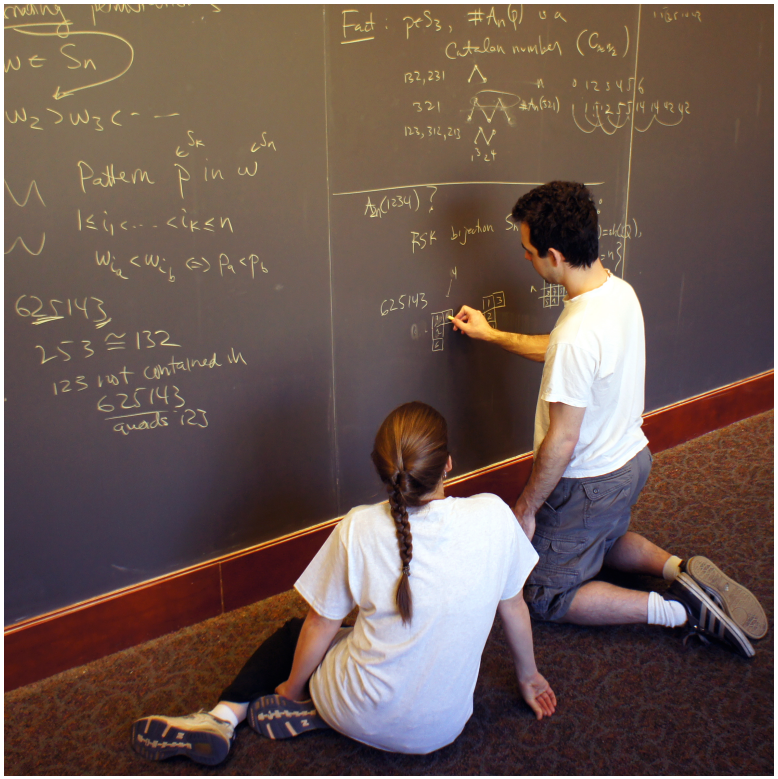
Teaching Students to Communicate as Mathematicians

November 27, 2018



Susan Ruff
Lecturer II
Writing, Rhetoric, and Professional Communication
Department of Comparative Media Studies/Writing
Massachusetts Institute of Technology

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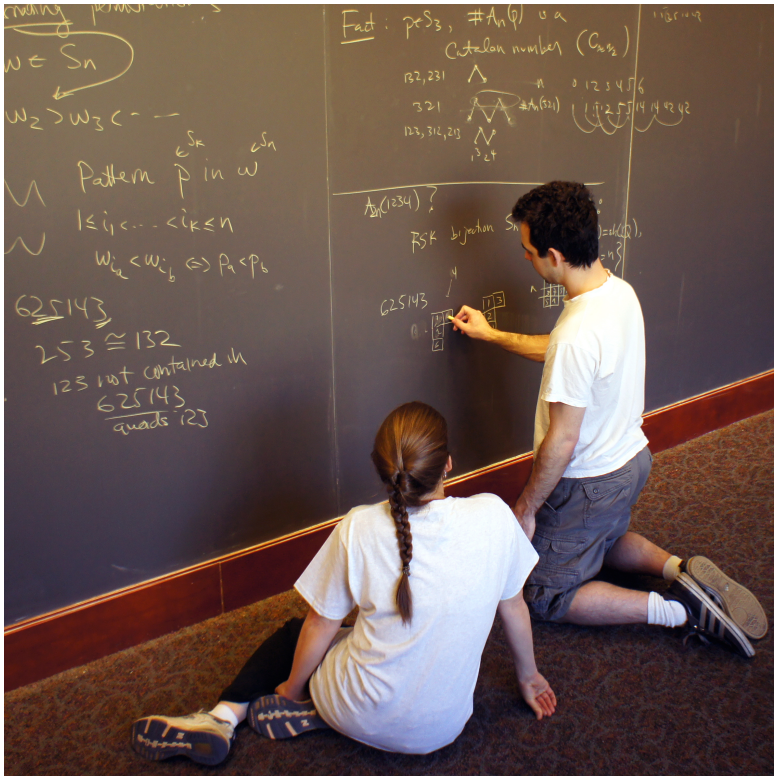

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15 communication-intensive subjects:

- 4 introductory-level subjects (mostly proof writing)
- 10 undergraduate seminars
- Project laboratory in mathematics

Why have students communicate mathematics?

- Writing to enable assessment
- Writing to learn mathematics
- Learning to write mathematics

Why have students communicate mathematics?

- Writing to enable assessment
- Writing to learn mathematics
- Learning to write mathematics

Q: Why do *you* have students communicate mathematics?

Why have students communicate mathematics?

- Writing to enable assessment
- Writing to learn mathematics
- Learning to write mathematics

Today's focus:

Learning to communicate effectively as mathematicians

Why have students communicate mathematics?

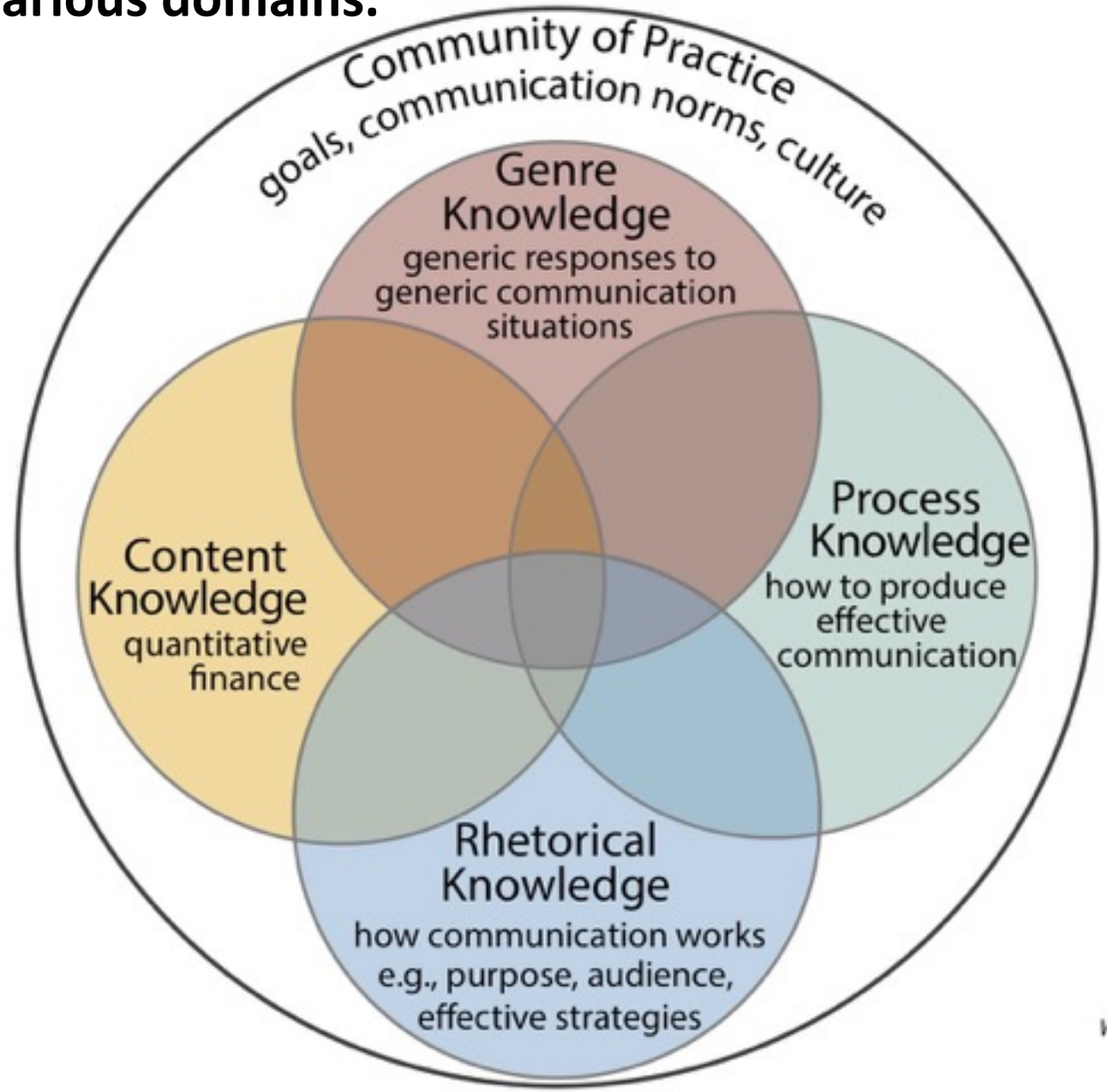
- Writing to enable assessment
- Writing to learn mathematics
- Learning to write mathematics

Today's focus:

Learning to communicate effectively as mathematicians

Q: What does it mean to communicate effectively as a mathematician?

