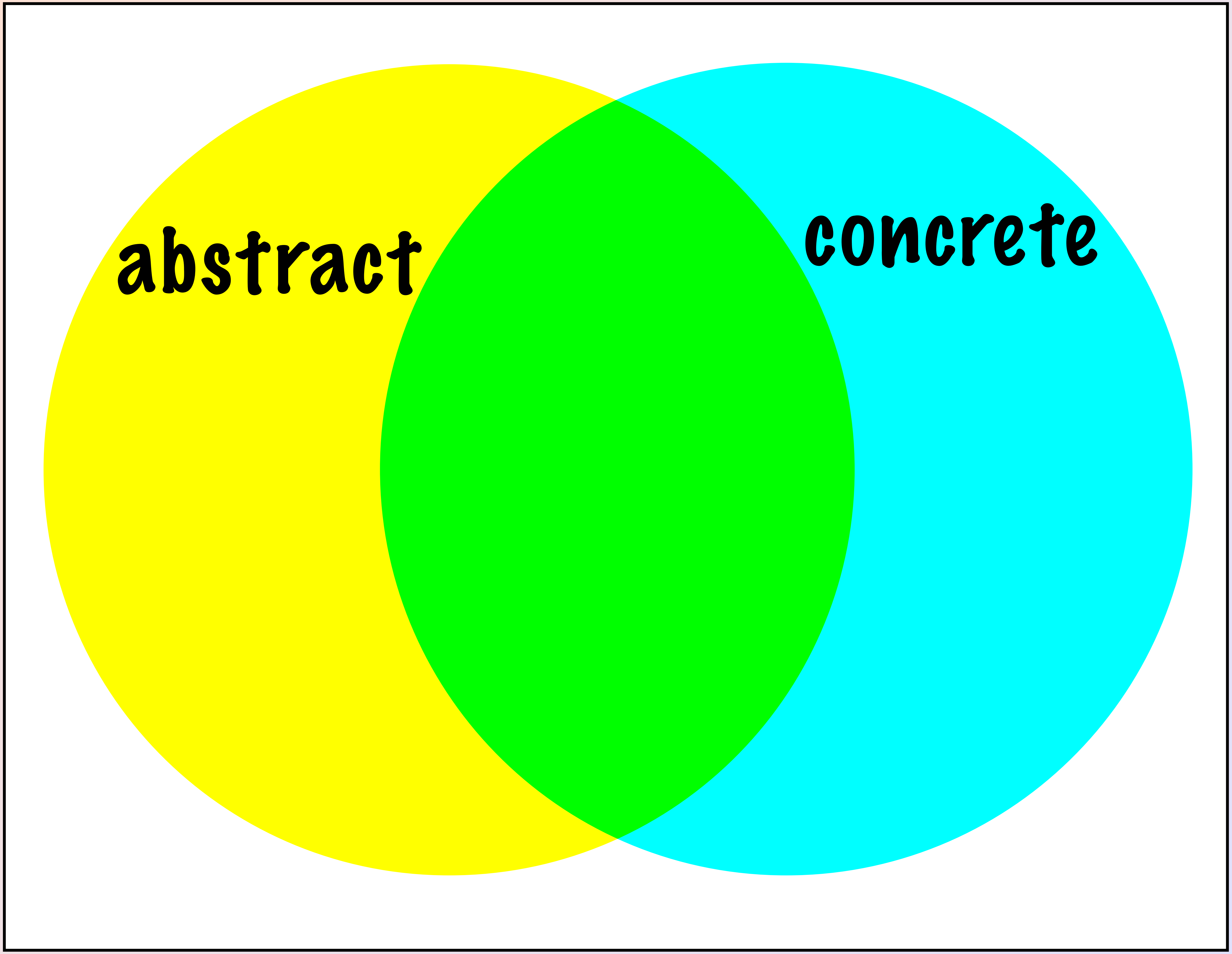
–Peter March, Dean
School of Arts and Sciences
Rutgers University

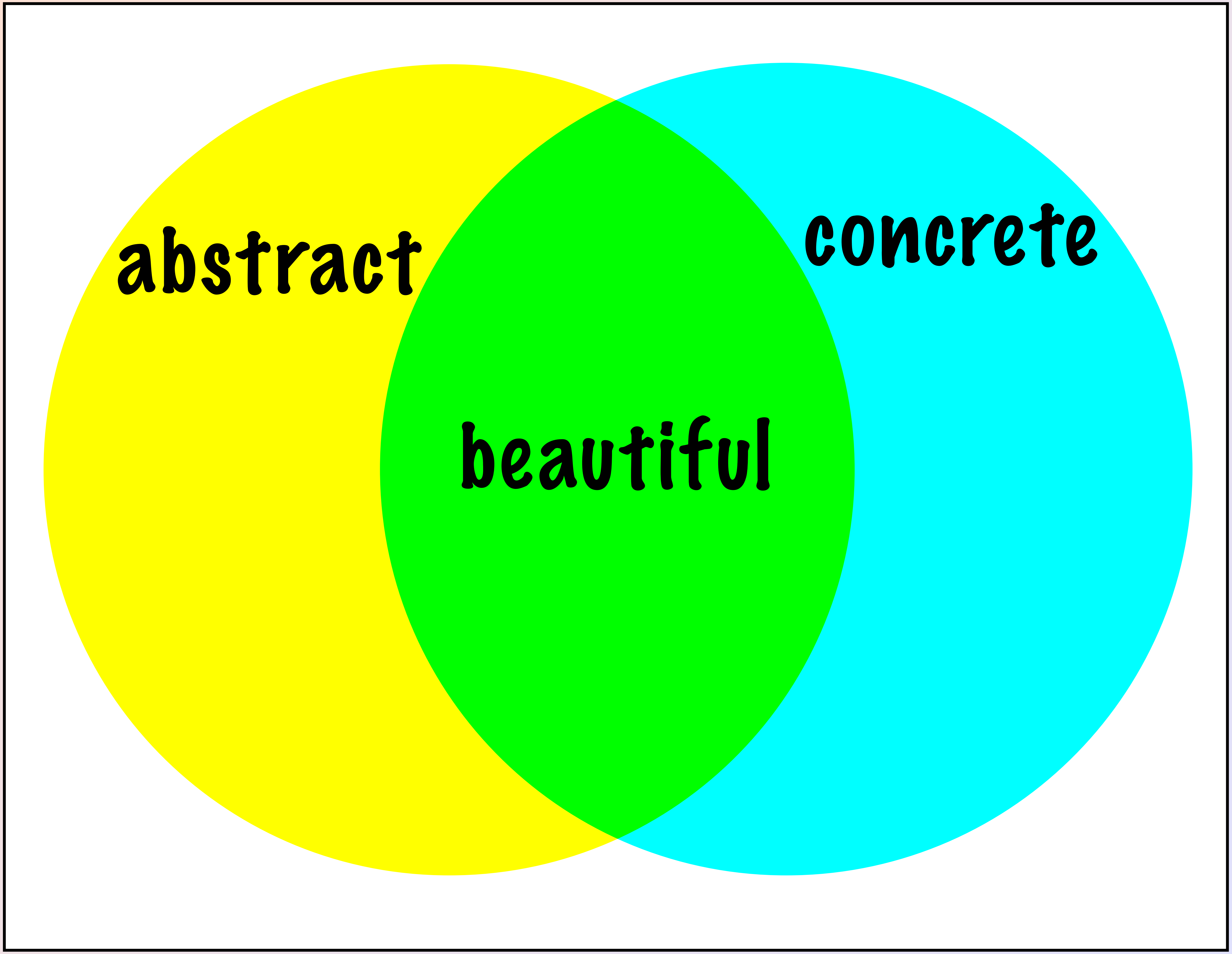

pure


pure
applied


pure
applied
contextual

abstract


abstract
concrete


abstract
beautiful
concrete

PART I.

CALCULUS: WHAT IT IS (The definition)

Definition:

Calculus *is* the set of mathematical manifestations and implications of the fact that:

If you know how fast you're going, then you know how far you'll get in a given amount of time.

In symbols:

$$\Delta S = v\,\Delta t$$

In symbols:

Δ denotes change

$$\Delta S = v\,\Delta t$$

In symbols:

Δ denotes change

$$\Delta S = v\,\Delta t$$

S denotes position

In symbols:

Δ denotes change

v denotes velocity

$$\Delta S = v\,\Delta t$$

S denotes position

In symbols:

Δ denotes change

v denotes velocity

$$\Delta S = v \underbrace{\Delta t}$$

S denotes position

Δt denotes elapsed time

More generally:

if a quantity y (like position) depends on another quantity x (like time), then the change in y equals the rate of change of y with respect to x, denoted y', times the change in x.

In symbols:

$$\Delta y = y' \Delta x$$

In symbols:

change in y

$$\overbrace{\Delta y} = y' \Delta x$$

In symbols:

change in y

$$\overbrace{\Delta y}=\underbrace{y'}\Delta x$$

rate of change of y
with respect to x

In symbols:

$$\overbrace{\Delta y}^{\text{change in } y} = \underbrace{y'}_{\substack{\text{rate of change of } y \\ \text{with respect to } x}} \overbrace{\Delta x}^{\text{change in } x}$$

In symbols:

$$\overbrace{\Delta y}^{\text{change in } y} = \underbrace{y'}_{\substack{\text{rate of change of } y \\ \text{with respect to } x}} \overbrace{\Delta x}^{\text{change in } x}$$

The prediction equation (PE)

Notes:

(1) Why "prediction"? Because (PE) allows us to step into the future, Δx units at a time!

(2) Typically, y' is itself changing, so (PE) requires some thought and interpretation (maybe the "$=$" there should be "$\approx$"). (This note represents the real *meat* of calculus.)

(3) If your future of interest is more than Δx units away, you could (a) choose a larger Δx, OR (b) apply (PE) iteratively!

Because y' itself varies, choice (b) here gives better approximations to the "true" future. (If your rate of change is constantly changing, then the more often you recalibrate it, the better.) This comes at the cost of increased computation, but this is less of a deal these days, with calculators and computers and the like.

(4) Of course, you *do* need to know "how fast you're going;" that is, you need to know y'!

Information about a rate of change y', or about several related such rates, is called a *dynamical system.*

MORAL: equipped with a dynamical system, initial conditions (so you know where to start), and (PE), we *can* predict.

