

Impacting students' practice of mathematics



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blogs.uoregon.edu/practiceofmathematics

Warm-up Activity

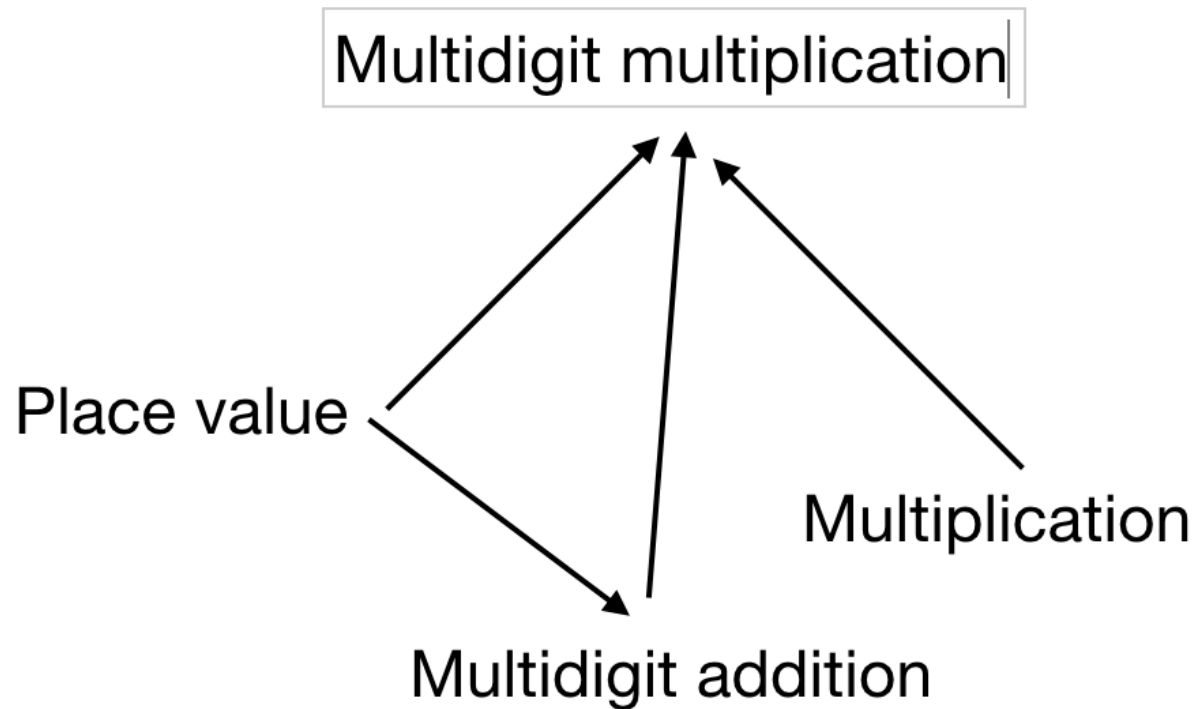


In which class, and for what purpose, have I used the following in-class question?

