Student-Centered Assessment of Mathematical Proficiency

Benjamin Braun
Associate Professor of Mathematics
University of Kentucky
benjamin.braun@uky.edu

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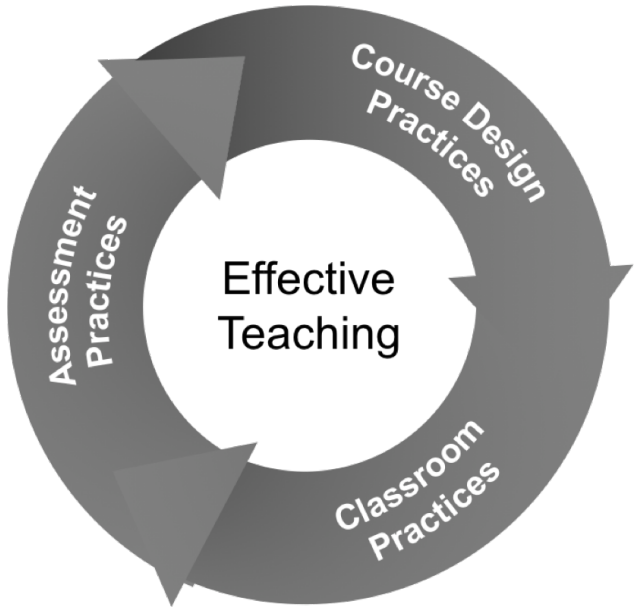
Note: This talk includes a brief segment about mental health and suicide rates among young people.

This talk is partially based on the *MAA Instructional Practices Guide*.



I was a member of the IP Guide Steering Committee and served as one of two lead writers for the chapter on Assessment.

There Are Many Existing Resources on Assessment

- MAA Assessment Practices in Undergraduate Mathematics (1999)
- MAA Supporting Assessment in Undergraduate Mathematics (2006)
- American Statistical Association Guidelines for Assessment and Instruction in Statistics Education (GAISE, 2016)
- Society for Industrial and Applied Mathematics Guidelines for Assessment and Instruction in Mathematical Modeling Education (GAIMME, 2016)
- MAA CUPM Curriculum Guide (2004 and 2015)
- CBMS Mathematical Education of Teachers II (2012)
- NCTM Principles to Actions (2014)

Defining Effective Assessment

Authentic Student Assessment Opens the Door to Difficult Discussions

Student Learning Outcomes

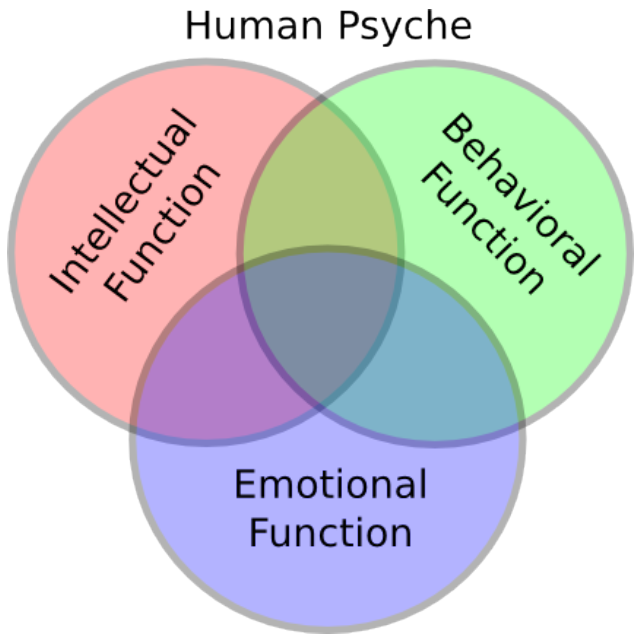
Setting Objectives for a Specific Assessment Item

Examples of Formative Assessments

Designing Summative Assessments

Three Psychological Domains

A useful oversimplification: contemporary psychology provides a basic framework of the human psyche with three domains.