Communicating effectively as a mathematician requires command of various domains.



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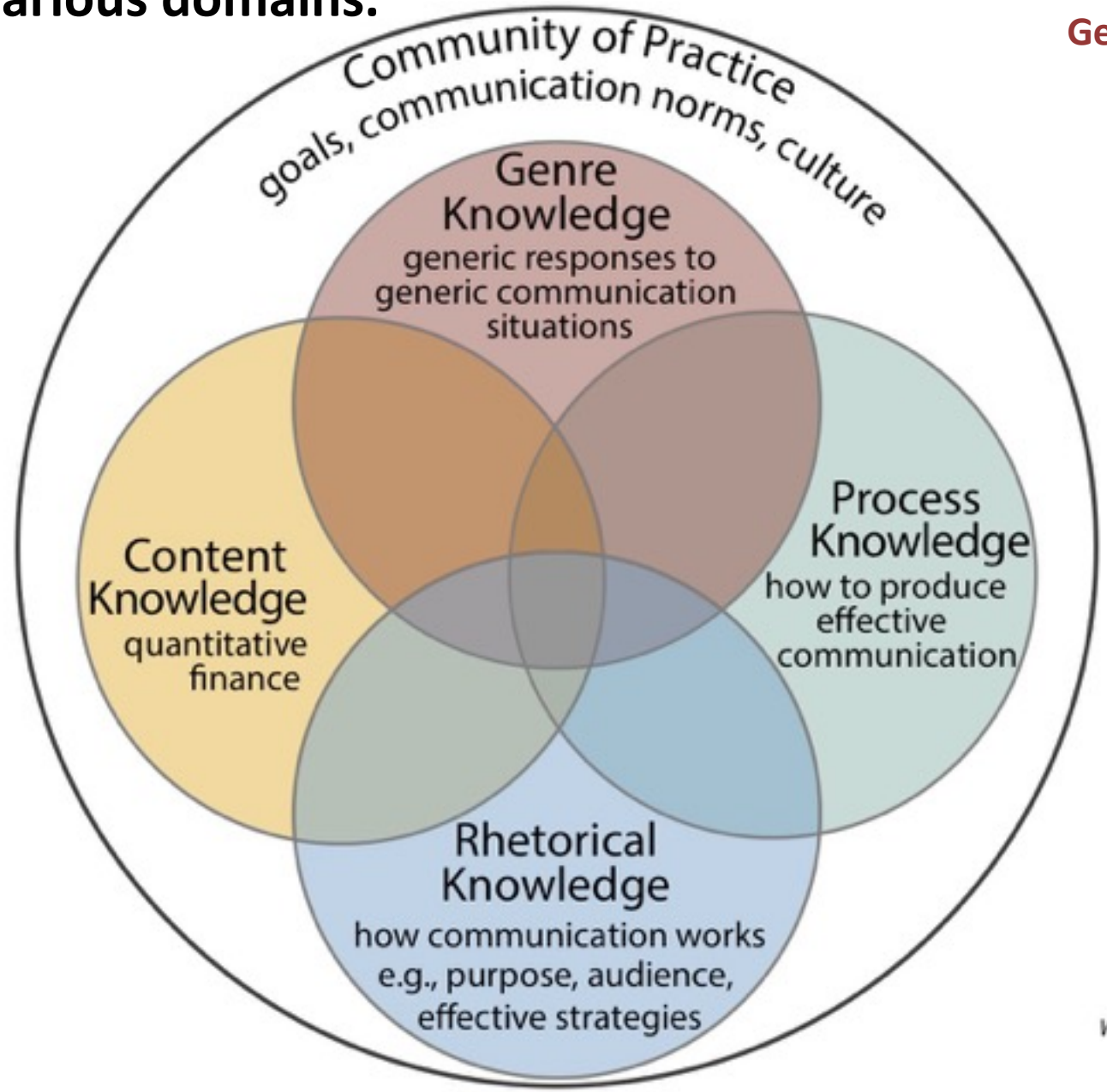

Genre system for research

- Funding proposal
- Notebook
- Meetings and emails with collaborators
- Colloqium talk
- Conf. presentation
- arXiv preprint
- Referee report
- Journal article
- Expository article

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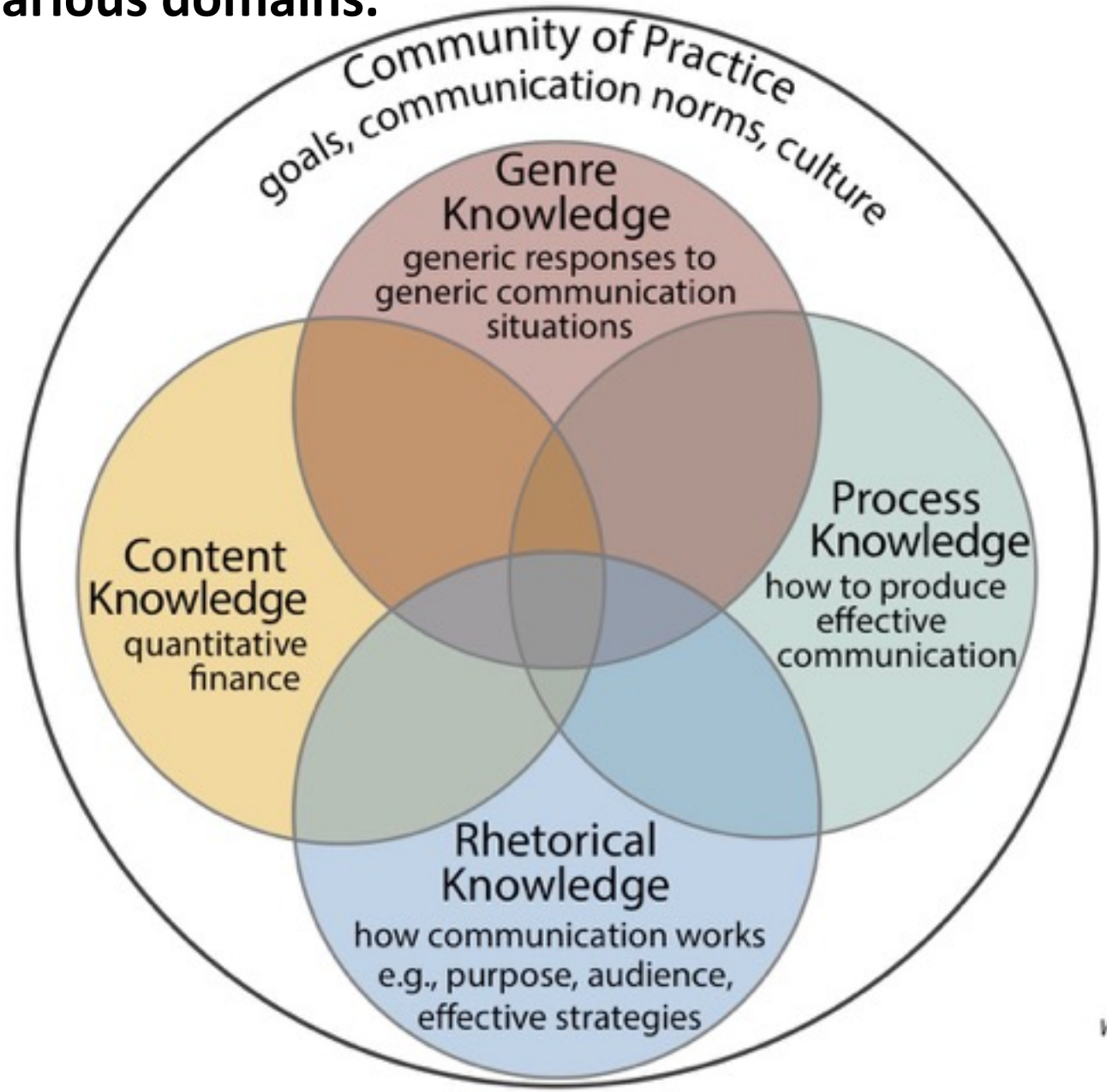
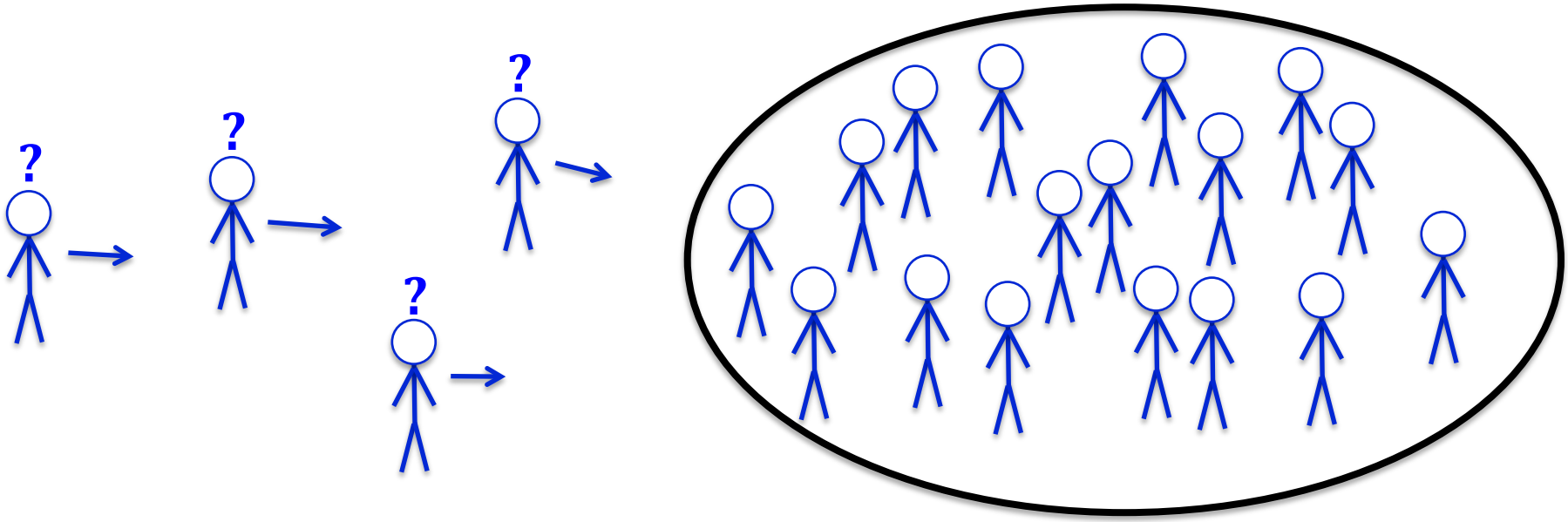