(5) As a formal equation, (PE) is behind just about every neat idea in calculus, including: average versus instantaneous rates of change, derivatives, linear approximation, The Chain Rule, l'Hôpital's rule, The Mean Value Theorem, Newton's Method, integrals, Taylor polynomials, Taylor series, etc., etc.

PART II.

CALCULUS: WHAT IT HAS BEEN

A brief, imagined history,

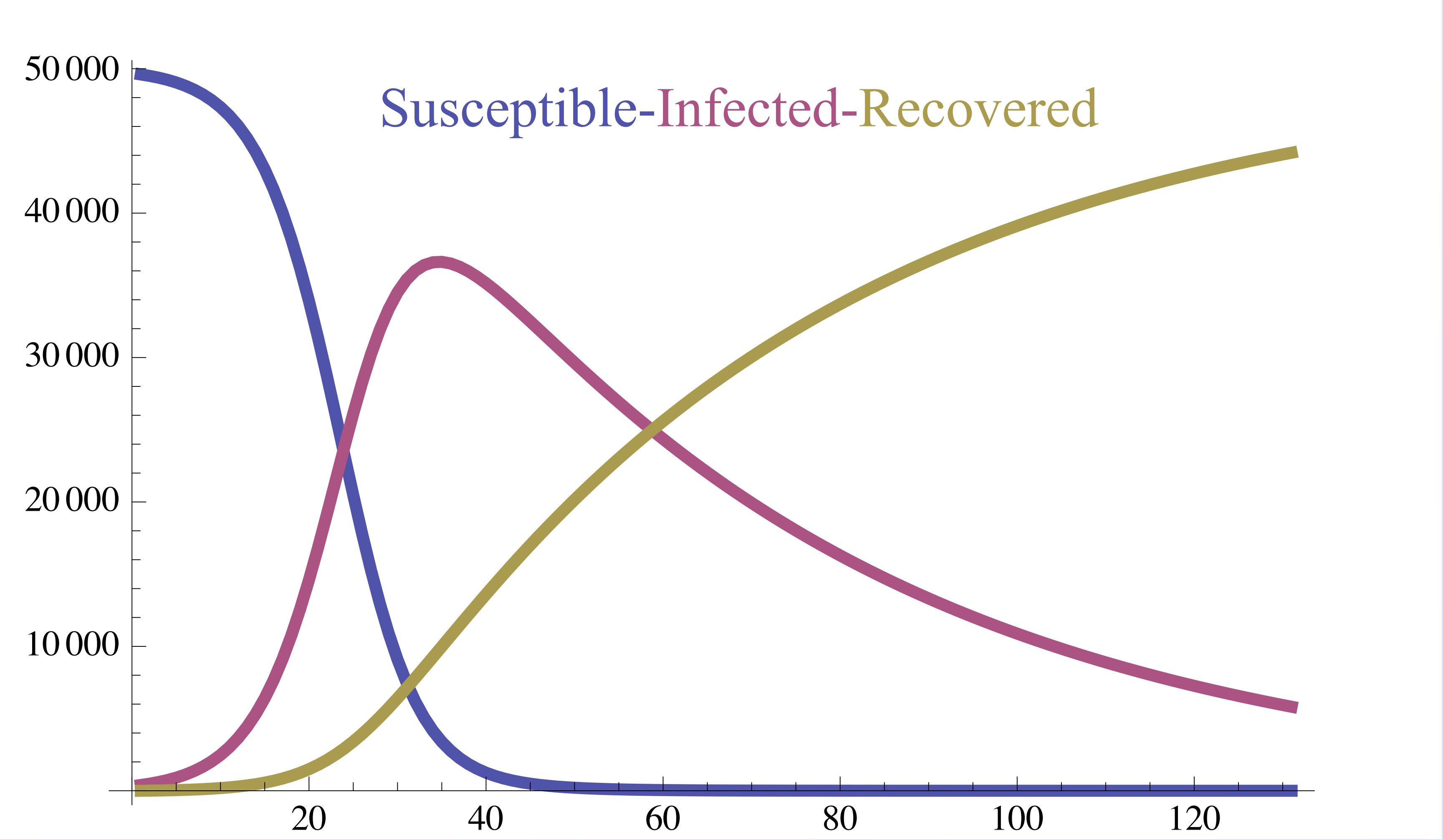
from SIR

to

S-I-R

from SIR Isaac Newton
to
S-I-R

from SIR Isaac Newton
to
S-I-R



Here's how it happened:

- Isaac Newton developed calculus (ca. 1666-1687) as a means of understanding motion, gravitation, and the like. (Calculus was developed more or less independently by Gottfried Leibniz, around the same time.)
- For the next ∼150 years, many (Euler, Lagrange, Gauss, Laplace, Poisson, etc.) used *and* developed calculus, in natural contexts.

- **Around 1820, in response to some bold mathematical claims made by the physicist Joseph Fourier (in the context of the theory of heat), a new era of formalism in mathematics arose, and gave rise to numerous entirely new mathematical subdisciplines. (Cf. Riemann, Cauchy, Weierstrass, Cantor, Lebesgue, etc., etc.)**

- **Formalism became the *metier* of choice in much of mathematics, and in particular, in the teaching of calculus.**
- **In this formal approach to calculus, the notions cited in note (5) above are often developed from axioms and definitions, more or less independently of each other, and generally without explicit reference to (PE). Moreover, dynamical systems are treated cursorily, and far into the semester, if at all.**

- **Back to the future/forward to the past:** **A new approach (with seeds in the 1980's text** *Calculus in Context* **by Callahan et al.). Here's the idea:**
  - **The course** *begins* **with a discussion of the spread of disease in a population, by stipulating assumptions about the behavior of the disease, and developing, from these assumptions, a dynamical system describing evolution of the susceptible, infected, and recovered subpopulations.**

Modeling a disease using SIR.

I) Initial set-up.

S: # of susceptibles
I: # of infected
R: # of recovered

S': rate of change of S
I': rate of change of I
R': rate of change of R

Assumptions:

- Everyone infected recovers eventually.
- The duration of infection is the same for everyone.
- Once recovered, you're immune and can't infect.
- Only a fraction of contacts with the disease cause infection.
- The units are:
  - days for time t;
  - people for S, I, and R;
  - people/day for S', I', and R'.

II) Thinking about the rates of change S', I', and R'.

(a) R':

Say the disease lasts k days. Then each day, on average, the number recovered will increase by $1/k$ times the size of the infected population. So

$$R' = \frac{1}{k} I = bI \qquad \text{where } b = 1/k.$$

Note: b is constant with respect to t. We say b is a parameter. We also say R' is proportional to I.

(b) S'.

Suppose:

(i) Each susceptible has contact with a fraction, call it p, of the infected population on a given day. Since the number of possible S-to-I contacts on a given day is $S \cdot I$, this means the number of actual S-to-I contacts on a given day is pSI.

(ii) A fraction, call it q, of such contacts yield infection.

Together, (i) and (ii) mean $qpSI$ new infections each day, meaning S decreases by $qpSI$ each day. So:

$$\boxed{S' = -qpSI = -aSI} \quad \text{where } a = pq.$$

(The minus sign reflects the decrease in S.)

Here a, p, and q are all parameters.

(c) I':

Assuming the total population $S+I+R$ stays constant, the changes in S, I, and R must cancel, meaning $S'+I'+R'=0$, so

$$I' = -S' - R'$$

or, by (a) and (b) above,

$$\boxed{I' = aSI - bI.}$$

III) SUMMARY

Under the assumptions described above, we have

$$\begin{aligned} S' &= -aSI \\ I' &= aSI - bI \\ R' &= bI \end{aligned}$$

SIR equations

Here:

- b (>0) is the recovery coefficient. Units: day^{-1}.
- a (>0) is the transmission coefficient. Units: 1/(person-day).

Question: so what?

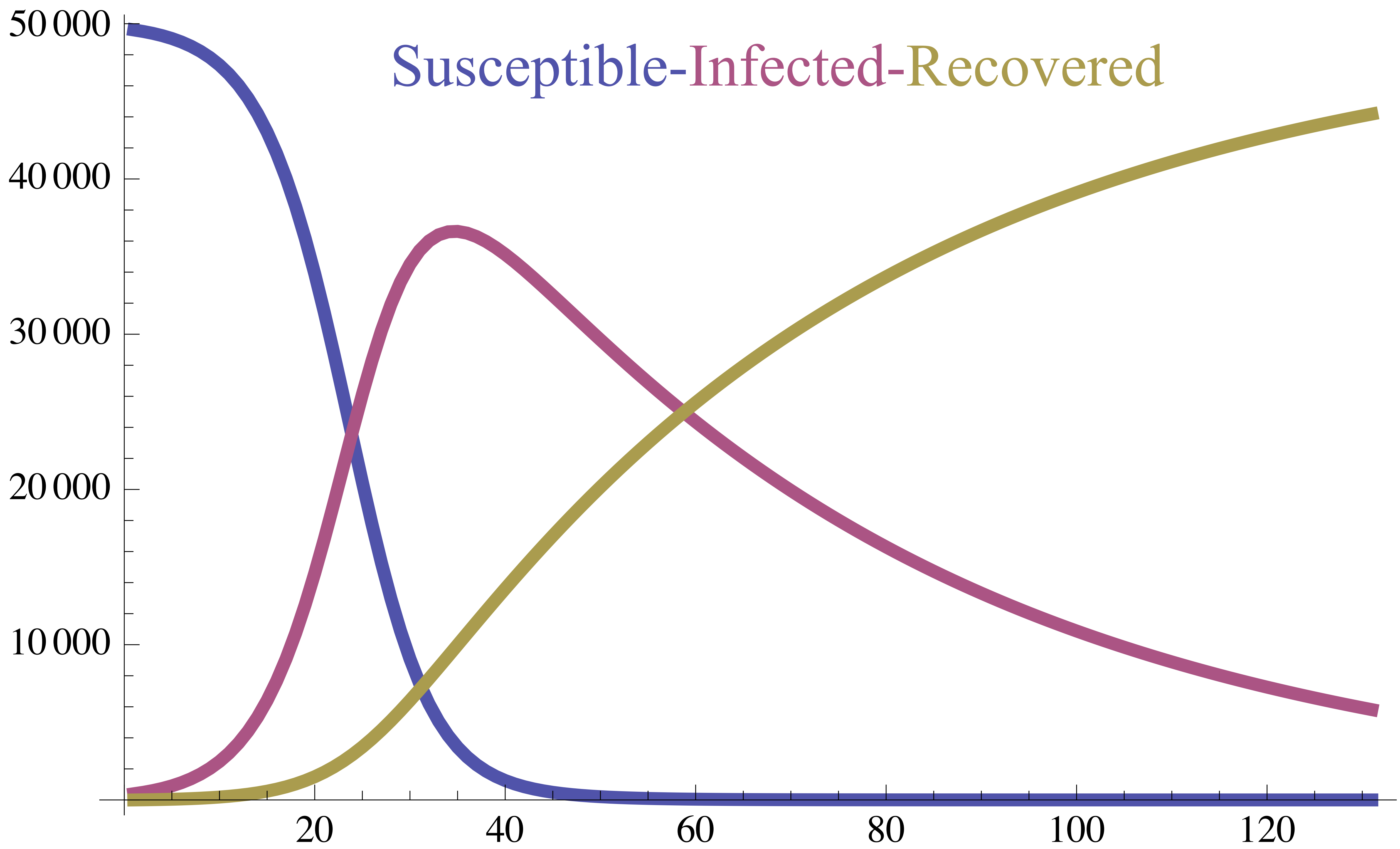
Answer: prediction. More on this soon.