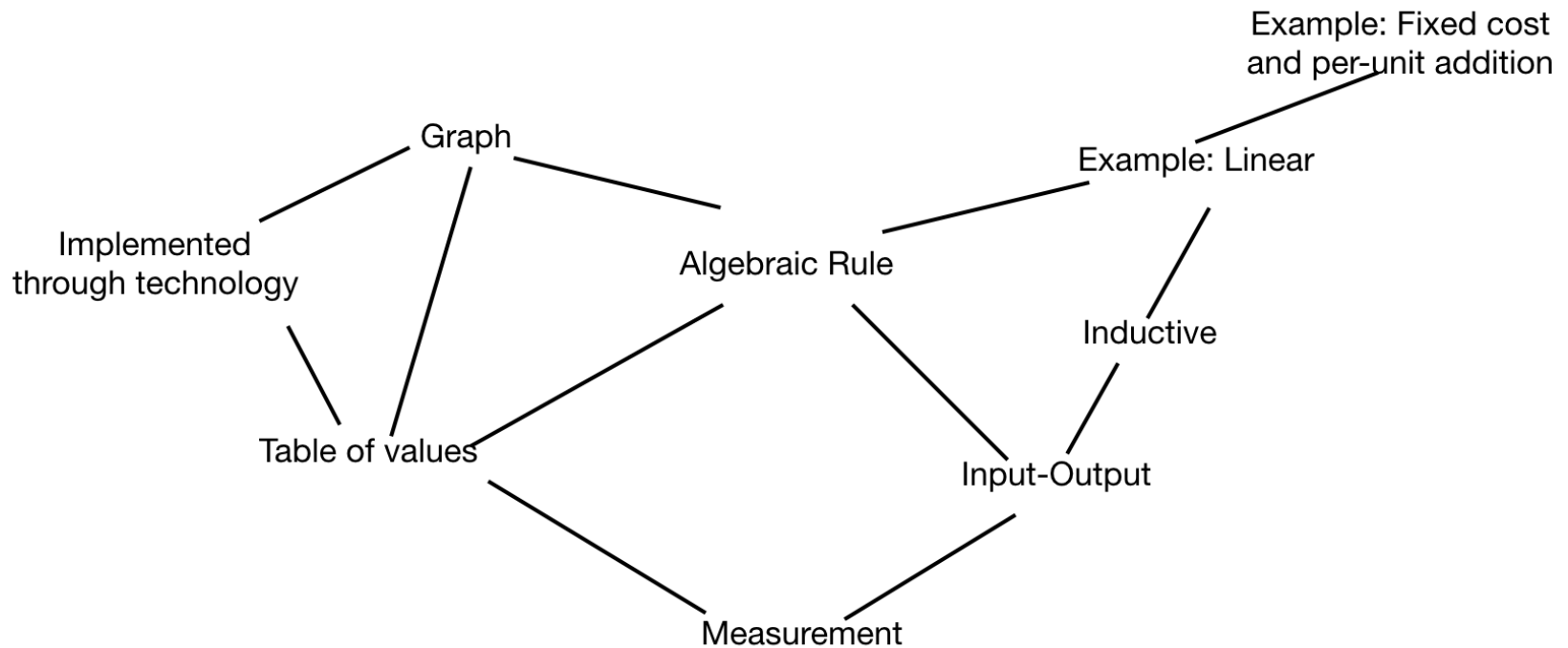
What does it mean when you enter $\sqrt{2}$ in your calculator and it says 1.4...? Give a complete explanation.

(Your answers in the chat box, please.)

Topologist's view: knowledge networks



Topologist's view: knowledge networks (function)



Topologist's view

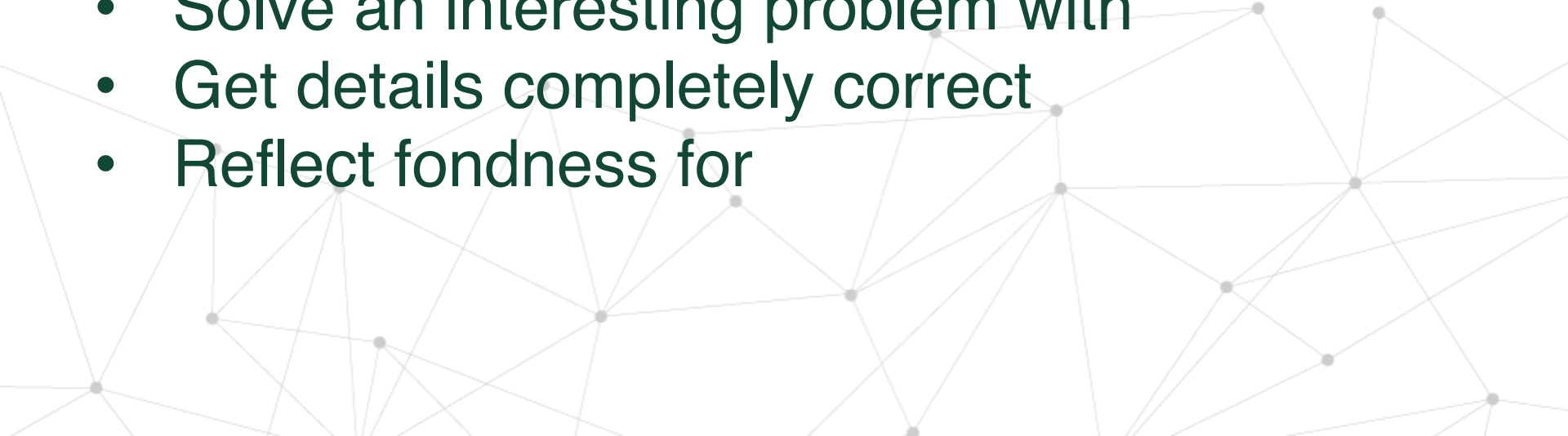
Proposition: a curriculum defines a path through the barycentric subdivision of the “knowledge complex”.