Q: Which domains challenge your students most?

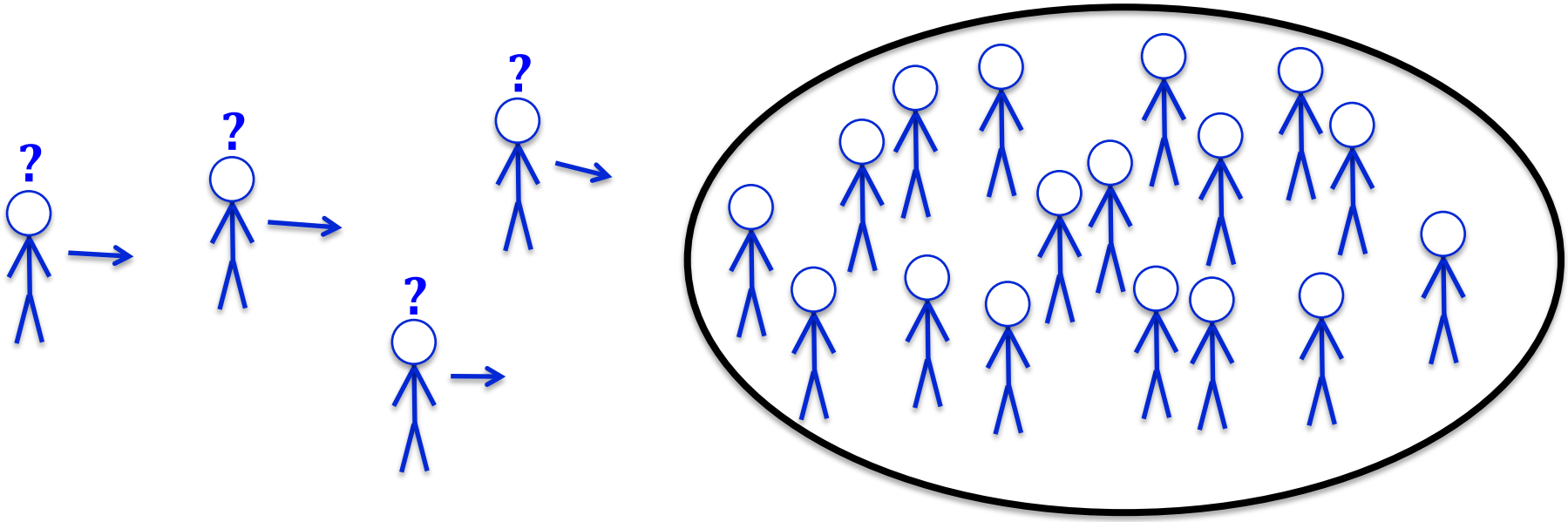
image based on ABeaufort, *College Writing and Beyond: A New Framework for University Writing Instruction*, Utah State UP, 2007 & EWenger, *Communities of Practice*, Cambridge University Press, 1998

How can students learn to communicate effectively as mathematicians?



Students **Community of Mathematicians**

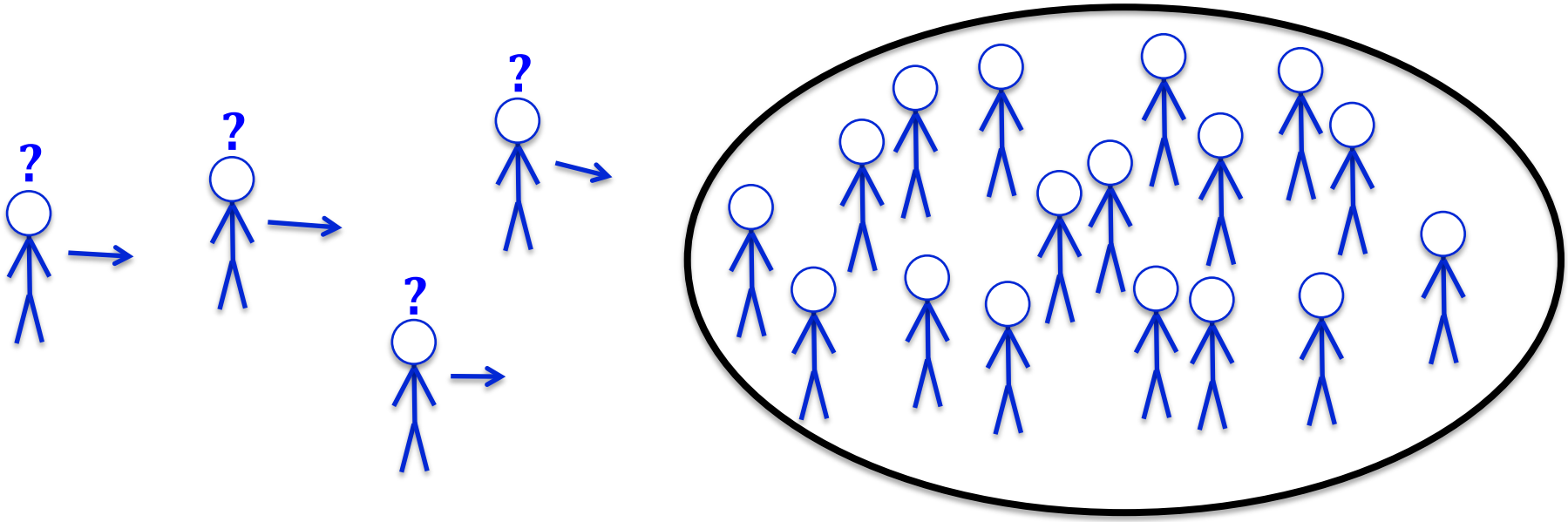
How **can students learn to communicate effectively as mathematicians?**



Students **Community of Mathematicians**

Q: How did you learn to communicate as a mathematician?