Many math courses focus primarily on "Intellectual" aspects of student learning. Effective teaching requires course design, classroom practices, and assessment practices that address all three domains.

This three-domain framework is reflected in contemporary descriptions of mathematical proficiency, e.g.

- 2001 National Research Council Report *Adding It Up*
- 2015 MAA CUPM Curriculum Guide

Defining Effective Assessment

Definition: Effective assessment occurs when we

1. clearly state high-quality goals for student learning that address the three psychological domains,
2. give students frequent informal feedback about their progress toward these goals, and
3. evaluate student growth and proficiency based on these goals.

The purpose of this talk is to elaborate on these three characteristics of effective assessment.

There are other issues that come up when discussing program-level assessment, e.g. assessing a major program or a set of sequenced courses.

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Many Factors Impact Student Learning

If a student does not have their basic needs met (food and housing security) and/or does not have their mental and physical health needs met and/or has not been supported by their educational experiences, it is difficult for them to learn math.

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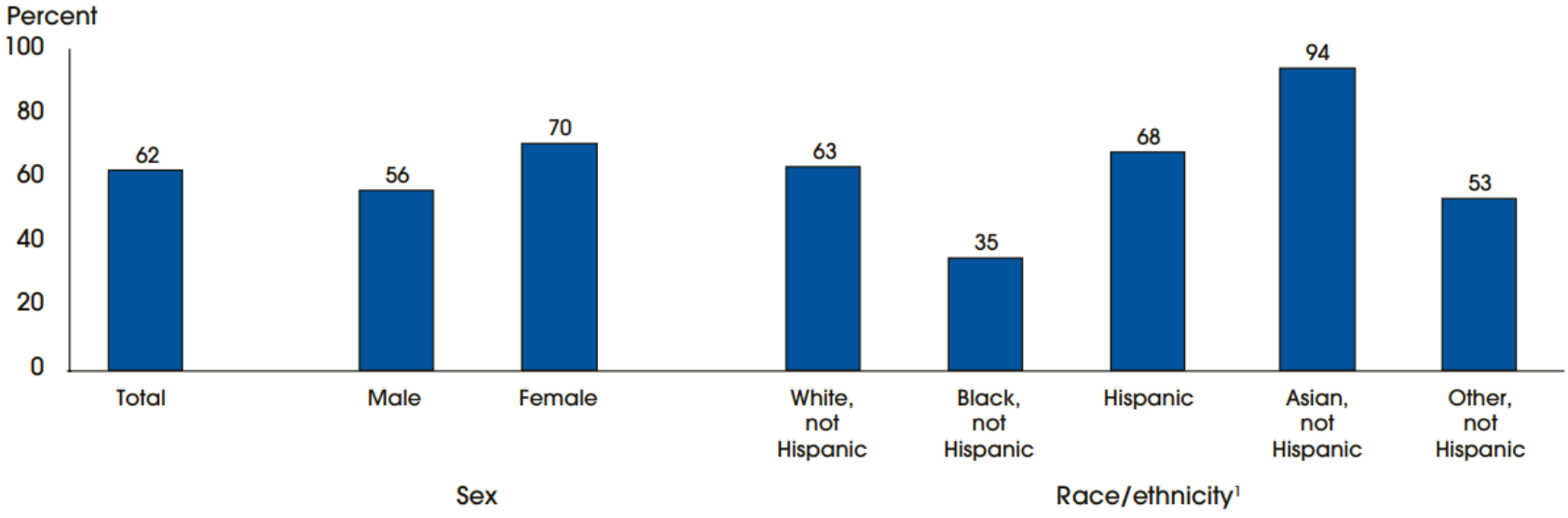
Here are a few examples of factors we do not talk about often enough, but that have a large impact in the lives of our students.