Now, we can chart evolution of the disease iteratively, as follows ("Euler's method"):

* **Use initial conditions on S, I, R, plus the dynamical system, to compute initial values of S', I', R'.**
* **Use the prediction equation to compute the net *change* in S, I, R over a time interval of length Δx.**
* **Add the net changes to the initial values to predict S, I, R Δx units later.**
* **Use the newly computed values as the new "initial" values, and repeat as necessary!!**

As noted above, good predictions require small Δx and therefore, if one is to predict far into the future, repeated calculation. You need a computer! So programming is taught to facilitate iterated implementation of (PE).

The resulting predictions are graphed, yielding solutions like the one depicted above:


Susceptible-Infected-Recovered
50 000
40 000
30 000
20 000
10 000
20
40
60
80
100
120

- **Now that the significance of (PE) has been firmly, contextually established, all of the above-noted neat consequences of (PE) can be developed and *really* appreciated.**

  (The "applications" are applied to the unfolding of the mathematics!)

EXAMPLES OF KEY CALCULUS CONCEPTS as consequences of the prediction equation

$$\Delta y \approx y' \Delta x.$$

We can:

DIVIDE BOTH SIDES BY Δx to get

$$y' \approx \frac{\Delta y}{\Delta x} :$$

Average rates of change approximate instantaneous rates of change.

Next, we can:

TAKE LIMITS AS $\Delta x \to 0$ to get

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} :$$

Definition of the derivative (=instantaneous rate of change).

Or, we can:

Evaluate the above equation $y' \approx \Delta y/\Delta x$ at the appropriate point c, and replace the "$\approx$" with "$=$," to get

$$y'(c) = \frac{\Delta y}{\Delta x}:$$

The Mean Value Theorem.

Or, we can:

Consider the ratio $\Delta y_2/\Delta y_1$, for two different functions y_1 and y_2 of x, to get

$$\frac{\Delta y_2}{\Delta y_1} \approx \frac{y_2' \Delta x}{y_1' \Delta x} = \frac{y_2'}{y_1'}.$$

If y_1 and y_2 are both zero at some common value of x, say at $x = a$, then we can write this as

$$\frac{y_2}{y_1} \approx \frac{y_2'}{y_1'}.$$

Taking limits as $x \to a$ turns the "$\approx$" into "$=$," and thus gives l'Hôpital's rule.

Finally (for now), we can:

Add up many many copies of the prediction equation $\Delta y \approx y'\Delta x$, over many many contiguous intervals from a to b (and take limits as $\Delta x \to 0$), to get

$$y(b) - y(a) = \int_a^b y'\,dx :$$

the Fundamental Theorem of Calculus.

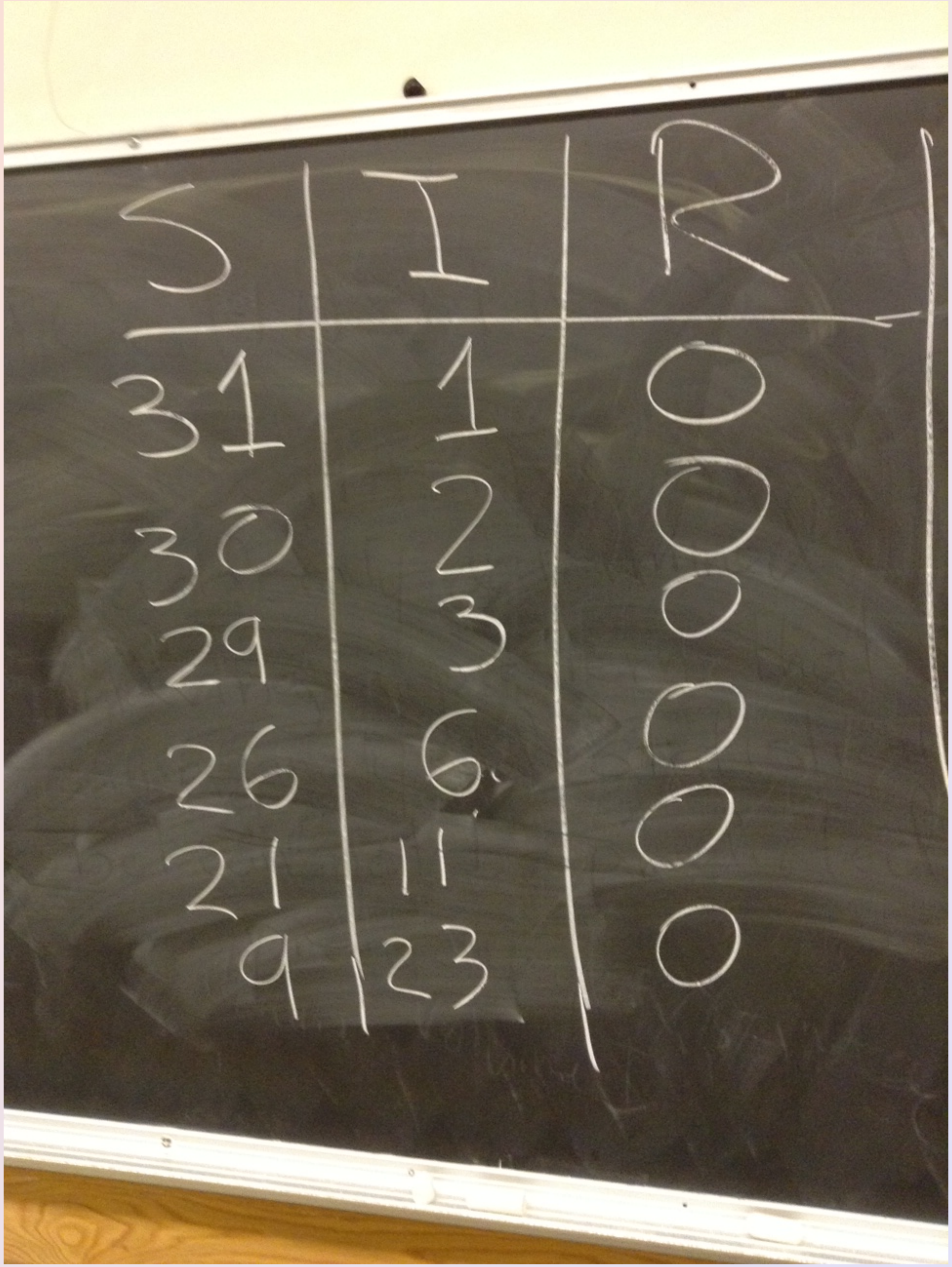
(In terms of computational thinking, the prediction equation is a sort of subroutine.)

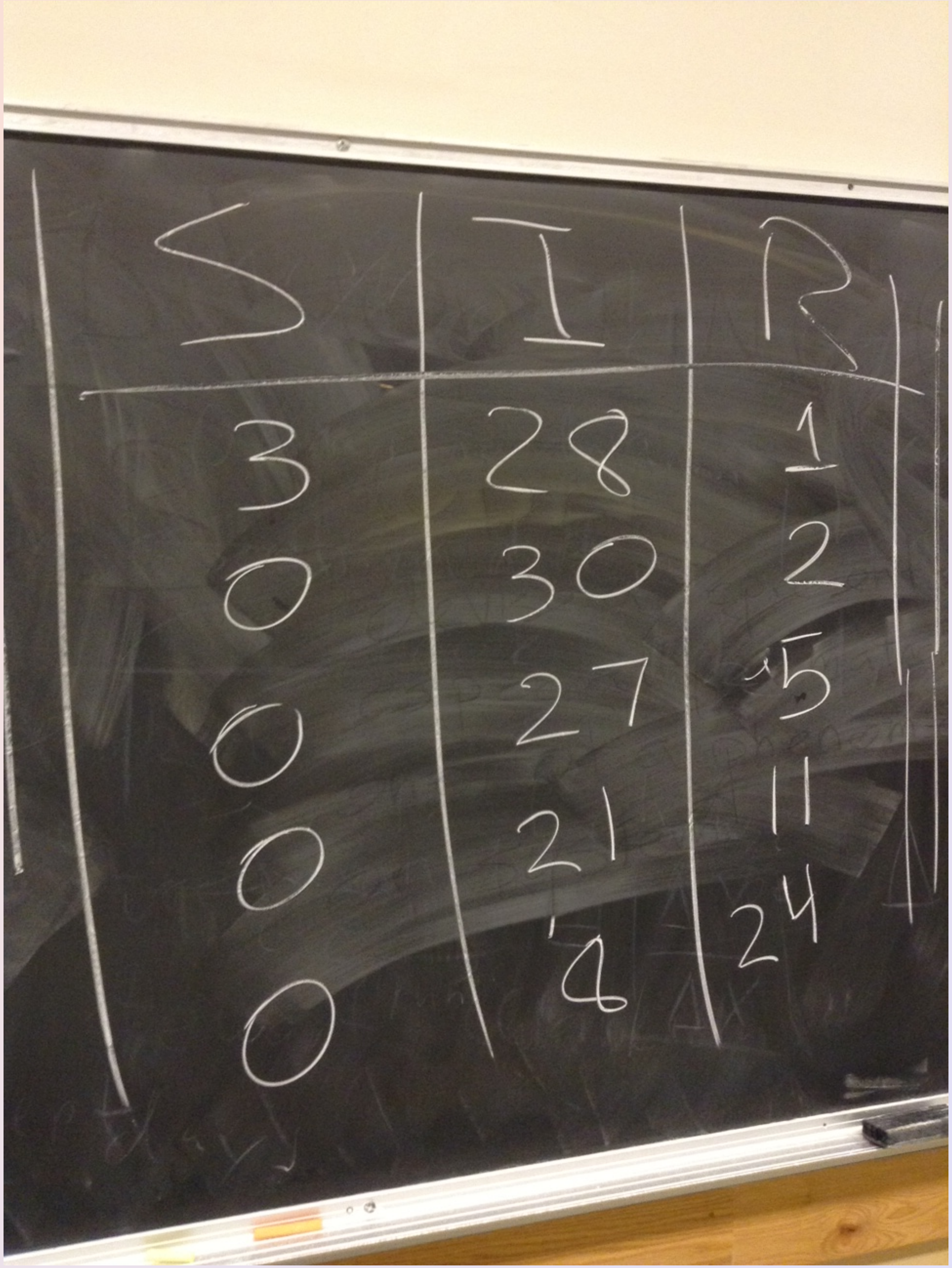
"A mathematical idea. . . is part of an internal network. . . . The degree of understanding is determined by the number and the strength of the connections."