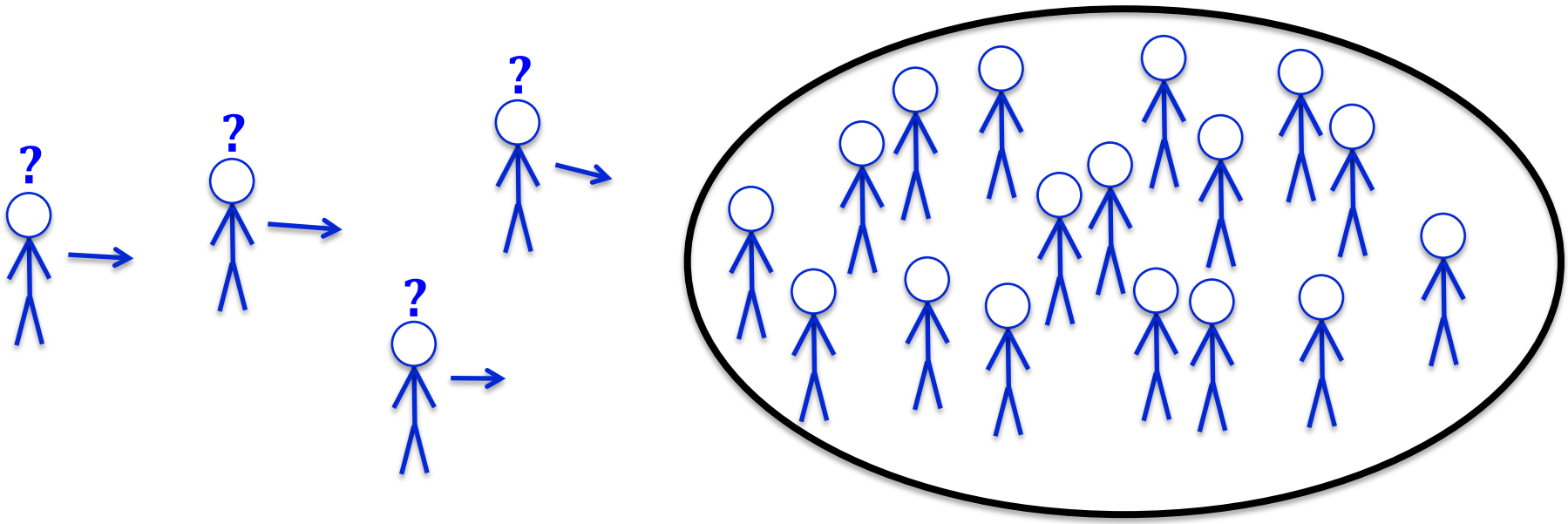
How can students learn to communicate effectively as mathematicians?



Students **Community of Mathematicians**

Apprentices learn via legitimate peripheral participation in the community of practice… (Lave & Wenger)

How can students learn to communicate effectively as mathematicians?



Students **Community of Mathematicians**

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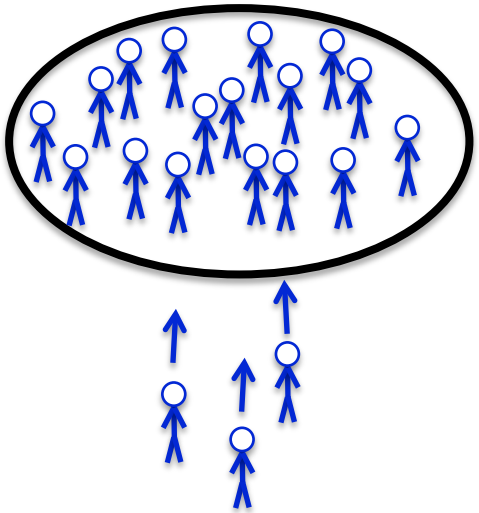
My takeaway: as much as feasible, have students communicate *as mathematicians* …and "read."

Project laboratory in mathematics



Teams of 3 research open-ended problems

Three projects during term

Write a paper for each project

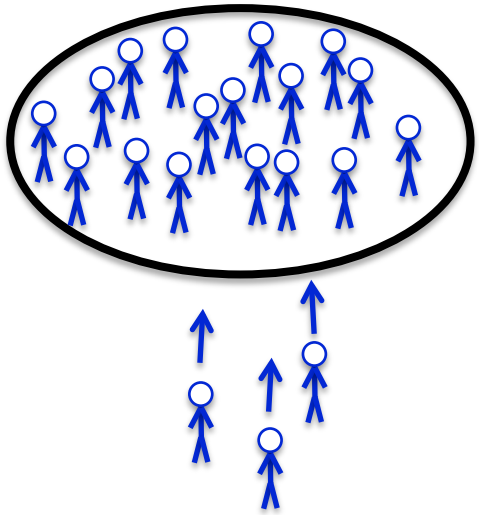
Present one project to classmates

Image courtesy of MIT Open Courseware

Undergraduate seminars

Students lecture to each other following a book or on topics of interest.

Write expository paper.



Introductory classes

Large faculty-led lectures on topic
(e.g., discrete mathematics)

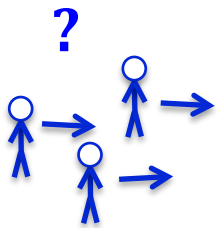
Smaller, more active “recitations”

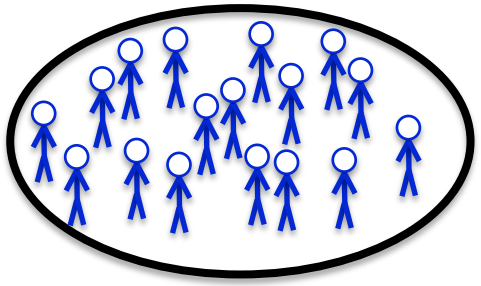
Writing = proving assigned statements

Legitimate peripheral participation?

Valuable?

Example: teaching “audience” via
hypothetical scenarios vs.
published versions of same result:
research article, Quanta article,
blog post





ON LARGE SUBSETS OF $\mathbb{F}_q^n$ WITH NO THREE-TERM ARITHMETIC PROGRESSION

JORDAN S. ELLENBERG AND DION GIJSWIJT

arXiv:1605.09223v1 [math.CO] 30 May 2016

Quanta magazine

Simple Set Game Proof Stuns Mathematicians

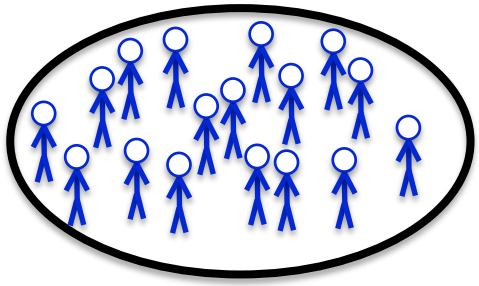
By Erica Klarreich

May 31, 2016

A new series of papers has settled a long-standing question related to the popular game in which players seek patterned sets of three cards.

Olena Shmahalo/Quanta Magazine

Introductory classes



Large faculty-led lectures on topic
(e.g., discrete mathematics)
Smaller, more active “recitations”
Writing = proving assigned statements

Legitimate peripheral participation?
Valuable?

Example: teaching “audience” via
hypothetical scenarios vs.
published versions of same result:
research article, Quanta article,
blog post

Q: How do/could you demystify for your students what it means to communicate as a mathematician?

ON LARGE SUBSETS OF $\mathbb{F}_q^n$ WITH NO THREE-TERM ARITHMETIC PROGRESSION

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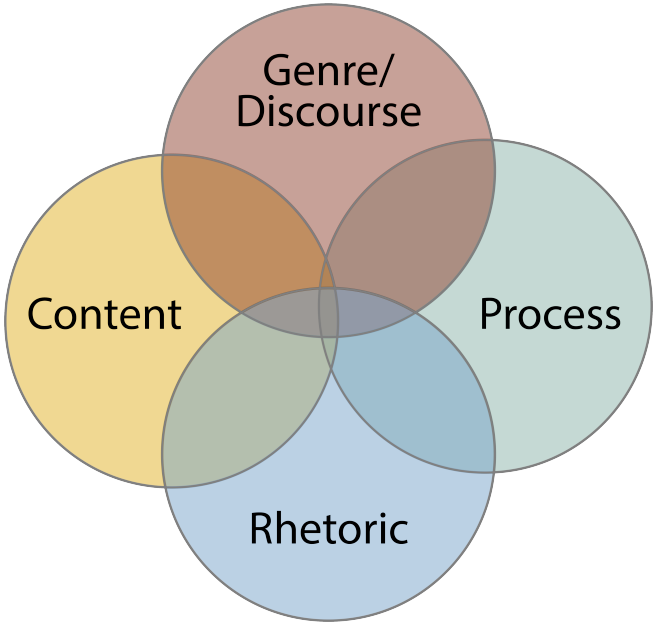
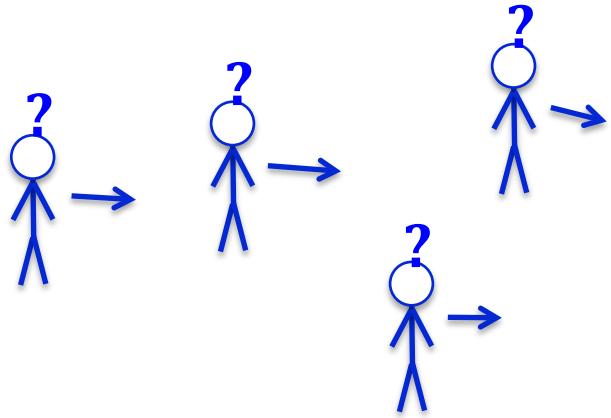
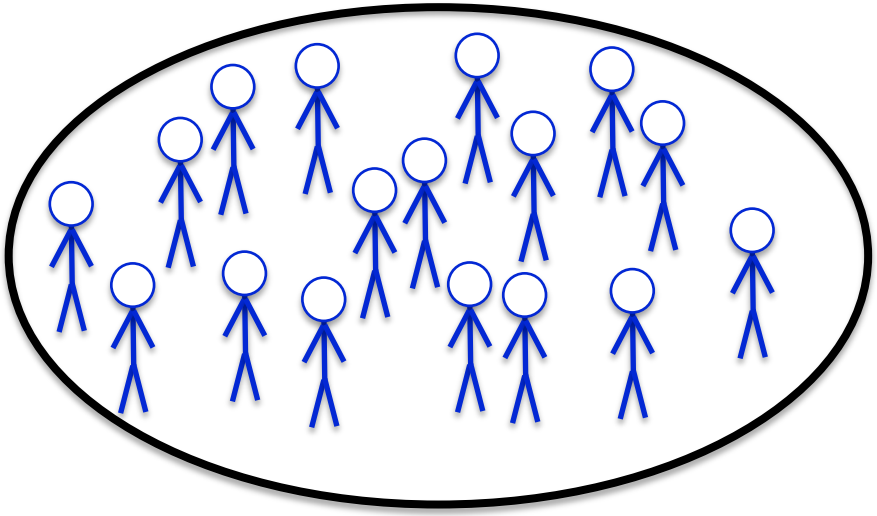
How do these concepts inform teaching?