Neglected variables: Practices



But there's something missing here: student actions! What do we want them to be able to DO with this knowledge:

- Recall (an algorithm)
 - Explain (prove!)
 - Use to understand the world
 - Solve an interesting problem with
 - Get details completely correct
 - Reflect fondness for
- 

I - Introduction to proofs course

Inspired by the PROMYS/ Ross programs



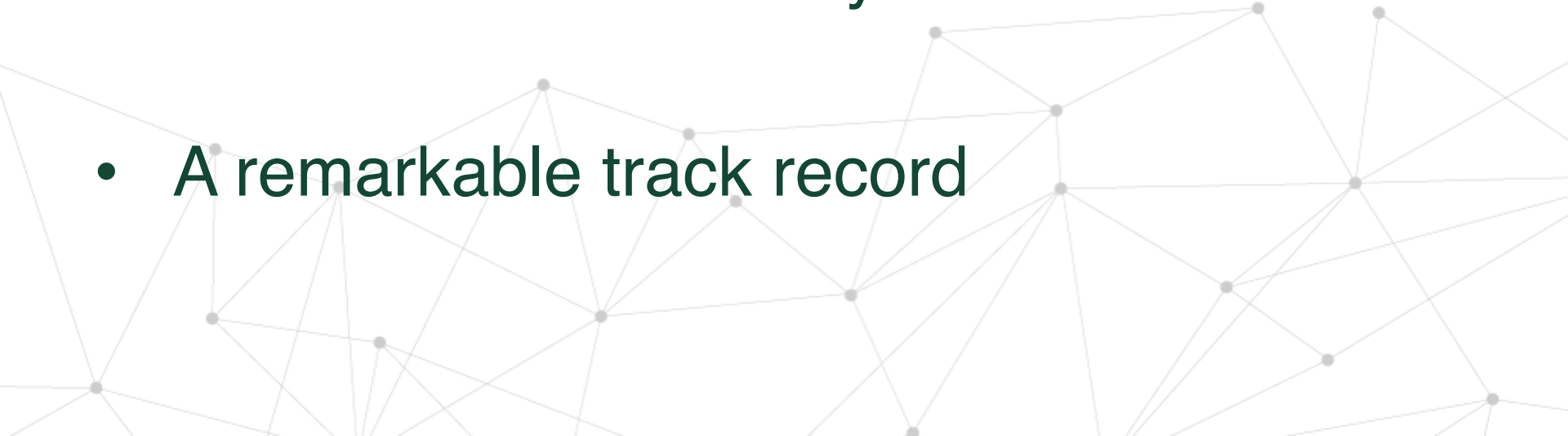
Arnold Ross



Glenn Stevens

Ross/ PROMYS programs



- Exploration
 - “Prove or disprove and salvage if possible”
 - Lecture trails worksheets
 - Attention to community
 - A remarkable track record
- 

Introduction to Proofs

Square root of two task.

What does it mean when you enter $\sqrt{2}$ in your calculator and it says 1.4...? Give a complete explanation.