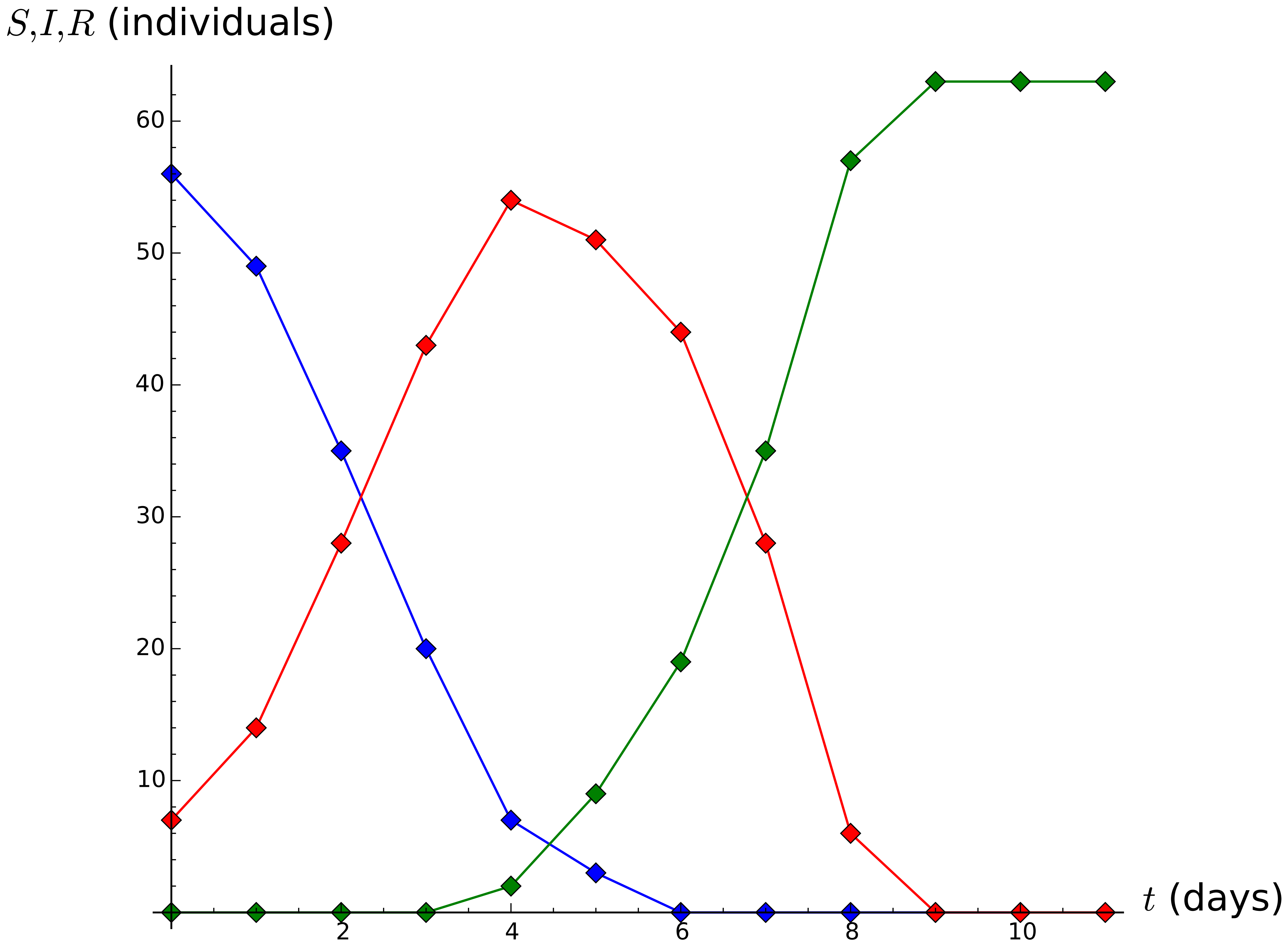
Learning and Teaching with Understanding,
Hiebert and Carpenter (1992)

PART III.

