Redesigning a large linear algebra service course - a travel report

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- Born in Bonn, Germany
- DPhil in Oxford in 2008
- At Illinois since 2010, now as an Associate Professor
- Research in Logic
- Director of the Illinois Geometry Lab (since 2018)

- Disclaimer: I am not an expert in math education

Linear algebra.

- The Department has many linear algebra courses (I counted at least 5) on various levels of sophistication.
- This talk is about updating the course “Applied Linear Algebra” (Math 415), the highest level linear algebra course for non-Math majors. Highest enrollment among linear algebra courses
- Introductory course emphasizing techniques of linear algebra with applications to engineering
- Taken by around 1600-2000 students a year.
- Required by many engineering majors
- Median student is a Junior

Math 415 in 2012/2013

- Substantial increasing in the number of engineering students
- Enrollment in Math 415 increased even more
- Old set up of running up to 8 sections with 40 students each (3hrs lecture, no discussion) became impossible
- The size of a section grew up to 120 students, creating a sub standard learning experience for students.

Other problems

- Insufficient coordination between sections
- No coordinated effort to include “real applications”
- No active learning

A common situation?

``The reasons students give for abandoning STEM majors point to the retention strategies that are needed. For example, high performing students frequently cite uninspiring introductory courses as factor in their choice to switch majors. And low performing students with a high interest and aptitude in STEM careers often have difficulty with the math required in introductory STEM courses with little help provided by their universities. Moreover, many students, and particularly members of groups underrepresented in STEM fields, cite an unwelcoming atmosphere from faculty in STEM courses as a reason for their departure.''

From the 2012 Report of President's Council of Advisors on Science and Technology

Marching orders (Matt Ando)

1. Close the excitement gap: identify exciting applications and activities that showcase the power of linear algebra in “real life” situations
2. Close the math gap: provide the students with an engaging learning environment that appears them well for future courses

“Classroom approaches that engage students in `active learning’ improve retention of information and critical thinking skills, compared with a sole reliance on lecturing, and increase persistence of students in STEM majors.” – 2012 PCAST

Important aspect: Listen to the students!

Math gap

New set up:

- 4 tightly coordinated lecture sections with 200-300 students each (3hrs/week)
- 30 discussion sections with 20-30 students (1hr/week)

joint effort: in particular, Marteen Bergvelt, and many others

(my estimate: roughly cost equivalent to having sections with 120 students each and no discussion section. Half as expensive as running sections with 40 students each and no discussion section).

- Effort to include active learning component in lectures and discussion sections

Active learning

In the Department:

- Active learning group work in Calculus discussion sections, group of 4 student working through a worksheet (2hrs/week)
- Merit Workshop (Jennifer McNeilly), supports students with high potential from traditionally underrepresented populations, based on Uri Treisman's collaborative learning model (4hrs/week)

In Math 415:

- Active learning during lecture through in class discussions/questions
- Active learning in discussion sections through group work or in class discussion/questions

Active learning in lecture

- Active learning group work currently unfeasible without further resources
  - No large enough active classroom
  - Not enough TAs

- Active learning through in class discussions/questions
  - Main tool: Fill-in lecture (guided) lecture notes
  - Definitions or theorems are already written down
  - Consists mainly of follow up questions about definitions and theorem
  - Mostly empty
  - Could be done without the instructor

Multiplication of matrices

We know how to multiply a matrix and a vector (of the right size!). Now we want to define how to multiply matrices by matrices.