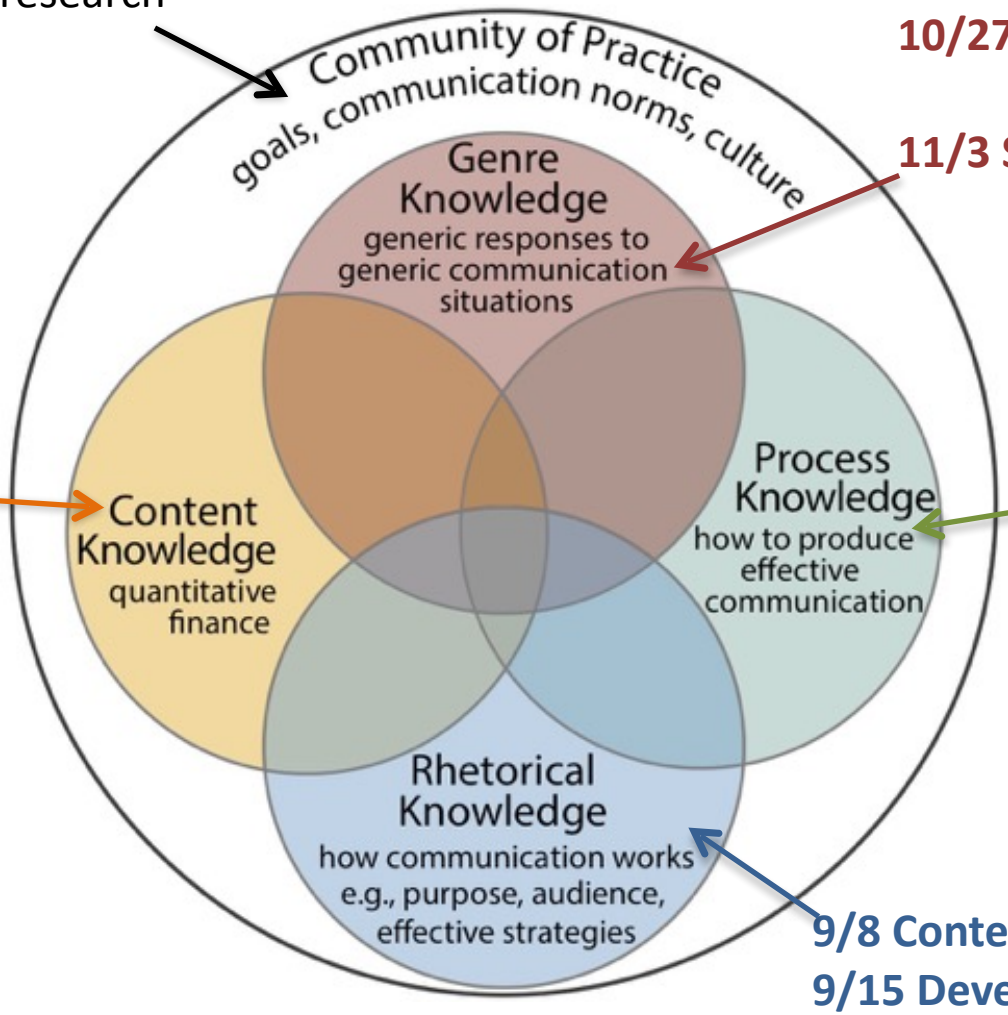
Students



Community of Mathematicians

Designing Curriculum

10/20 Rhetorical context in industry
11/17 Reproducible research



10/27 Model documentation

11/3 Statistical analyses

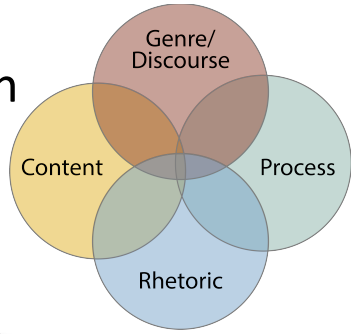
9/8 linear algebra
9/15 fixed income assets
9/22 forward rate agreements
10/13 quiz review
10/27 regulation
11/3 statistics

9/8 Sweave
9/22 Tools for figures
12/1 Informal peer critique

9/8 Context/Summary
9/15 Development
9/22 Figure Design
10/6 Flow
12/1 Elevator Pitch

Example is from Topics in Math with Applications to Finance

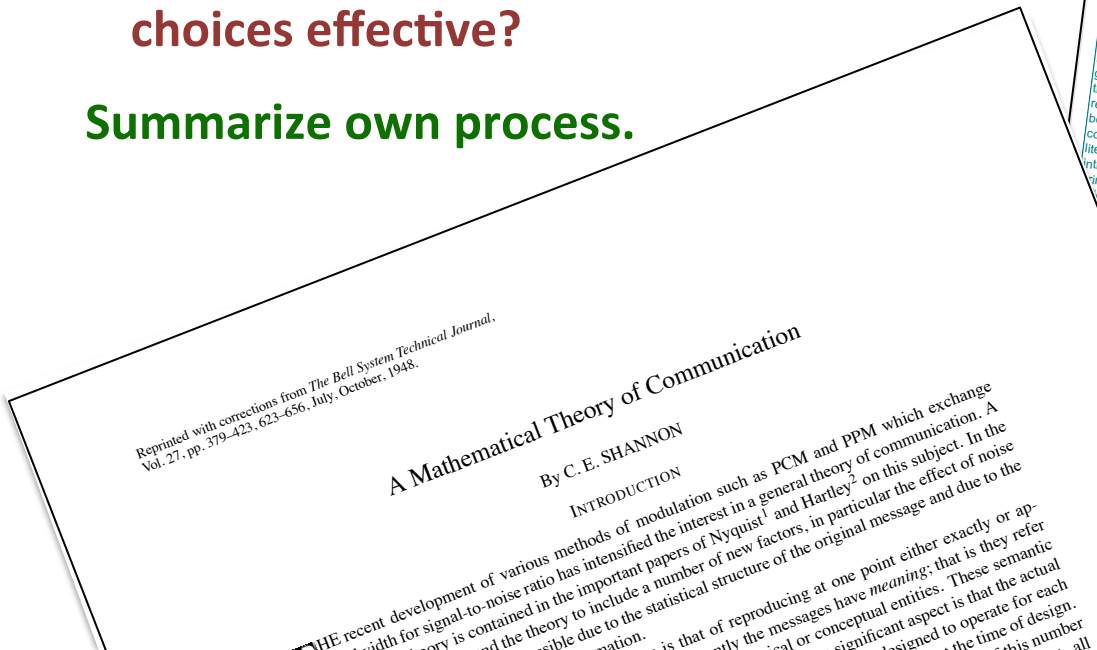
Teaching Paper Writing Reading Assignment + Discussion



Choose a published paper that reinforces course content.

- What is the purpose of the paper?
- What strategies does the author use to
  - –convince readers?
  - –help readers understand?
  - –interest readers?
- Which conventions noted on "Maximum Overhang" does the author follow? Are these choices effective?

Summarize own process.



Reprinted with corrections from *The Bell System Technical Journal*, Vol. 27, pp. 379–423, 623–656, July, October, 1948.