- Many students have no idea what is being asked for.
- Most students write down only

$$1.4 < \sqrt{2} < 1.5$$
$$1.96 < 2 < 2.25$$



Square root of two task – purpose and design

Shout out to



Purposes

- create disequilibrium
- spur discussion of verbal nature of proofs
- spur discussion of “reversing steps”

Design

- intentional lack of scaffolding
- soliciting a common (incomplete, in this case) response, for further discussion.

Square root of two task

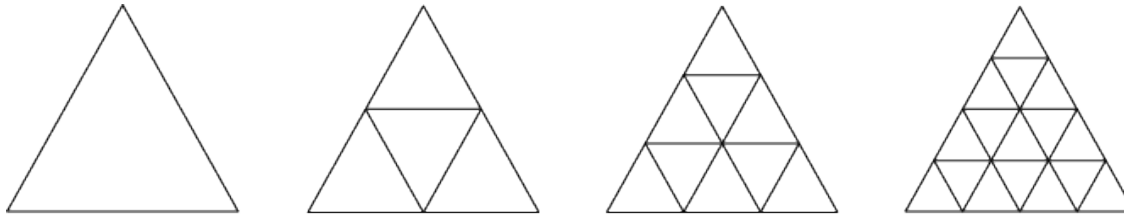


No new content is involved, though some deepening of latent knowledge of real number system.

Almost all cognitive load is in practices rather than content.



Triangle counting task (revised)



1. What do you notice? What do you wonder?
2. Give a precise description of a way to generate and continue this series of figures as well as the related series where the size of the smallest triangle remains constant.
3. Count the number of smallest triangles in each figure and make a conjecture as to what the number of smallest triangles will be in the n th figure.
4. Prove that your conjecture holds.
5. Make a conjecture about and try prove something you noticed or wondered about.

Triangle counting task

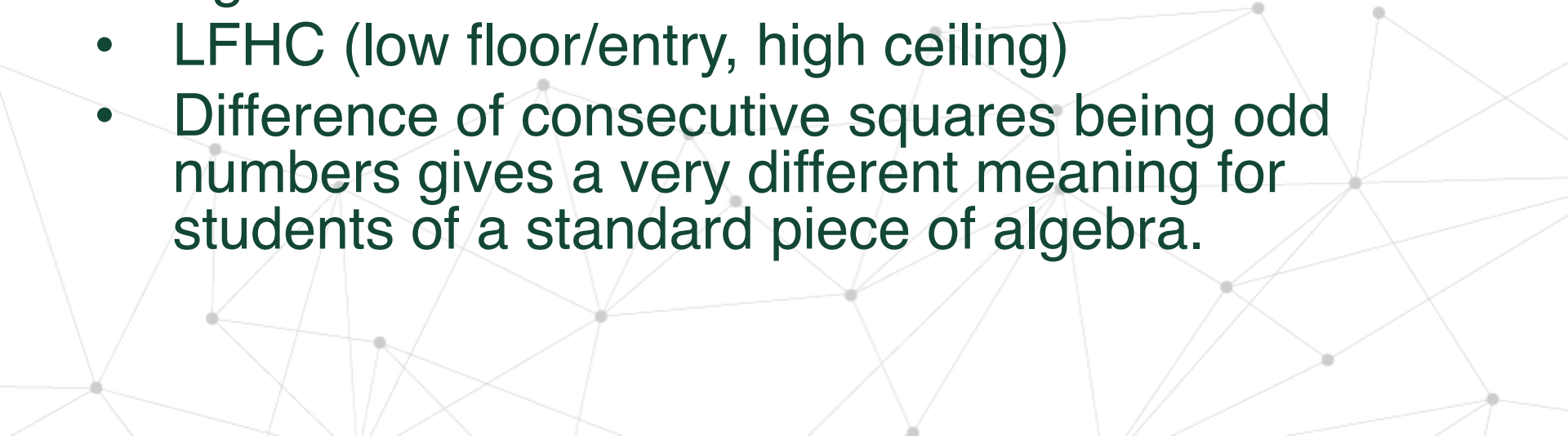
Purpose and design



Purposes:

- Engage students in conjecture
- Preview work on induction and arithmetic sequences

Design:

- LFHC (low floor/entry, high ceiling)
 - Difference of consecutive squares being odd numbers gives a very different meaning for students of a standard piece of algebra.
- 

Not entirely Seinfeldian



A thread running through the course is continued fraction representation of Golden ratio.