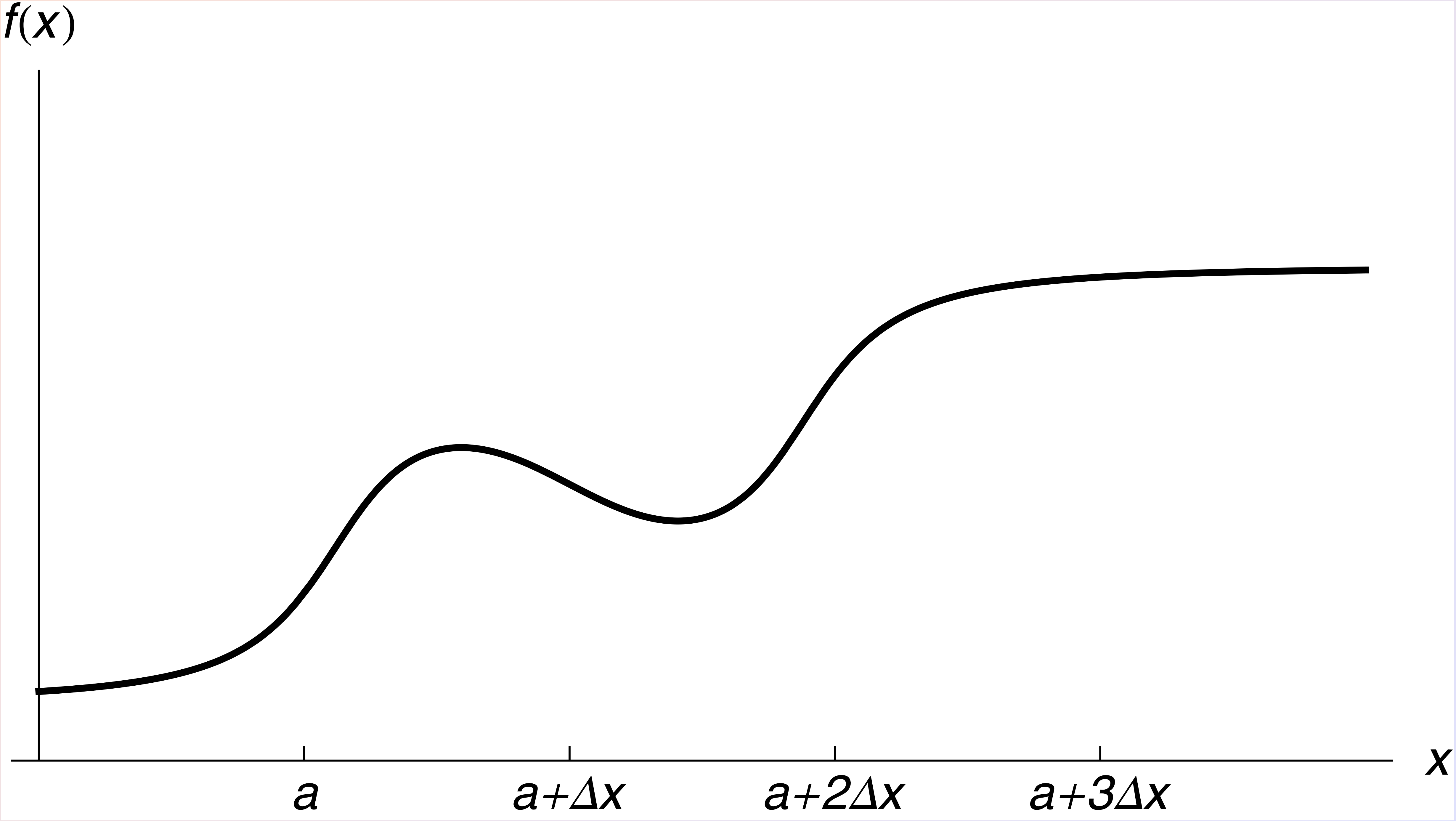
SOME SAMPLE CSM TASKS


S I R
31 1 0
30 2 0
29 3 0
26 6 0
21 11 0
9 23 0

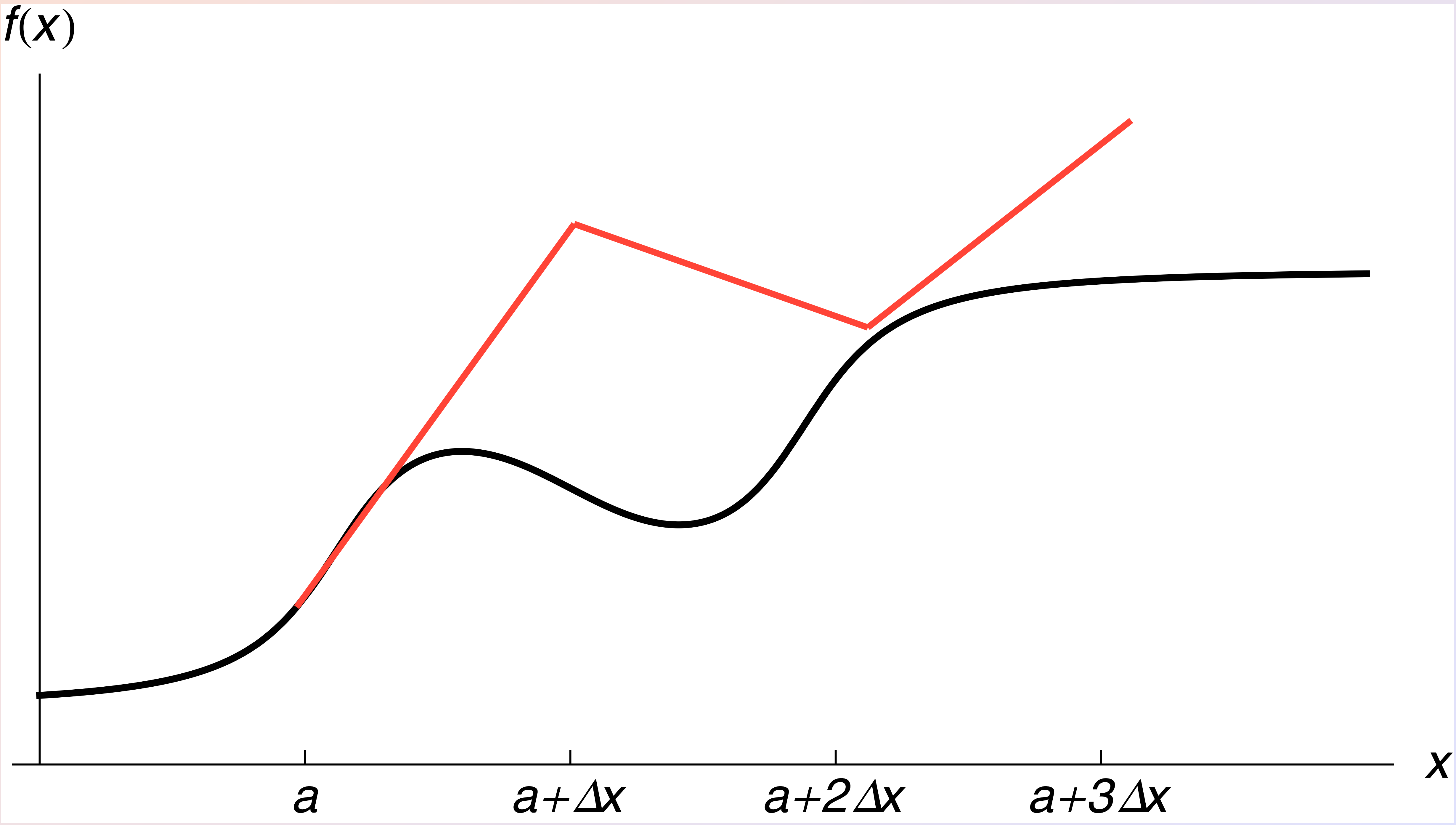

S I R
3 28 1
0 30 2
0 27 5
0 21 11
0 4 24


S,I,R (individuals)
60
50
40
30
20
10
2
4
6
8
10
t (days)

On the graph below, draw a picture indicating what approximation to $f(a+\Delta x)$, $f(a+2\Delta x)$, and $f(a+3\Delta x)$ you would get using Euler's method (starting at $x=a$ and using stepsize Δx). Please *explain your picture and your ideas briefly* in the space below.



On the graph below, draw a picture indicating what approximation to $f(a + \Delta x)$, $f(a + 2\Delta x)$, and $f(a + 3\Delta x)$ you would get using Euler's method (starting at $x = a$ and using stepsize Δx). Please *explain your picture and your ideas briefly* in the space below.



Today's tutorial: fermentation

Wine is produced by adding yeast to grape juice. The yeast, Y, digests the sugar, S, in the juice, yielding alcohol, A, as a by-product. The alcohol is, in turn, toxic to the yeast. In this way, the yeast, alcohol, and sugar interact until equilibrium is produced.

Create a system of differential equations that model the interaction of Y, A, and S, assuming that:

Today's tutorial: fermentation

- In the absence of other factors, yeast grows logistically, with carrying capacity proportional to the amount of sugar present.
- Yeast dies at a rate proportional to the amount of alcohol present *times* the amount of yeast present.
- Alcohol is produced at a rate proportional to the amount of yeast present.
- Sugar is consumed at a rate proportional to the amount of yeast present.

$\frac{dY}{dt} =$	$kY\left(1-\frac{Y}{bS}\right)$	$-cAY$
$\frac{dA}{dt} =$	eY	$-dYS$
$-\frac{g}{1+hSY}$	$\frac{dS}{dt} =$	$-fY$
$+gS$	$kY\left(1-bSY\right)$	$+\alpha$

Other dynamical systems studied in the course:

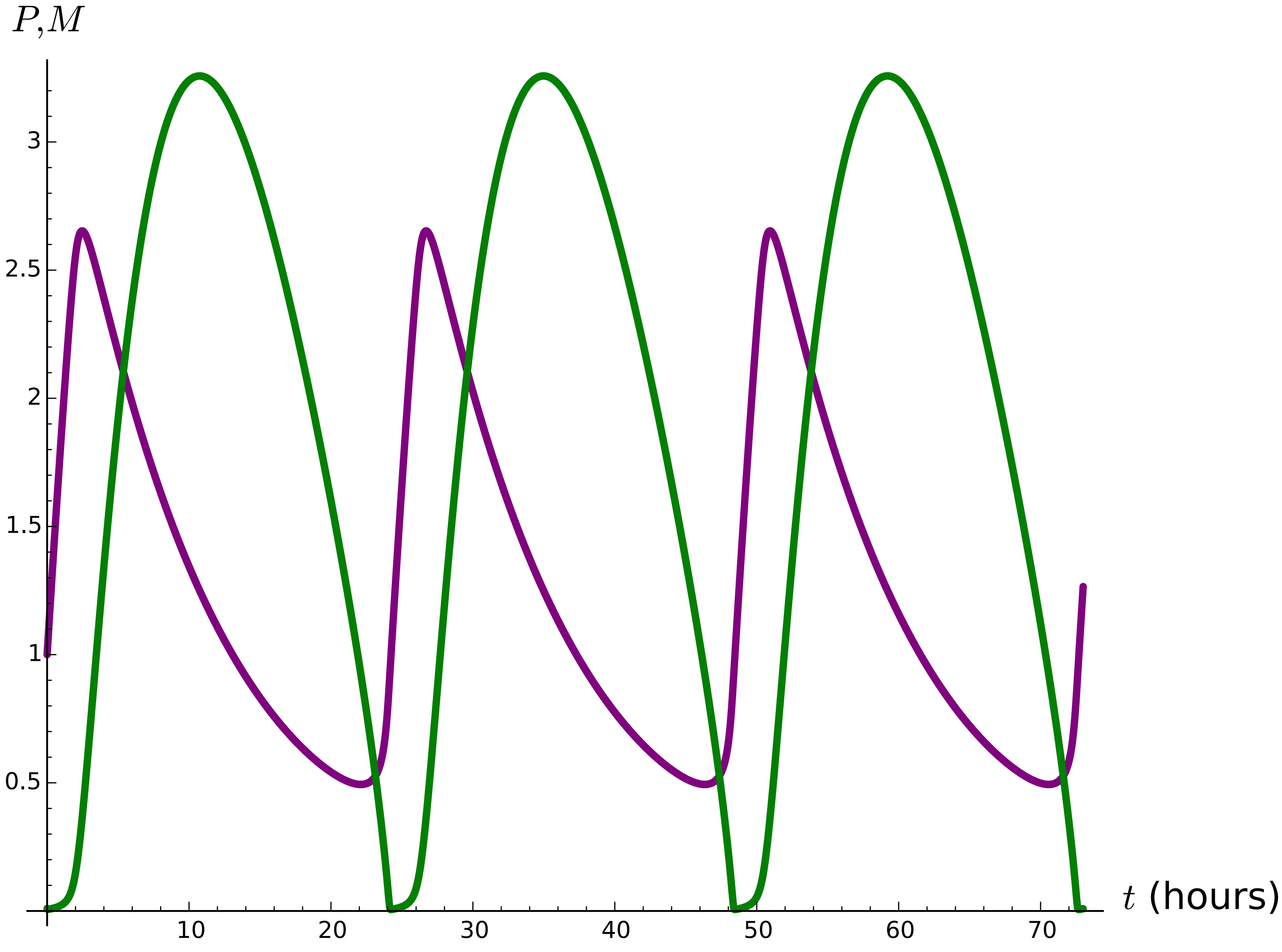
- Lotka-Volterra equations for predator/prey populations.
- Hodgkin-Huxley model for neural impulses.
- Circadian rhythms: interaction of per/tim mRNA with PER/TIM proteins.

$$
(CR)\begin{cases}
\dfrac{dM}{dt} = \overset{(i)}{\dfrac{a}{1+bP_2^2}} \overset{(ii)}{- cM} \\[2ex]
\dfrac{dP_1}{dt} = \overset{(iii)}{dM} \overset{(iv)}{- \dfrac{eP_1}{f+P_1+gP_2}} \overset{(v)}{-hP_1} \overset{(vi)}{-2kP_1^2} \overset{(vii)}{+2\ell P_2} \\[2ex]
\dfrac{dP_2}{dt} = \overset{(viii)}{-\dfrac{mP_2}{f+P_1+gP_2}} \overset{(ix)}{-nP_2} \overset{(x)}{+kP_1^2} \overset{(xi)}{-\ell P_2}.
\end{cases}
$$