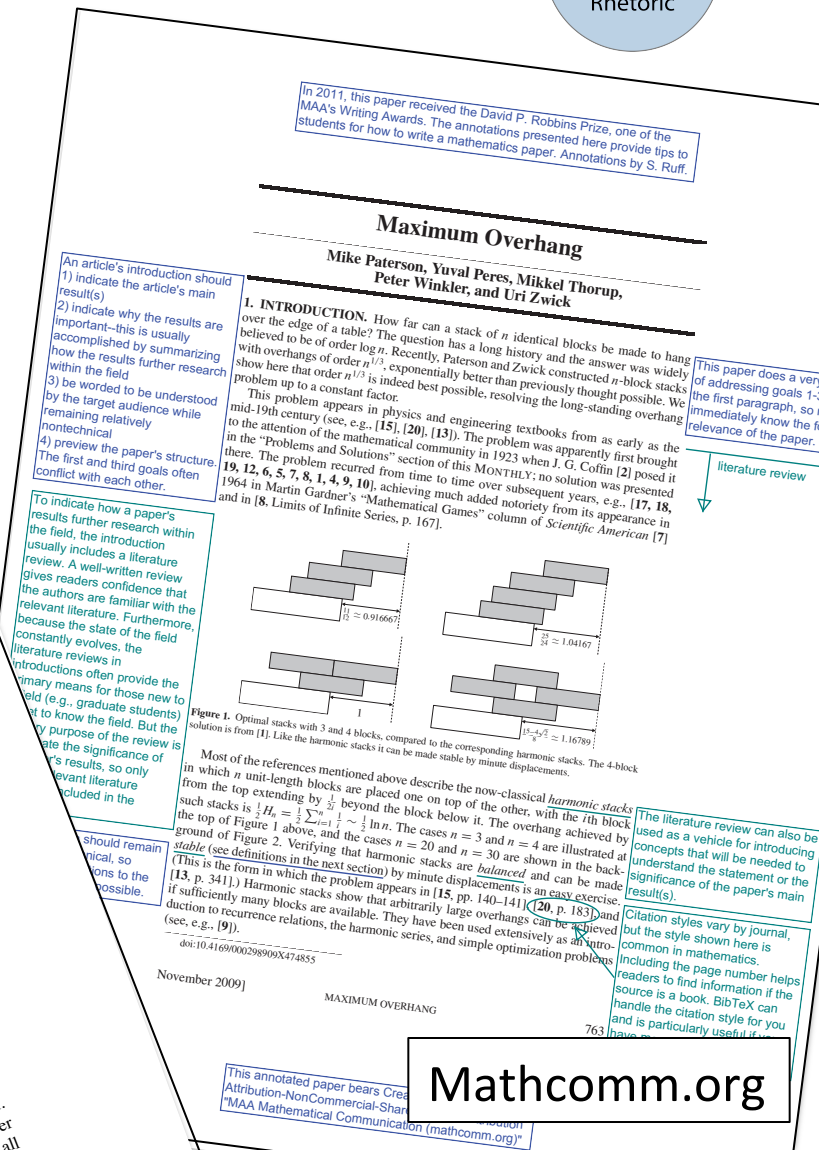
A Mathematical Theory of Communication

By C. E. SHANNON

INTRODUCTION

THE recent development of various methods of modulation such as PCM and PPM which exchange bandwidth for signal-to-noise ratio has intensified the interest in a general theory of communication. A basis for such a theory is contained in the important papers of Nyquist[1] and Hartley[2] on this subject. In the present paper we will extend the theory to include a number of new factors, in particular the effect of noise in the channel and the savings possible due to the statistical structure of the original message and due to the characteristics of the ultimate receiver.

The fundamental problem of reproducing at one point either exactly or approximately a message selected at another point. Frequently the messages have *meaning*; that is they refer to or are correlated according to some system with certain physical or conceptual entities. These semantic aspects of communication are irrelevant to the engineering problem. The significant aspect is that the actual message is one selected from a set of possible messages. The system must be designed to operate for each possible selection, not just the one which will actually be chosen since this is unknown at the time of design.



In 2011, this paper received the David P. Robbins Prize, one of the MAA's Writing Awards. The annotations presented here provide tips to students for how to write a mathematics paper. Annotations by S. Ruff.

Maximum Overhang

Mike Paterson, Yuval Peres, Mikkel Thorup, Peter Winkler, and Uri Zwick

An article's introduction should 1) indicate the article's main result(s) 2) indicate why the results are important--this is usually accomplished by summarizing how the results further research within the field 3) be worded to be understood by the target audience while remaining relatively nontechnical 4) preview the paper's structure. The first and third goals often conflict with each other.

This paper does a very of addressing goals 1-3 the first paragraph, so immediately know the relevance of the paper.

literature review

To indicate how a paper's results further research within the field, the introduction usually includes a literature review. A well-written review gives readers confidence that the authors are familiar with the relevant literature. Furthermore, because the state of the field constantly evolves, the literature reviews in introductions often provide the primary means for those new to field (e.g., graduate students) to know the field. But the purpose of the review is ate the significance of 's results, so only evant literature cluded in the

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1. INTRODUCTION. How far can a stack of n identical blocks be made to hang over the edge of a table? The question has a long history and the answer was widely believed to be of order $\log n$. Recently, Paterson and Zwick constructed n-block stacks with overhangs of order $n^{1/3}$, exponentially better than previously thought possible. We show here that order $n^{1/3}$ is indeed best possible, resolving the long-standing overhang problem up to a constant factor.

This problem appears in physics and engineering textbooks from as early as the mid-19th century (see, e.g., [**15**], [**20**], [**13**]). The problem was apparently first brought to the attention of the mathematical community in 1923 when J. G. Coffin [**2**] posed it in the "Problems and Solutions" section of this MONTHLY; no solution was presented there. The problem recurred from time to time over subsequent years, e.g., [**17, 18, 19, 12, 6, 5, 7, 8, 1, 4, 9, 10**], achieving much added notoriety from its appearance in 1964 in Martin Gardner's "Mathematical Games" column of *Scientific American* [**7**] and in [**8**. Limits of Infinite Series, p. 167].

Figure 1. Optimal stacks with 3 and 4 blocks, compared to the corresponding harmonic stacks. The 4-block solution is from [**1**]. Like the harmonic stacks it can be made stable by minute displacements.

Most of the references mentioned above describe the now-classical *harmonic stacks* in which n unit-length blocks are placed one on top of the other, with the ith block from the top extending by $\frac{1}{2i}$ beyond the block below it. The overhang achieved by such stacks is $\frac{1}{2}H_n = \frac{1}{2}\sum_{i=1}^{n} \frac{1}{i} \sim \frac{1}{2}\ln n$. The cases $n = 3$ and $n = 4$ are illustrated at the top of Figure 1 above, and the cases $n = 20$ and $n = 30$ are shown in the background of Figure 2. Verifying that harmonic stacks are *balanced* and can be made *stable* (see definitions in the next section) by minute displacements is an easy exercise. (This is the form in which the problem appears in [**15**, pp. 140–141] and [**20**, p. 183] and [**13**, p. 341].) Harmonic stacks show that arbitrarily large overhangs can be achieved if sufficiently many blocks are available. They have been used extensively as an introduction to recurrence relations, the harmonic series, and simple optimization problems (see, e.g., [**9**]).

The literature review can also be used as a vehicle for introducing concepts that will be needed to understand the statement or the significance of the paper's main result(s).

Citation styles vary by journal, but the style shown here is common in mathematics. Including the page number helps readers to find information if the source is a book. BibTeX can handle the citation style for you and is particularly useful if

doi:10.4169/000298909X474855

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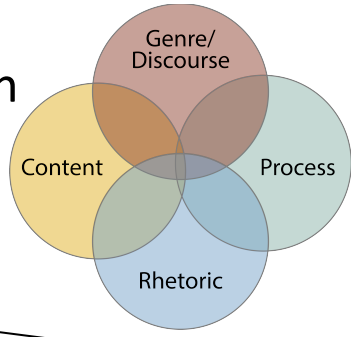
Teaching Paper Writing Reading Assignment + Discussion



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What strategies does the author use to

–convince readers?