We Are Part of a Biased System

Figure 2. Percentage of the ECLS-K spring 2000 first-grade cohort in the top two fifth-grade mathematics score quintiles who were enrolled in algebra or a course more advanced than algebra in the eighth grade, by sex and race/ethnicity: Spring 2004 and spring 2007



[1] Black, not Hispanic includes African American. Hispanic includes Latino. Other, not Hispanic includes Native Hawaiians, Pacific Islanders, American Indians, Alaska Natives, and non-Hispanic students of two or more races.
NOTE: The estimates represent all U.S. students who attended first grade in the spring of 2000 and then were in a U.S. eighth grade in the 2006–07 school year. "Algebra or a course more advanced than algebra" includes algebra I, integrated or sequential mathematics, algebra II, and geometry. Estimates were weighted by C7CPTM0. Standard errors are available upon request.
SOURCE: U.S. Department of Education, National Center for Education Statistics, Early Childhood Longitudinal Study, Kindergarten Class of 1998–99 (ECLS-K), Kindergarten-Eighth Grade Full Sample Public-Use Data File.

From "Eighth-Grade Algebra: Findings From the Eighth-Grade Round of the Early Childhood Longitudinal Study, Kindergarten Class of 1998-99 (ECLS-K)," https://nces.ed.gov/pubsearch/pubsinfo.asp?pubid=2010016.

We Are Part of a Biased System

Rochelle Gutiérrez' Four Dimensions of Equity details four key aspects of the educational process that require attention.

1. **Access:** Ability to gain intellectual and physical access to mathematical ideas and mathematical teaching and learning spaces
2. **Achievement:** Success in mathematics as traditionally measured
3. **Identity:** Who students are and who they become through their participation in mathematics
4. **Power:** Attending to the distribution of power between instructor and student, between students, and between students and mathematics

I also recommend learning about her work on "Rehumanizing Mathematics".

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https://www.cdc.gov/injury/wisqars/LeadingCauses.html
https://www.nimh.nih.gov/health/topics/suicide-prevention/index.shtml

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We can ensure that we are active participants in campus-wide student support systems.

- Invite your campus Counseling Center, CARE/Community of Concern/etc program, Bias Incident Reporting Program, Violence Prevention Program, Disability Resource Center, and/or other student support units to present about their services at a faculty meeting or department seminar.
- Stay informed regarding how faculty at your institution can serve as a connection between students in need or in crisis and support systems on campus.
- Strive to ensure that conversations about difficult/uncomfortable topics are possible and normal in your department.

An aside: At most colleges and universities, almost every student takes a course in mathematics. This means the math community has a unique opportunity to be a leading collaborator with campus counseling centers, food pantries, health systems, PreK-12 education, etc.

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Key point: Effective assessment practices addressing intellectual, behavioral, and emotional aspects of mathematics lead to an expanded understanding of our students and their experiences.

This opens the door to a deeper understanding of our students, and of the supports and barriers for their mathematical learning.

Returning Our Attention to Effective Assessment

Effective assessment occurs when we

1. clearly state high-quality goals for student learning that address the three psychological domains,
2. give students frequent informal feedback about their progress toward these goals, and
3. evaluate student growth and proficiency based on these goals.

Student Learning Outcomes

Each course should have a comprehensive set of Student Learning Outcomes (SLOs) *for that course* addressing all three of the intellectual, behavioral, and emotional domains of human functioning.

Example: College Algebra SLOs, Part I

Students will:

- Use multiple perspectives (symbolic, numeric, graphic, and verbal) to explore elementary functions
- Algebraically solve linear, quadratic, exponential, logarithmic, and power equations
- Sketch polynomial and rational functions using a graphing calculator.
- Identify and algebraically find important characteristics of these graphs such as intercepts, vertical asymptotes, and horizontal asymptotes.
- Recognize and use standard transformations with graphs of elementary functions
- Use and solve systems of equations to model real world situations

College Algebra SLOs, Part II

Students will have opportunities to engage in the following mathematical practices:

- Being persistent, working through perceived failure
- Collaborating productively with a team
- Developing a personal framework of problem solving techniques (e.g. to make sense of problems, sketch and label diagrams, restate and clarify questions, identify variables and parameters, and use analytical, numerical, and graphical solution methods)
- Creating, interpreting, and revising real-world models and solutions of problems

Two Types of Assessment

Formative Assessment: Alan Schoenfeld informally defines formative assessment as

> *examinations or performance opportunities the primary purpose of which is to provide student and teachers feedback about the student's current state, while there are still opportunities for student improvement.*

Summative Assessment: Summative assessment is conducted with the purpose of evaluating student growth and/or proficiency with regard to one or more learning outcomes.

Growth and Proficiency

- When measuring student *growth*, we evaluate how far students have progressed compared to their starting point.
- When measuring student *proficiency*, we evaluate students against a specific SLO, regardless of their starting point.

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In courses for STEM majors, such as calculus, linear algebra, and differential equations, it is reasonable for proficiency to be the primary goal. In courses for non-STEM majors, such as general education or quantitative literacy courses, it is equally reasonable for growth to be the focus. Sometimes, growth and proficiency might be given equal weight when assessment methods are chosen.

For any given course, a clear and well-articulated decision of how to balance the assessment of growth and proficiency should be made.

Objectives vs Outcomes

Your course SLOs specify the overall learning goals for students.

When using a specific assessment item, e.g. a homework problem, a small-group activity during class, an exam problem, a writing assignment, etc., you need to have a clear understanding of your objective *for that specific item*.

In other words, each assessment that you conduct with students should:

- be aligned with one or more of your overall SLOs,
- be either formative or summative in nature, and
- focus on either growth or proficiency.

Recap

What we've discussed so far:

- Student Learning Outcomes
- Formative and Summative Assessment
- Growth and Proficiency
- SLOs vs Assessment item objectives

Example #1: Think-Pair-Share

Technique for a Number Theory class: Ask students to use Euclid's proof of the infinitude of primes to produce as many new prime numbers as possible starting with only the prime 5. Students have three minutes to compute independently, then three minutes spent comparing their results with one or two of their neighbors in class, discussing the reason for why their lists are the same or different. A subset of the students are then asked to share the results of their conversations in order to start a whole-class discussion.

Example #2: Online homework (unlimited attempts)

An example of formative assessment that is often not recognized as formative: online homework systems where students have unlimited attempts to answer the problem. This provides feedback to students (correct or incorrect) and can be used to help students:

- serve as instructional resources for one another; and
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It is a useful habit to reflect on your regular teaching practices to identify "secret" formative assessment you do — many people don't think of online homework as a feedback mechanism for students. Viewing these practices explicitly as formative assessment techniques can make them more effective and lead to new ideas for how to incorporate them in a course.

Example #3: Reflective Essays

Short reflective essays about challenges in the course promote development in both the emotional and behavioral domains.

- Write several paragraphs on the following topic: what was the most challenging aspect for you regarding [TOPIC]? What made this difficult for you? Did you overcome the challenge, or are you still struggling with it?

This can be graded based on completion, i.e. if students write several paragraphs that address these questions then they receive full credit for the problem.

Example #4: IBL-style small group activity

Technique: Assign students to small groups. Give each group a theorem (or problem) with a 15-line proof (or solution) where each line is separately cut out and mixed together, where the proof (or solution) has one fixable error. Students must first collaboratively reconstruct the proof, then identify and correct the error. The instructor spends the class time circling the room, listening to student conversations, offering clarification and answering questions, etc.

Example #5: Critical Reviews of Reading

To promote critical analysis skills and develop students' reading abilities, have students write a review of selected readings from your course text.

- Write a three page critical review of [ASSIGNED READING].
- Imagine that you are writing your review for a journal for undergraduates in mathematics and the sciences.
- You must address the mathematical depth and mathematical style of [ASSIGNED READING] in addition to other topics.

In my courses, short essays are graded using a rubric with five criteria: Writing Style, Arrangement and Development, Editing and Conventions, Mathematical Depth, Mathematical Style.