Definition. Let A be an $m \times n$-matrix and let B be an $n \times p$-matrix. We define

$$AB := [A\mathbf{b}_1 \quad A\mathbf{b}_2 \quad \cdots \quad A\mathbf{b}_p]$$

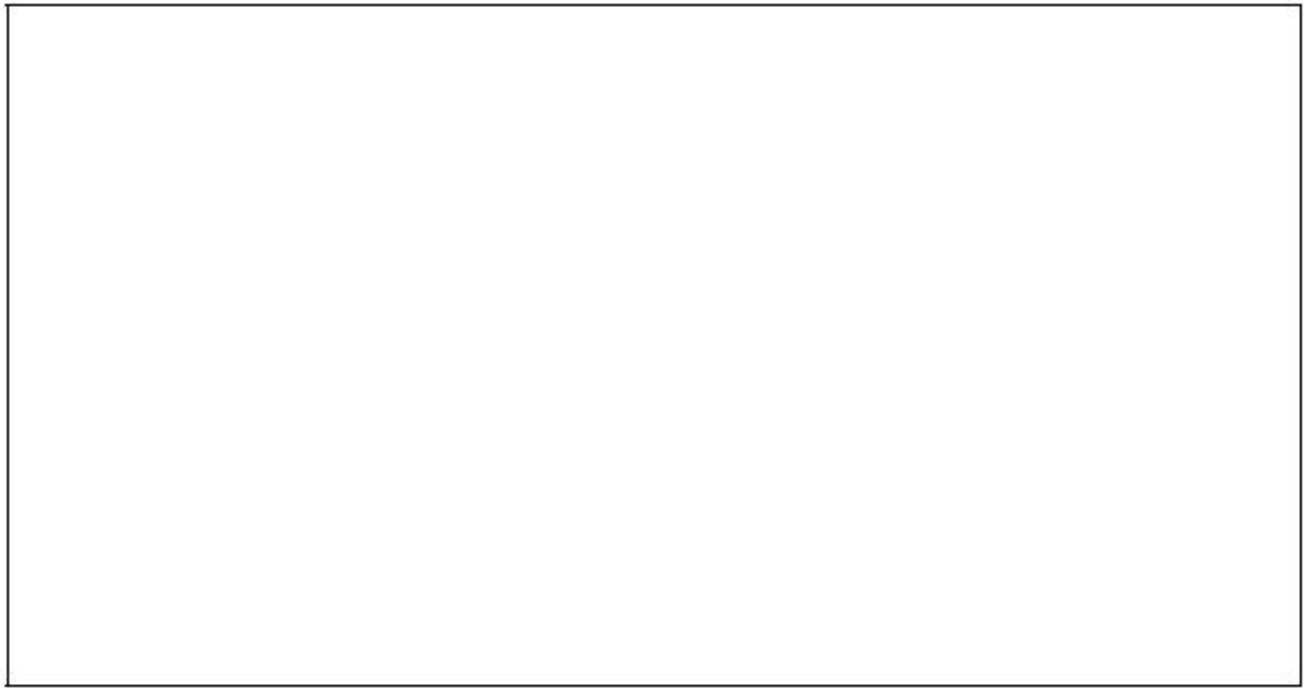
Example 1. Compute AB where

$$A = \begin{bmatrix} 4 & -2 \\ 3 & -5 \\ 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & -3 \\ 6 & -7 \end{bmatrix}$$

Solution.

Example 2. Use your geometric understanding to find the eigenvectors and the eigenvalues of $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

Solution.