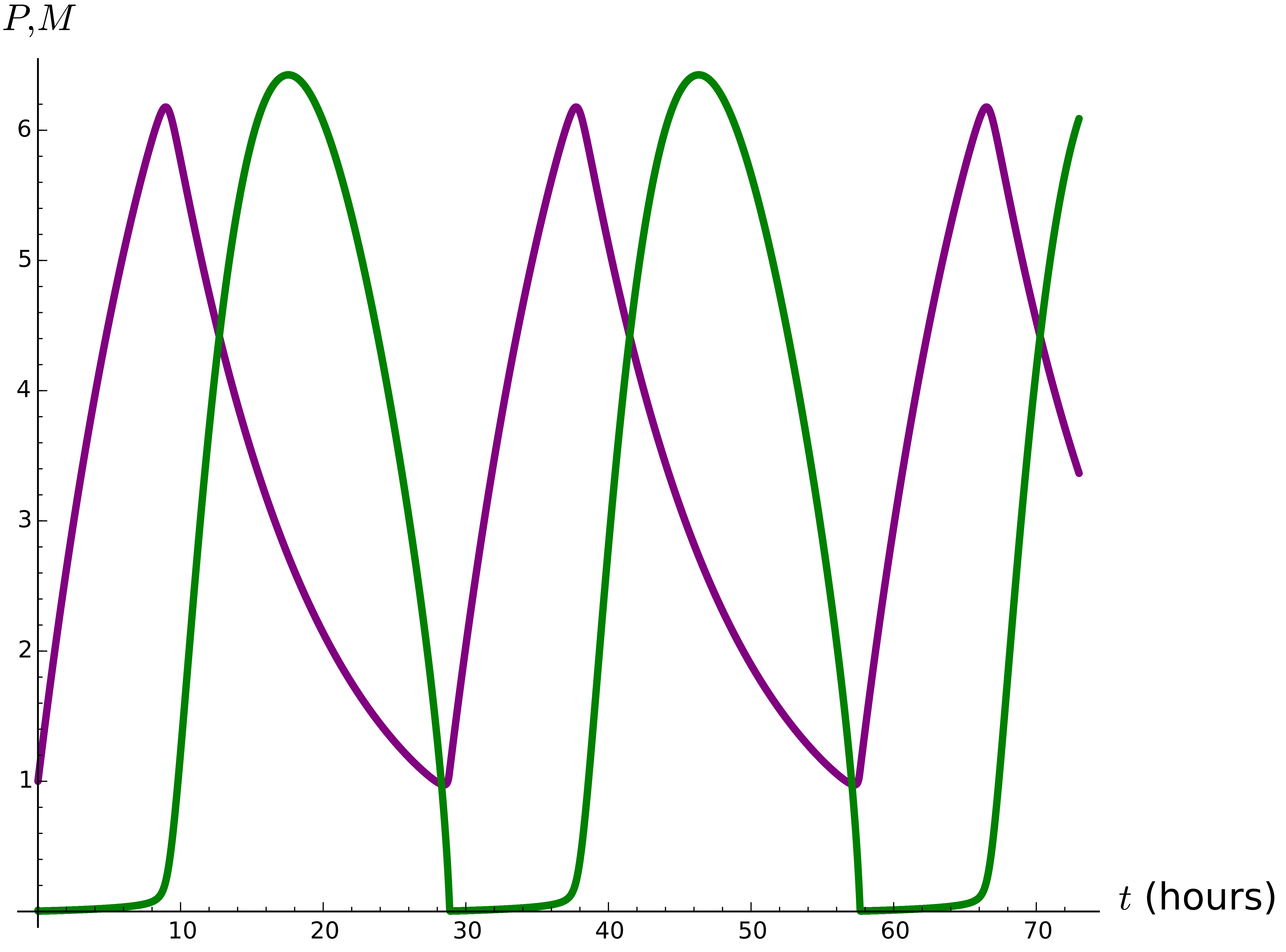
Analysis of terms:

(i) P_2 inhibits production of M: (i) tells us that more P_2 means slower growth of M.

(ii) M is degraded/used up at a rate proportional to the amount of M present.



P,M
3
2.5
2
1.5
1
0.5
10
20
30
40
50
60
70
t (hours)



P,M
6
5
4
3
2
1
10
20
30
40
50
60
70
t (hours)

CLOSING THOUGHT:

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Math is a creative endeavor!!!

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CSM “DIY Dynamical system” term project:

CLOSING THOUGHT:

Math is a creative endeavor!!!

CSM "DIY Dynamical system" term project:

model *whatever you want* **with a dynamical system!!**

Prof. Stade
12-9-15

A Sample DIY Modeling Mini-Project

We analyzed a race between three extremely mental athletes. Our first competitor is Discouraged Dan, a snail who enters as the undisputed underdog. Everybody loves a Cinderella story, but for this snail who prefers to go by "DD", an upset is unlikely. Next we have a veteran crowd favorite, Timmy Two-Toes, the sloth. He is one of the animal kingdom's most respected low-speed athletes, and is racing for his world record 10th win. Rounding out the pack is Greasy Garry, a turtle who makes up for his lack of speed with cunning, and trickery. As Timmy once said, "If Garry ain't cheatin', then you've got the wrong Garry, because this Gary is seriously always cheating. "
As mentioned, these are incredibly mental athletes, whose performances depend heavily on their states of mind.

Here's what we know:
--We can count on Greasy Garry timing the starting shot all too well, and getting a (relative) running start, at a (relatively) high speed of 15 meters/hour.
--Dan complains his way into a sliding start, and with a boost from Tanner Ten Point, the buck, our underdog snail starts the race at a cool 10 meters/hour.
--We see so many athletes use the "everyone does it" excuse, but expect Timmy Two-Toes to start standing still, in a display of true veteran class.
--Absent of other factors, Dan's speed increases logistically with a growth rate of .001, with a maximum speed that is proportional to Garry's speed. The mental aspect of racing comes into play here. Discouraged Dan doesn't get his name for just any reason. The faster his opponents get, the worse he performs. His maximum performance is essentially limited by Garry's success. Discouraged Dan really looks up to Timmy, and is negatively affected by the sloth's stellar performance. Specifically, he loses speed at a rate proportional to .001 of Timmy's speed, in proportion to his own speed.
--Timmy displays the extreme drive necessary for success in the competitive low-speed racing world. He couldn't care less about the slow snail to his right. But, Garry got a head start, and Timmy Two Toes is motivated and prepared. The determined sloth increases his speed at a rate proportional to 2% of Garry's speed.
--Garry normally couldn't keep the speed with which he started. Absent outside factors, he would lose his speed at a rate of .25 meters/hour per foot. However, he is a competitive animal as well. As Timmy gets started, and especially as he accelerates quickly behind, Garry speeds up accordingly. In an attempt to hold his lead, Garry increases his speed at a rate proportional to 4% of Timmy's speed.

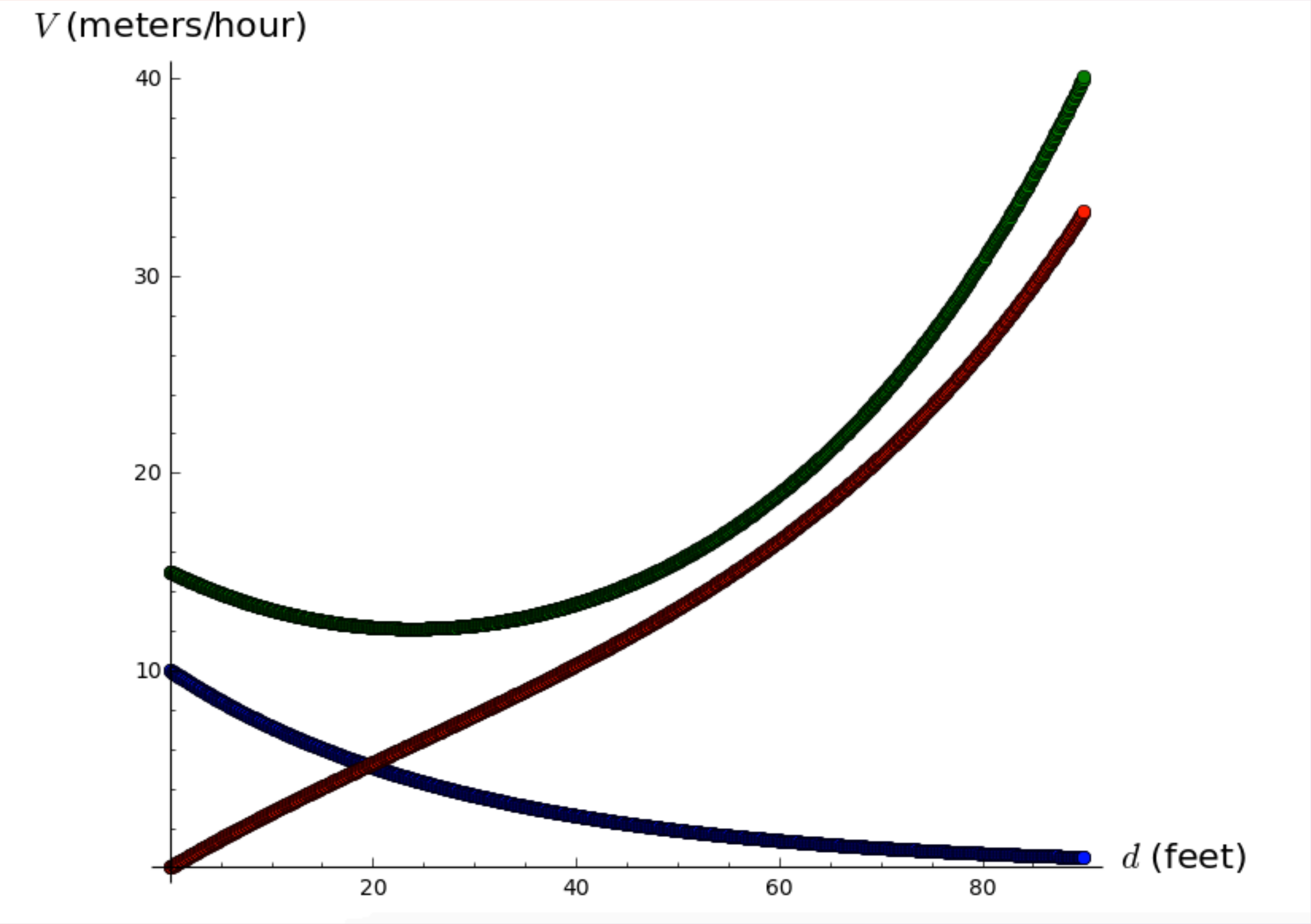
Differential Equations

```
Dprime=k*D*(1-D/(b*G))-(a*A*D)
Tprime=z*G
Gprime=-e+(.04*T)
```

Sage Code

```
# Race Speed Analysis

# Starting and ending points, stepsize, and total number of observation points
dstart=0
dfin=90
stepsize=0.1
length=(dfin-dstart)/stepsize+1

# Next, specify the intial speed of Dan; and the growth rate k; initial speed of Garry;initial speed
of Timmy; and constants a, z, and e
D=10
T=0
G=15
a=.001
b=.4
z=.02
e=.25
k=.001
d=dstart

# Next we create lists to store our computed values of d and D and T and G
Dvalues=[]
Tvalues=[]
Gvalues=[]
dvalues=[]



for i in range(length):

    # Store current values
    Dvalues.append(D)
    Tvalues.append(T)
    Gvalues.append(G)
    dvalues.append(d)
```

Graph and Analysis



Garry got his head start, but then began to slow, as predicted. But, once Timmy started to accelerate we see Garry pick his speed up. Timmy has the fastest acceleration, as the slope of his speed is the greatest, but with Garry's head start, Timmy never catches up. His speed never even reaches a higher point than Garry's. Dan, our discouraged snail, lives up to his name. He too had a head start, but it didn't do him much good. As soon as he saw his competitors take off, he decelerated quite rapidly. Timmy Two-Toes will live to race another day, and hopefully get his world record. But for now, Greasy Garry is the low speed champion of the animal kingdom.