The Challenge of Summative Assessment

Recall: *Summative assessment* is conducted with the purpose of evaluating student growth and/or proficiency with regard to one or more learning outcomes.

Most of the assessments that we typically see in math courses, such as exams, quizzes, and homework (when graded after only one attempt) fall within this context.

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Two major challenges are:

- to design a meaningful overall evaluation scheme, i.e. course grading system. (We will not focus on this today.)
- to create and select problems/tasks with a clear sense of what the problem/task:
  - is *intended* to assess (this can be done)
  - *actually* assesses (this is usually difficult to determine)

Creating and Selecting Effective Problems/Tasks

A well-known framework for analyzing problems given to students is Bloom's taxonomy. Bloom's work originally outlined multiple levels of skills in the cognitive domain of learning, increasing from simple to complex. These are described by six skill levels:

- knowledge
- comprehension
- application
- analysis
- synthesis
- evaluation

Revised Bloom's Taxonomy

Bloom's taxonomy has been extended by researchers in educational psychology to more robust frameworks. Anderson et al. introduced a two-dimensional extension of Bloom's taxonomy.

The first dimension consists of a cognitive process dimension similar to Bloom's taxonomy (serving the behavioral domain), while the second consists of a knowledge dimension (serving the intellectual domain).

When evaluating a problem or task using this taxonomy, the cognitive process is represented by the verb used when specifying the task (what the student is doing) and the knowledge process dimension corresponds to the noun (what kind of knowledge the student is working with).

Knowledge Dimension (Intellectual)

- Factual Knowledge: The basic elements that students must know to be acquainted with a discipline or solve problems in it.
- Conceptual Knowledge: The interrelationships among the basic elements within a larger structure that enable them to function together.
- Procedural Knowledge: How to do something; methods of inquiry, and criteria for using skills, algorithms, techniques, and methods.
- Metacognitive Knowledge: Knowledge of cognition in general as well as awareness and knowledge of ones own cognition.

Cognitive Process Dimension (Behavioral)

- Remember: Retrieving relevant knowledge from long-term memory.
- Understand: Determining the meaning of instructional messages, including oral, written, and graphic communication.
- Apply: Carrying out or using a procedure in a given situation.
- Analyze: Breaking material into its constituent parts and detecting how the parts relate to one another and to an overall structure or purpose.
- Evaluate: Making judgments based on criteria and standards.
- Create: Putting elements together to form a novel, coherent whole or make an original product.

Recommended Activity

Each assessment problem or task will fall into one or two of the boxes on this table:

	Remember	Understand	Apply	Analyze	Evaluate	Create
Factual						
Conceptual						
Procedural						
Metacognitive						

It is a useful exercise to collaboratively create problems that address a single SLO but fit in different boxes in this taxonomy.

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Remark: Some of the terms in this taxonomy are defined by the psychologists in ways that subtly differ from their use in mathematics, e.g., "understand". So, it is most practical to use this taxonomy somewhat flexibly, making minor adaptations as needed to fit your context.

A Few Questions to Ask About Your Summative Assessment

1. What cultural assumptions am I making regarding my students? E.g., when writing word problems?
2. Are my questions clear to students whose first language is not English?
3. Are my assessments accessible for students with learning disabilities, for blind or visually impaired students, or for students with other (visible or invisible) disabilities?
4. How often am I giving summative assessments? Infrequently (high stakes) or frequently (low stakes)?
5. Am I allowing students to revise summative assessment items? Why or why not?
6. How do the choices I am making with regard to summative assessment structures align or not align with my Student Learning Outcomes?

Summary

1. Effective assessment involves:
   - 1.1 Broad SLOs
   - 1.2 Formative assessment
   - 1.3 Summative assessment
2. Authentic student-centered assessment is more than "just the math," and frequently brings up complex and challenging issues impacting students.

Thanks for listening!

Questions? Further thoughts?