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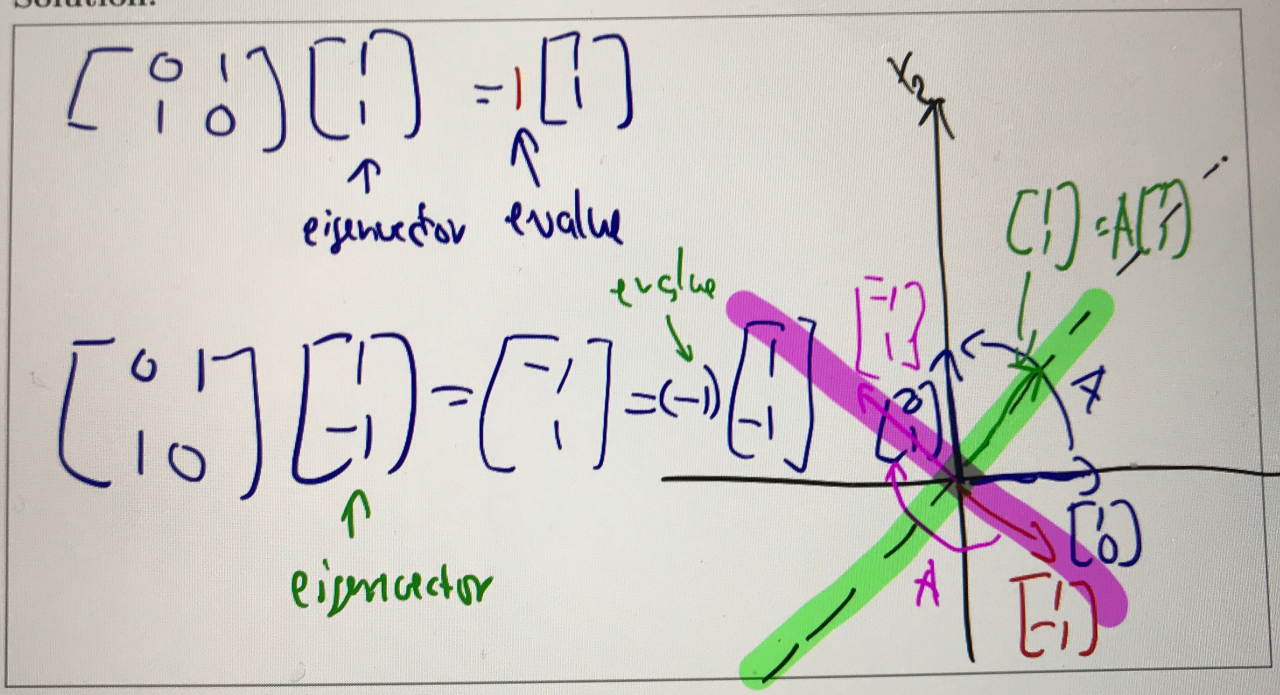


Example 3. Use your geometric understanding to find the eigenvectors and the eigen-

Active learning in lecture

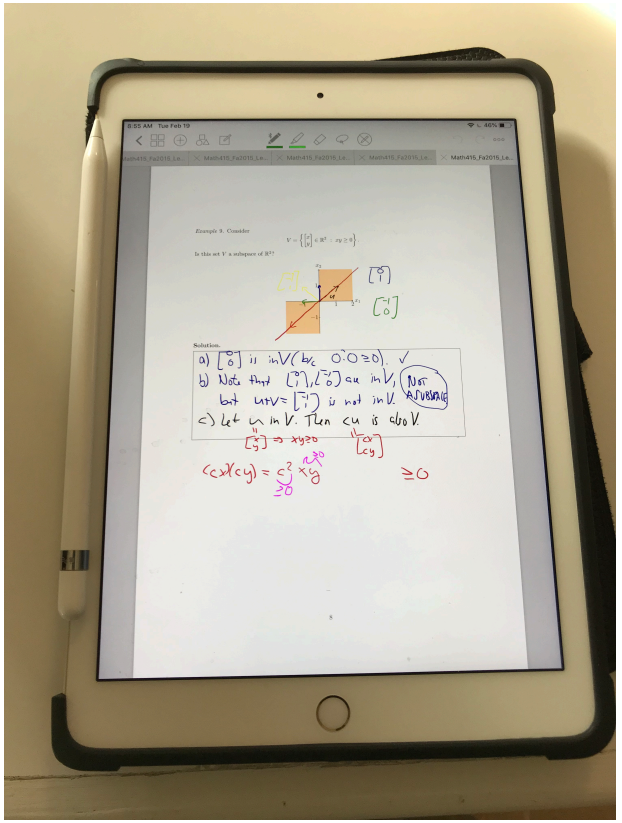
- Students receive fill-in notes before class
- Students could do them on their own
- Provided fill-out notes (and videos) after class

- Many ways of using such notes
  - Group work or individual works on the notes or part of the notes during class
  - Class-wide discussion between students (with or without the instructors input)
  - “Discussion section” style instructor/student interaction
  - Classical lecture style

Important: Take your time.