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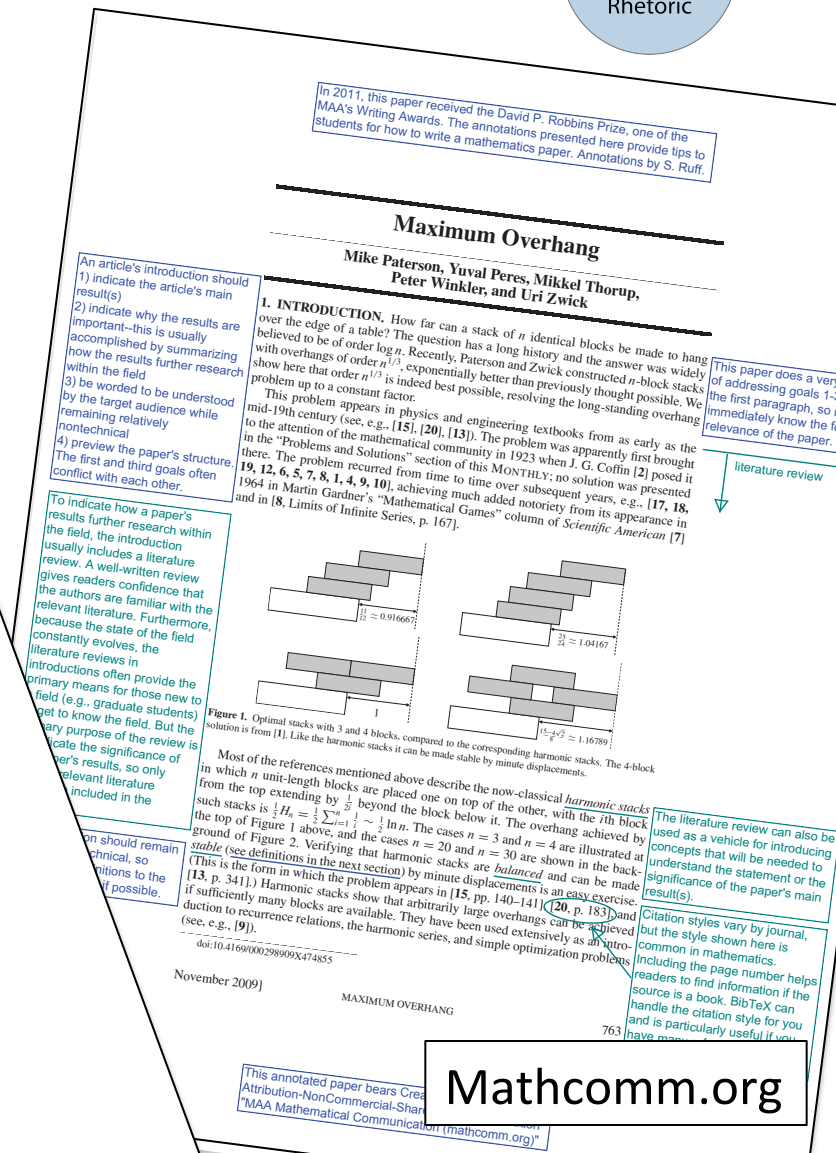
Which conventions noted on "Maximum Overhang" does the author follow? Are these choices effective?

Summarize own process.

What are some well-written papers that you could annotate or have students analyze?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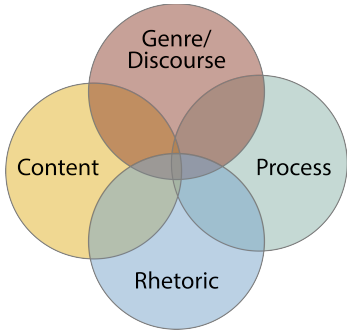
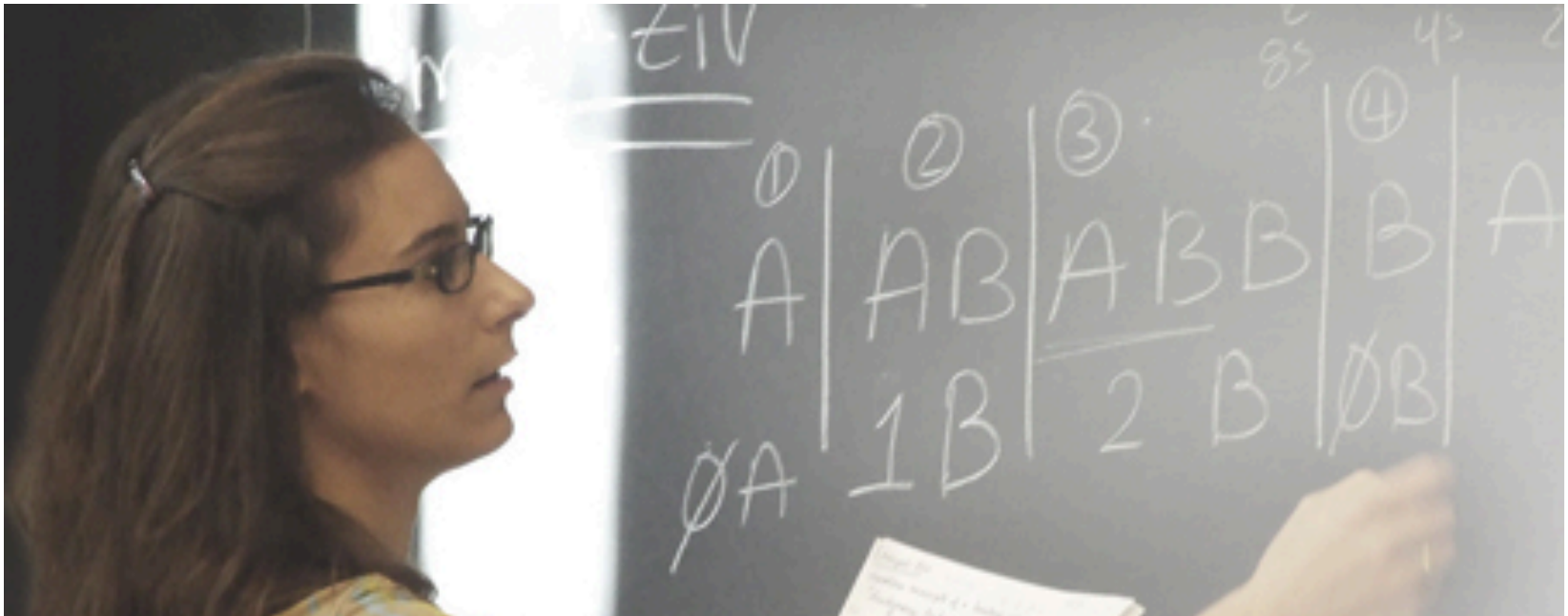
Designing Assignments:
Seminar presentations + associated genre system



Process is scaffolded into assignment sequence:

- Content review with course lead
- Practice presentation with me
- Write presentation abstract for classmates
- Present to classmates
- Classmates provide feedback
- Write lecture notes for classmates

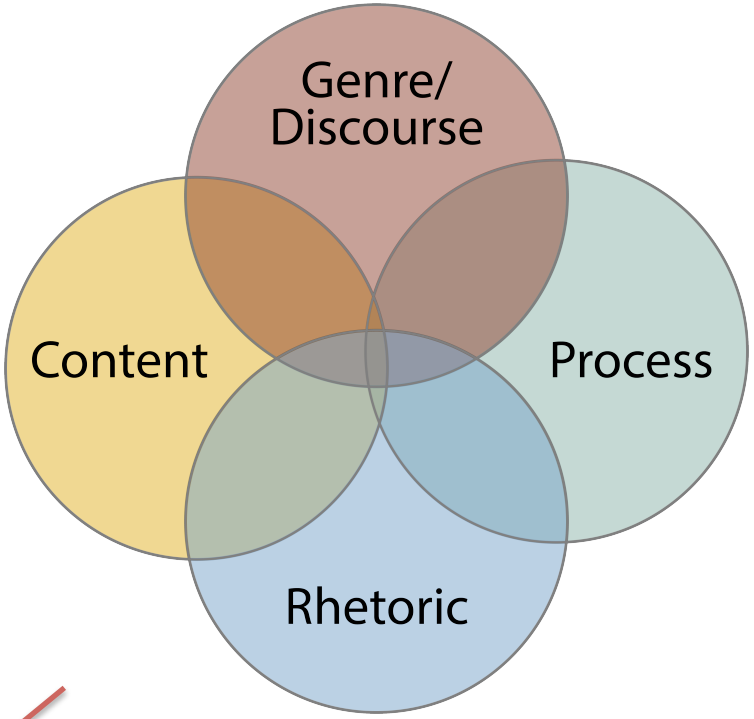


Providing Feedback

It's conventional to write the introduction as though readers haven't seen the abstract.

Discussing this baby case does a good job of achieving the purposes of an introduction while avoiding technicalities.

~~You keep making sign errors.~~



Add to your editing checklist: check signs throughout

Why can we swap the integrals here?

You could write the point of each paragraph in the margin to create a "retroactive outline" that's likely to reveal ways to restructure.

~~Insert "right-to-left"~~

I don't understand: are you working left-to-right or right-to left?

Has your audience seen this approach before?
If not, will they need to see a concrete example,
or is it obvious enough that citation is sufficient?

We're teachers not editors

Grading

Content

Genre/ Discourse

Rhetoric

Process

Grading Rubric for 18.821 Papers (20 points total)

Spring, 2018

Mathematical Correctness and Vision (10)

9–10 The students discovered something remarkable and provided exceptionally elegant explanations of the phenomena they identified.