This brings together conjecture, induction, estimates and properties of convergent sequences.



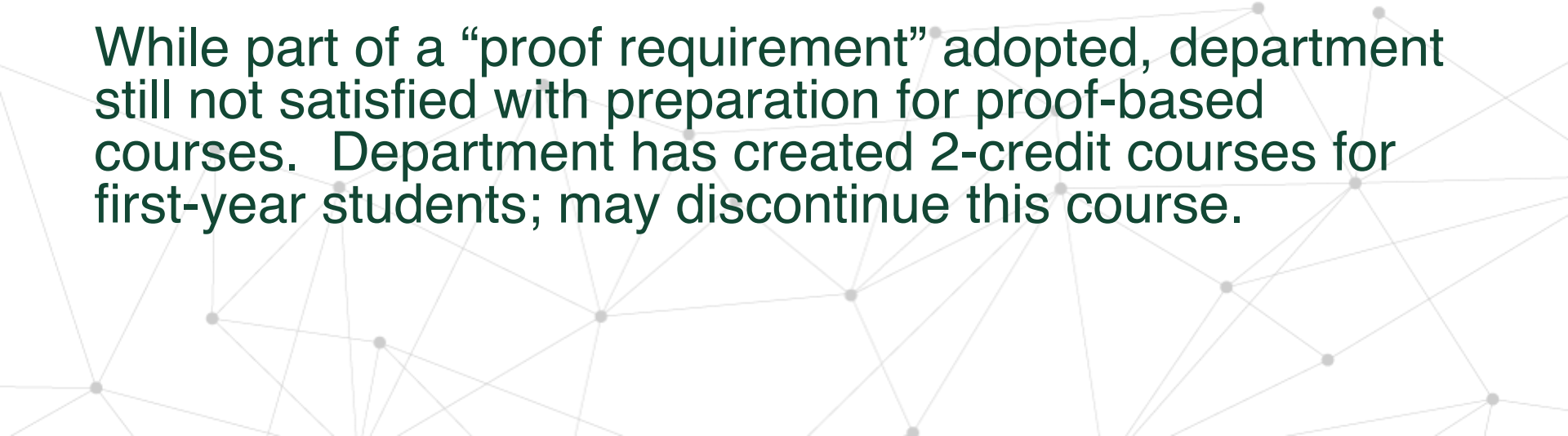
Lack of dissemination



Anecdotal success.

Nothing but choice of book (D'Angelo-West, which wasn't always followed) passed on to most colleagues teaching the course in following decade. In particular, these tasks not shared. Hardly, if at all, taught as developed.


While part of a “proof requirement” adopted, department still not satisfied with preparation for proof-based courses. Department has created 2-credit courses for first-year students; may discontinue this course.