Active learning in lecture

Technology/infrastructure helps



Active learning in discussions

- Department has strong success in other courses with this, and almost all our TAs are experienced with this
- Usually group work on a worksheet
- We have been experimenting with this in Math 415, but students (and TAs) always seemed eager to go back to classical discussion section setting.
- Why? I don't know.
  - Just 1hr of discussion section?
  - More mature students in compare to calculus?
  - After the active learning lecture students just want to ask questions?
- Students love the discussion sections (very high satisfaction)

A TA's experience (Ruth Luo)

- … I begin my discussion sections by asking an open question: what do you want to go over today? In the first days of the semester, the students are almost silent. A couple weeks in, they are asking to go over so many questions or concepts that there simply isn't enough time to cover them all! …
- …even though I am the one standing in the front of the room and writing on the chalkboard, I only speak half the time.
- I state the problem and write it on the board and then ask the students what to do next.
- Most of the problems we talk about in class are worked out by several students who volunteer their ideas or solutions.

A TA's experience (Ruth Luo)

- I believe that students can thrive in this collaborative environment. Moreover, because they have the opportunity to see the worksheet beforehand.
- those who have spent time working through it can confidently offer suggestions or solutions which boosts the general class morale.
- This is what I strive for when I teach: each class should be both informative and rewarding to each student.

- I would be very happy to replace my lectures with a group work set up.
- I am not so sure about the discussion sections.

Technology helps

- Easy computational online homework (reviewing what was covered in lecture) due the night before discussion section
- Harder conceptional multiple-choice online homework due shortly after the discussion
- Helps to make sure students are prepared and engaged during discussion section
- We use WebAssign using our own questions

Excitement gap

Goal: identify exciting applications and activities that showcase the power of linear algebra in “real life” situations

- Discussion with engineering faculty about what should be taught in a Linear Algebra course (Matt West, Luke Olson,…)
- The number one answer: “SVD”.
- Students in this course have very different backgrounds.
- Very few specific demands about content came up
- There was more a common desire for a well-taught linear algebra (preferably earlier in the curriculum = not post Calc III)