7–8 The students discovered something substantial and explained convincingly the phenomena they found (i.e., proofs are rigorous; conjectures are supported with convincing evidence).

5–7 The students made substantial progress and offered explanations for the phenomena they identified (i.e., claims are rigorously stated and support goes beyond a few specific examples).

3–5 The students gave a good expository description of the problem and of the most interesting aspects of the phenomena they found (e.g., conjectures are stated).

1–3 The students described the problem and found some immediately apparent aspects of it.

Exposition (6)

6 The paper is exceptionally interesting and engaging.

5 The paper is easy to read and understand and is well suited to the target audience (peers of the authors). The paper is consistent and cohesive (not just 3 parts pasted together); the paper is focused and structured and the structure is communicated to readers; new ideas are introduced efficiently and with proper motivation; displays and examples are well chosen to aid understanding; mathematical language and notation are used appropriately; citations clearly acknowledge any sources used; writing is accurate, appropriately concise, and carefully proofread.

4 Many of the criteria for a grade of 5 are met. The paper is sufficiently clear that peers can easily discern what was intended whenever expository roughness is encountered.

3–4 Peers must expend some effort to discern what was intended when expository roughness is encountered.

1–3 Substantial effort is needed to discern what was intended.

Research and Writing Process (4) All teammates contributed substantively and to the writing and attended all meetings. The draft was complete and careful revision took into account but was not limited to the feedback of course staff and of teammates.

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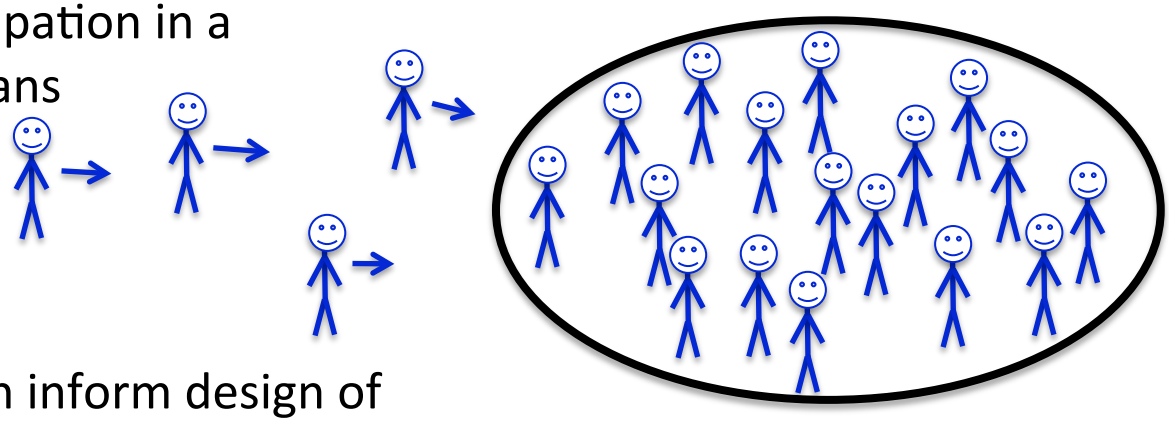
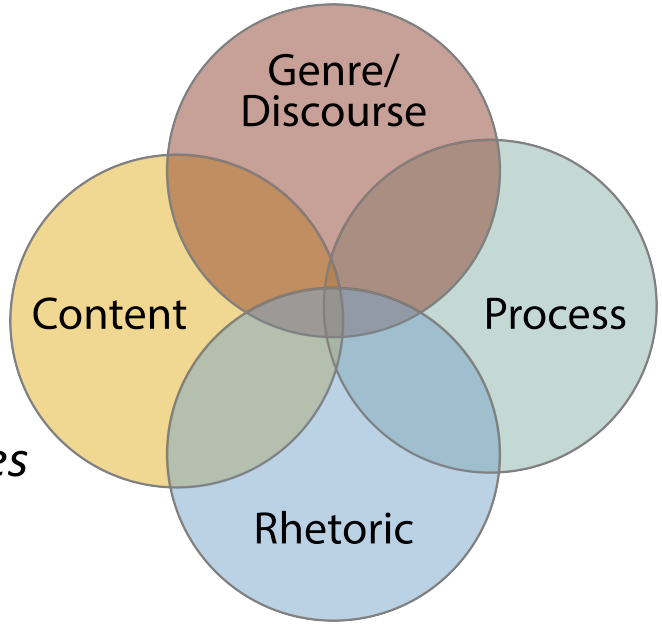
Teaching Students to Communicate as Mathematicians

Demystify communication of mathematicians, e.g., via legitimate peripheral participation in a community of mathematicians

…and reading.



The knowledge domains can inform design of curriculum, instruction, assignments, feedback, and grading.



Some Resources

MAA Mathematical Communication
mathcomm.org

Bahls, *Student Writing in the Quantitative Disciplines*

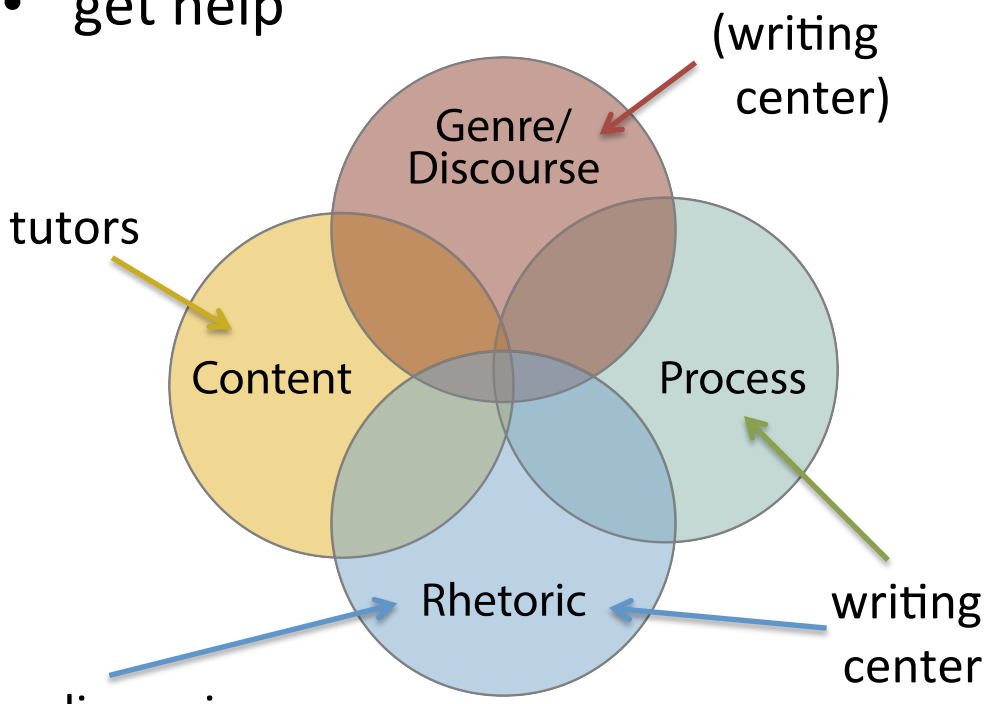
Gopen & Swan, "The Science of Scientific Writing"
American Scientist, 1990.

Wolfe, *Team Writing: A Guide to Working in Groups*

Thank you

Crafting feedback takes time

- prioritize (for yourself & students)
- give class-wide feedback
- meet with students
- be kind (to yourself & students)
- get help



If audience is peers,
peers can comment on effectiveness for audience

Student Name removed

This is a strong start, and you've clearly put a great de
paper. I particularly appreciate that these proofs, whic
in a way that is accessible to the reader. Your formatti
LaTeX environments to set apart things clearly? I also
where sections of proofs begin and end, and what stra
have any questions. I'm happy to meet to discuss you

Edge Probability of a Rand
of the Existence of Iso

Good abstract.

Abstract: This paper analyze
cliques in random graphs. I show tha
existence of an isolated vertex at ln
also show the size of the largest expe
order of 2log(n) (log base 2), and I
that the random graph G(n,p) is ex

Strong introduction.
Gets right to the point.

Section 1. Introduction. A r
tices in which every possible edge is
independently of each other. Rand
structures such as social networks
the graphs are so large that globa
nections are probabilistic. Studyin
understand structures within the r
In this paper, I will examine 3 rela
probability of isolated vertices and
$G(n,p)$. Since random graphs can
edges not present, I will use the ex
vertices to understand how the e
$G(n,p)$.

See note P4.

In Section 2, I will analyze th
that $G(n,p)$ has an isolated verte
incident to it. I will show that if
G has an isolated vertex goes to
some $c < 1$, the probability that