The Demographic of the AirPod

The scenario we chose to model is the demographic of Apple's infamous AirPods. This phenomenon started about a year ago as AirPods began to get more exposure due to celebrities and influencers. Since then, it has been an ongoing meme that the population of people who don't own AirPods (The Average Joe) are of a lower class than that of the population that do (The Transcended). Due to this joke, there has been a surge in sales of Apple's AirPods, which in turn has lead to a decrease in the Average Joe population and an increase in the Transcended population. Because of this massive popularity, there are some AirPod owners that have grown tired of the hype and have decided to retire their AirPods in favor of less mainstream earbuds (The Over-Its). In the equations below we have attempted to model this scenario.

Initial Conditions and Parameters:

J - 1000 individuals (The Average Joe) Population of people who don't own AirPods

T - 500 individuals (The Transcended) Population of people who own AirPods

O - 0 individuals (The Over-Its) Population of people who have purchased AirPods, but are now over them

a = 0.0003 $(\text{individuals*days})^{-1}$ (Ownership Coefficient) Reflects the spread of AirPod exposure via pop culture (memes)

b = 0.05 days^{-1} (Renegade Coefficient) Reflects people who have grown tired of the hype and abandoned the Transcended population

c = 1000 individuals (Carrying Capacity) Reflects the max number people who can own AirPods

u = 0.08 days^{-1} (Updated Hardware Coefficient) Reflects the new versions drawing in Average Joes to purchase/repurchase AirPods

Explanation of Scenario Using Differential Equations

The rate at which the Average Joe population decreases is proportional to the Average Joe population and logistic in the Transcended population with carrying capacity equal to the amount of people who can own AirPods, ‘c’. This represents the rate at which a proportion of the Average Joe population obtain AirPods, hence becoming Transcended. The Transcended population grows at the same rate that the Average Joe population decreases. The Transcended population rate is also affected by the Renegade Coefficient, ‘b’, where after a certain amount of days a proportion of the Transcended population is taken in by the Over-It population. The Over-It population then grows at a rate proportional to the Transcended population. At a point in the Transcended population, the Updated Hardware Coefficient ‘u’ of the Over-It population is added back into the Average Joe population, which represents a proportion of the Over-It population becoming susceptible to purchasing new AirPods after Apple releases a new edition. The spread of AirPod exposure in pop culture though memes can increase the Transcended population and take from Average Joe population, which is reflected by the Ownership Coefficient ‘a’. Updated Hardware, ‘u’, increases the Average Joe population and could negatively affect the Over-It population.

Differential Equations

$J' = -a(J*T)(1-(T/c)) + u*O$

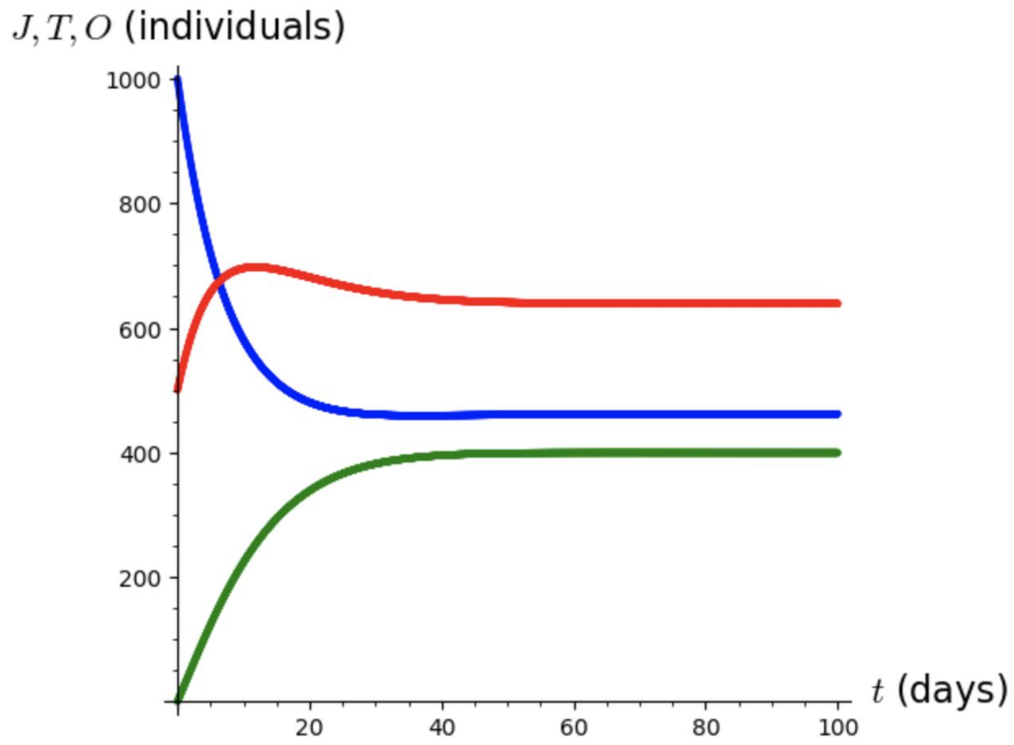
$T' = a(J*T)(1-(T/c)) - b*T$

$O' = b*T - u*O$

Sage Code

```
    # JTO program, for studying an epidemic of AirPods using Euler's method

    # First, specify the starting and ending points, stepsize, and total number of observation points
    tstart=0
    tfin=100
    stepsize=0.05
    length=((tfin-tstart)/stepsize)+1

    # Next, specify values of parameters, and initial values of variables

    a=0.0003
    b=.05
    c=1000
    u=.08
    J=1000
    T=500
    O=0
    t=tstart

    # Set up empty lists for the values we're about to compute

    Jvalues=[]
    Tvalues=[]
    Ovalues=[]
    tvalues=[]

    # The following loop does three things:
    # (1) stores the current values of J, T, O, and t into the lists created above;
    # (2) computes the next values of J, T, O using Euler's method;
    # (3) increases t by the stepsize

    for i in range(length):

        # Store current values

        Jvalues.append(J)
        Tvalues.append(T)
        Ovalues.append(O)
        tvalues.append(t)



    # Compute rates of change using JTO equations

    Jprime=-a*(J*T)*(1-(T/c))+u*O
    Tprime=a*J*T*(1-(T/c))-b*T
    Oprime=b*T-u*O

    # Net change equals rate of change times stepsize

    DeltaJ=Jprime*stepsize
    DeltaT=Tprime*stepsize
    DeltaO=Oprime*stepsize

    # New values equal current values plus net change

    J=J+DeltaJ
    T=T+DeltaT
    O=O+DeltaO
    t=t+stepsize

    # Next time through the loop, the above new values play the role of current values

# Zip the t values with the J/T/O values into lists of ordered pairs, and create plots of these

Jplot=list_plot(list(zip(tvalues,Jvalues)),marker='o',color='blue')
Tplot=list_plot(list(zip(tvalues,Tvalues)),marker='o',color='red')
Oplot=list_plot(list(zip(tvalues,Ovalues)),marker='o',color='green')

# Now plot the computed J,T,O values together on a single graph, with axes labelled appropriately

JTOgraph=Jplot+Tplot+Oplot
show(JTOgraph,axes_labels=['$t$ (days)','$J,T,O$ (individuals)'])
```

Graph



Blue Line = Average Joe Population

Red Line = Transcended Population

Green Line = Over-It Population

Analysis of Results

We saw over the course of 100 days the relationships between the Average Joe, Transcended and Over-It populations in relation to the ownership of AirPods. At t=0, the Average Joe population began at 1000 individuals and then rapidly decreased until t=20 days at 450 individuals, leveling off for the remaining 80 days. The initial condition of the Transcended population was 500 individuals at t=0 days. Around t=10, this population peaks at 700

Setup:

We propose a term project involving the relationships between the money (M) in your bank account starting on payday, the mindset of frugality (F) while trying to conserve this money, and the stress (S) it may all cause. The money in your bank account decreases at a constant rate. Money has a minimum value of 0. Frugality increases at a rate dependant on both F and M. In particular: F' is proportional to M; F' is logistic in F, with carrying capacity given by F=15, since any more actions would be detrimental to an individual's health. Stress has a base value of 10 stressful thoughts a day since all college students deal with stress on a daily basis. Stress decreases at a rate proportional to both frugality and money left in the bank account. On a graph, the X-axis would be in units of time from payday (0 days) to day before payday (13 days). The Y-axis would be in units of thoughts and actions. Money is put into units of dollars in the bank account divided by 10. F' is affected by both money and the carrying capacity. $\frac{F}{15}$ represents how close the F value is to the carrying capacity and ensures that the carrying capacity will not be exceeded. The $\cos(2\pi t)$ in the S' equation changes stress to fluctuate throughout, this is part of the S' equation because stress changes throughout the day, and does not stay constant.