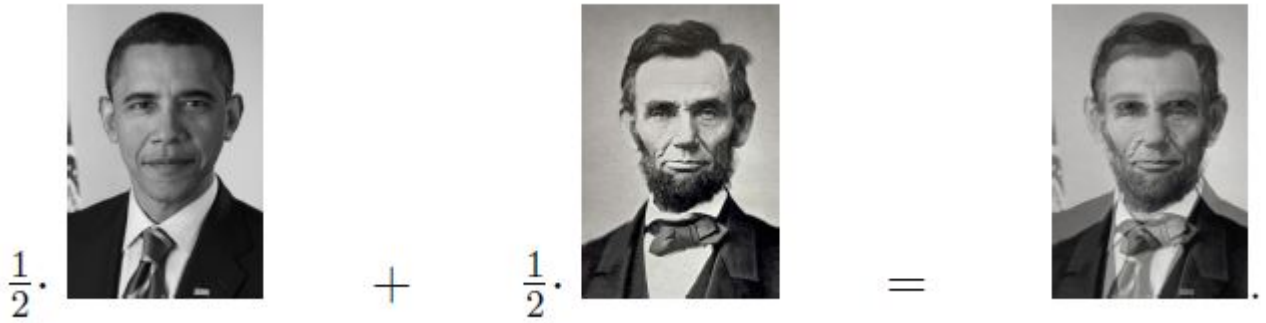
Applications

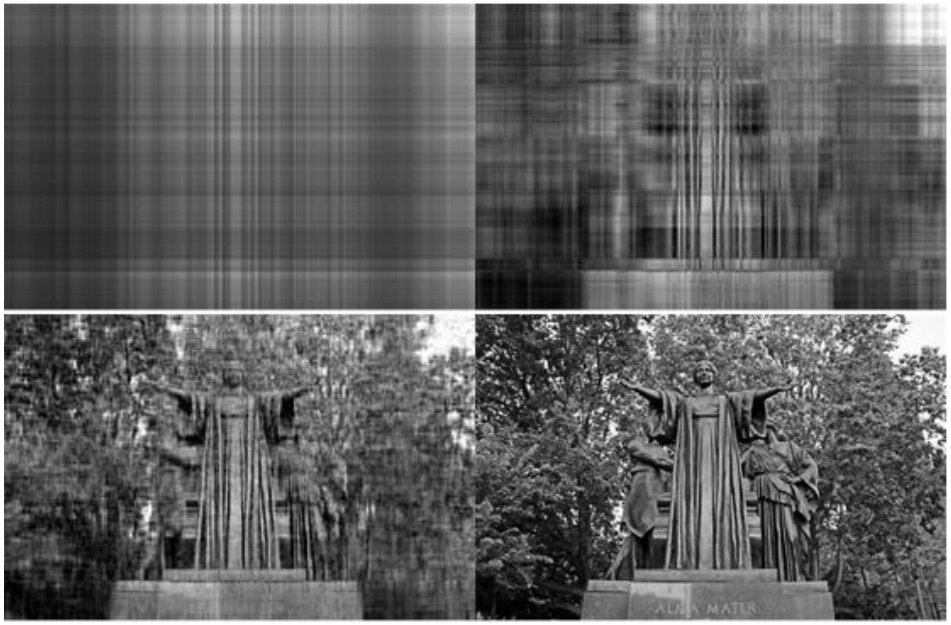
- Image manipulation
- PageRank, Markov chain
- Network/Graph Theory
- Image compression (JPEG)
- Image compression using SVD
- Face recognition using SVD

…

The list is endless.

Applications





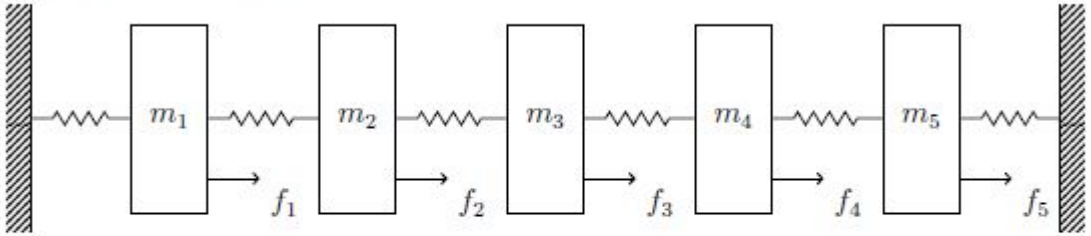
Applications

- Keep applications in class simple – if you have to read up how the application works, it may already be to complicated.
- Students like modern, technology driven applications – Leontief model not so popular
- Applications are not necessarily about knowledge transfer, but surely about motivating students.
- Provide Mathematica worksheets whenever possible

Idea/Goal: Include many (in particular the more sophisticated) applications as active learning group sessions.

Applications

Consider the following spring-mass system, consisting of five masses and six springs fixed between two walls.



We denote

- by f_i a (steady) applied force on mass i,
- by u_i the displacement of the mass i.

Note that positive values of u_i correspond to displacement away from the wall on left

Student evaluation

- Online feedback forms and focus group meetings with students each semester
- Students strongly prefer fill-in guided lecture over traditional blackboard lecture
- Students rate discussion sections very high (substantially above the already strong ratings for the course overall)
- Students very comfortable with online homework
- Substantial part of the students comfortable with replacing lectures with videos, while keeping discussion sections

- Better learning outcomes?

Active learning after class

- Important aspect: Help the student when the student is engaging with the material
- To me, this is as important during class as it is after class.

What to do?

- Piazza – help the student and help students help each other at all times.
- Provide spaces in which students can work while helps is available.

What's left to do

- Extend the computational component
  - we already use Mathematica worksheets when discussing applications
  - Make this a bigger part of the course
  - Student feedback was pretty negative on any extension in this direction

- Increase active learning group work
  - My dream: One hour dedicated each week for students to work out an application of linear algebra and implement/calculate it on a computer.
  - This has to come out of lecture time.