II - Mathematics for pre-service elementary teachers

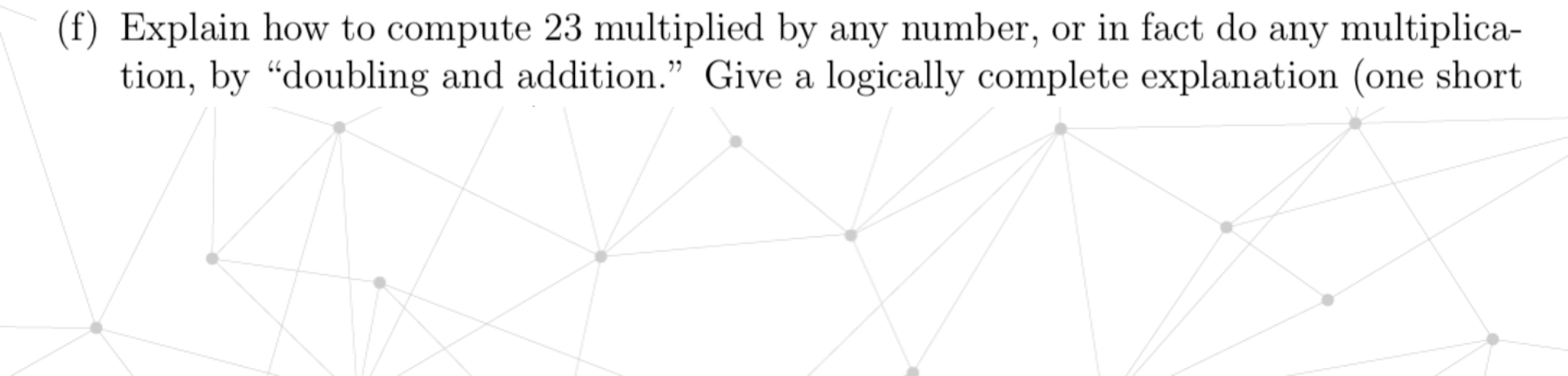


Similar to “Introduction to proofs” in need for mathematical reasoning, now codified through the arguments given in the Common Core State Standards for Mathematics (which can be read as a mathematical, a pedagogical and a policy document).



Double 23 Task



- (20 points) (a) Take the number 23 and write it, then double it, then double that, etc. a total of six times.
- (b) Take the first number on your list (namely 23), the fourth number and the sixth number, and add them together.
- (c) Compare what you got in the previous step with 23×41 . Explain.
- (d) Using your list, can you quickly calculate 23×18 ?
- (e) Extend your list if necessary to use it to calculate 23×75 .
- (f) Explain how to compute 23 multiplied by any number, or in fact do any multiplication, by “doubling and addition.” Give a logically complete explanation (one short
- 

Double 23 Task – purpose and design

Purposes:

- present a mystery
- put properties, namely distributivity, to use, instead of just naming
- discussion of how distributive law and factoring are the same equality)
- make use of base-two (cf. Russian Peasant algorithm).

Design:

- in class, so minimal scaffolding can be given as needed.
- wide range of success possible, from “Yes, I could do this for any number” to naming variables for full argument.

Fraction multiplication error task



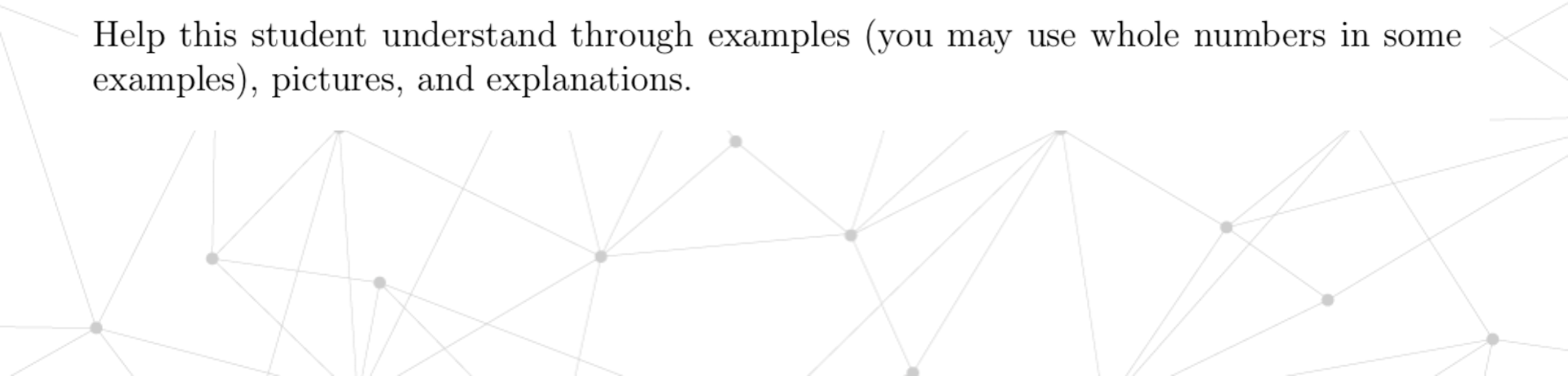
(8 points) What error is a student consistently making in the following? In particular, which algorithm is this student confusing with the multiplication algorithm?