Initial Conditions

$M=33.3 \ \frac{Dollars\ \ in\ Bank\ Account}{10}$

$F=1 \ \frac{Frugal\ Actions}{Day}$

$S=10 \ \frac{Stressful\ Thoughts}{Day}$

Equations:

$F' = a*M*F*(1-(\frac{F}{15}))$

$M' = -c+r*F$

$S' = -e*(F-M)+h*\cos(2*\pi*t)$

Variables and Values:

$M = \frac{Dollars\ in\ Bank\ Account}{10}$

$F = \frac{Frugal\ Actions}{Day}$

$S = \frac{Stressful\ Thoughts}{Day}$

a= Proportional constant between frugality and stress

c= Monetary Constant ($\frac{Dollars\ Spent}{Day}$)

e= Constant Stress Parameter

h= Constant Parameter

t= Time passed (Hours)

Code:

```
# First, specify the starting and ending points, stepsize, and total number of observation points

tstart=0
tfin=13
stepsize=0.042
length=((tfin-tstart)/stepsize)+1

# Next, specify values of parameters, and initial values of variables

a=.03
```

Graphical Representation:

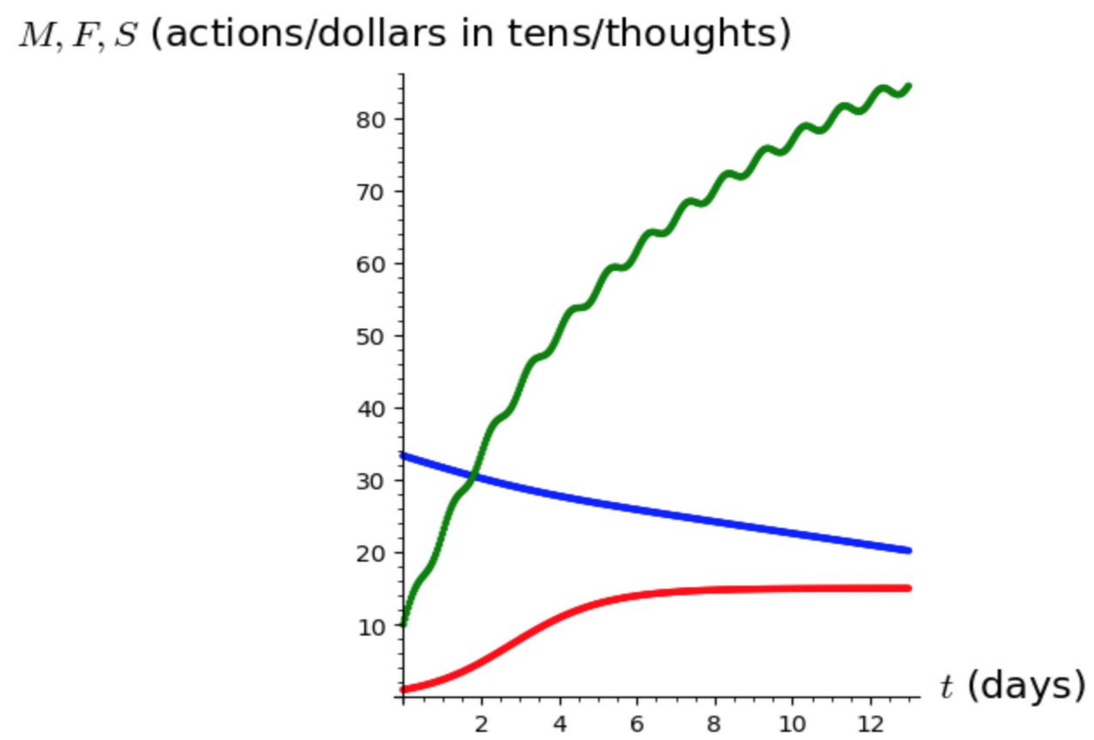


Figure 1: The **blue** line represents the amount of money left in an individual's bank account. The **red** line represents an individual's frugality actions. The **green** line represents an individual's stress levels.

Analysis

In the figure above the blue line, representing money, continually decreases over a 13 day period. The red line, or frugality, increases from zero to 15 in approximately 5 days and then levels off once reaching the carrying capacity of 15. The green line, or stress, increases with small spikes and dips, which represent daily differentiation in stress levels. From this model, we can see that as the amount of money decreases, the amount of frugality actions increases until frugality hits the carrying capacity. As money decreases, stress increases but sees increases due to daily life stress factors, so a clear relationship cannot be determined. However a trend could be possible. Overall, stress and frugality both increase over time as money decreases.

CSM Mini Project #6

Creative Mini Project- Final Draft

🔥💯Super-Lit Flu💯🔥

December 6th, 2016

Math 1310 - Section 001

Instructor: Eric Stade

m = 0.018307 *Percentage of population that dies/ Year* (The death rate)

b(t) = 0.25+sin(180πt) *Number of People that get infected/ season*(The force of infection with a seasonal contract rate)

γ = 0.07143 or (the recovery coefficient)

Units

N = Students

m =

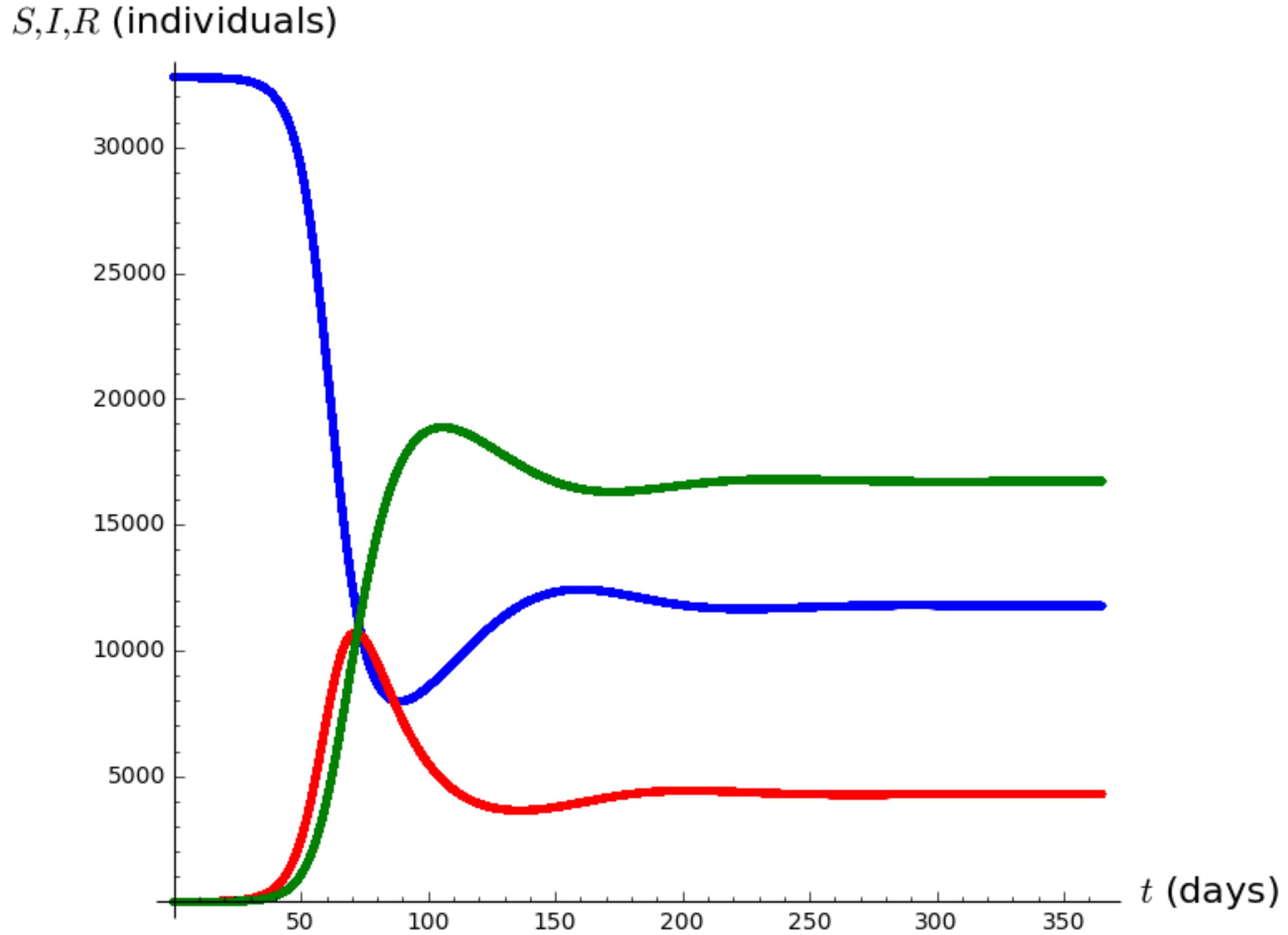
b(t) = Students *

γ =

3. Write a Sage program that solves your initial value problem numerically, using Euler's method

```
sage: # First, specify the starting and ending points, stepsize, and total number of observation points
sage: tstart=0
sage: tfin=365
sage: stepsize=0.001
sage: length=((tfin-tstart)/stepsize)+1
sage: # Next, specify values of parameters, and initial values of variables
sage: m=0.018307
sage: g=1/14
sage: N=32775
sage: S=32774
sage: I=1
sage: R=0
sage: t=tstart
sage: b=1
sage: # Set up empty lists for the values we're about to compute
sage: bvalues=[]
sage: Svalues=[]
sage: Ivalues=[]
sage: Rvalues=[]
sage: tvalues=[]
sage: # The following loop does three things:
sage: # (1) stores the current values of S, I, R, b, and t into the lists created above;
sage: # (2) computes the next values of S, I, R, and b using Euler's method;
sage: # (3) increases t by the stepsize
sage: for i in range(length):
...       # Store current values
...       bvalues.append(b)
...       Svalues.append(S)
...       Ivalues.append(I)
...       Rvalues.append(R)
...       tvalues.append(t)
...       # Compute rates of change using SIR equations
...       b=0.25+sin(565.2*(t))
...       Sprime=(m*N)-(m*S)-(b*(I/N)*S)
```

Number of Undergrads: Susceptible to, Infected with, and Recovered from the Super-Lit Flu over the course of one year at CU Boulder



Results:

32,774 students started out susceptible and were infected at a rate of 25%, where only one individual, Cameron T. Rammington, was infected with the super-lit flu. Over the course of one year, we see that the peak of the infection where ~11000 individuals are infected occurs at ~85 days. After 85 days, the amount of people infected with the disease decreases to ~4000 individuals at ~130 days before slowly increasing again, and then leveling of at around ~4000 individuals from ~240 days onward. At t=0 days, 32,774 individuals are susceptible to the super-lit flu, and radically decreases to ~800 individuals susceptible at ~90 days. From t=90 to t=150 days, the number of susceptible individuals increases to ~12,000

FIN