$$\frac{2}{3} \times \frac{5}{6} = \frac{20}{6} = \frac{10}{3}$$

$$\frac{1}{5} \times \frac{3}{2} = \frac{30}{10} = 3$$

$$\frac{3}{4} \times \frac{5}{4} = \frac{15}{4}$$

Help this student understand through examples (you may use whole numbers in some examples), pictures, and explanations.



Fraction multiplication error task

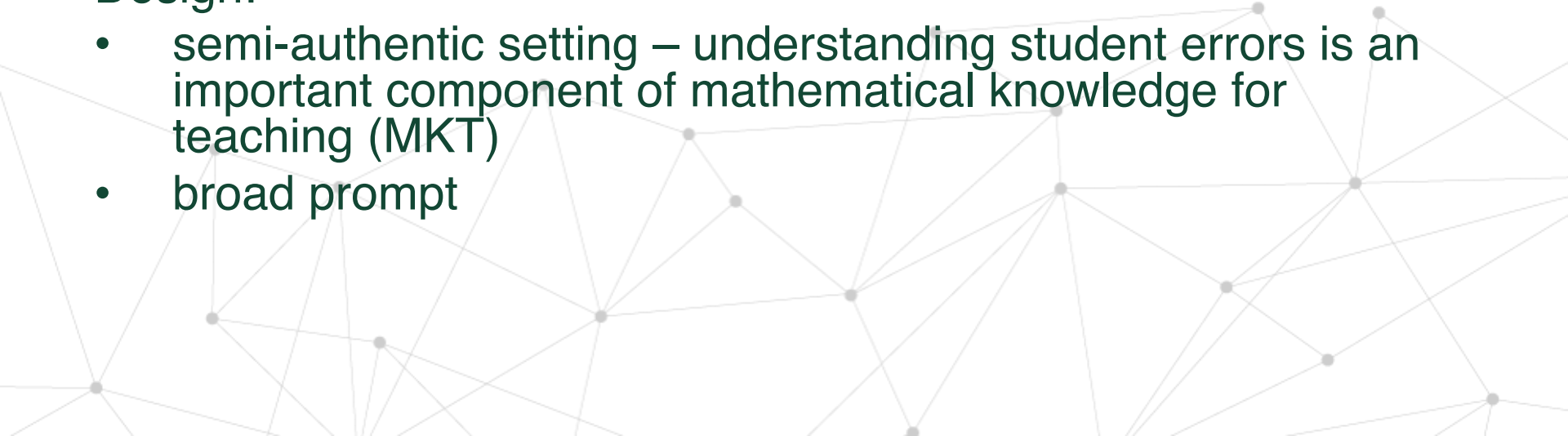
Purpose and design



Purpose:

- culmination for fraction addition and multiplication (students should use visual models, other meanings (“ $\frac{2}{3}$ times means $\frac{2}{3}$ of”) etc.
- opens wide avenues for discussion (why don’t we use a common denominator for multiplication?).

Design:

- semi-authentic setting – understanding student errors is an important component of mathematical knowledge for teaching (MKT)
 - broad prompt
- 

Course Outcomes



In short: focus on deep understanding of the number-related progressions in the CCSSM (Google “IME progressions”), using both numbers and variables (culminating with base b -imals!).

Tossed out: formal logic, sets, puzzles, cramming in all possible K-8 topics.




Course Outcomes



More at:

<https://blogs.uoregon.edu/practiceofmathematics/resources/>


By far my (our!) most mature course (re-)development.



Assessment



Two instructors, including myself, have switched to a portfolio assessment system, based on HW and exam items, which requires students to reflect on what they've learned and categorize their work.



Dissemination




Strong local dissemination:

- Shared Dropbox folder with activities
- Set of notes (unpublished)
- Support for new instructors, in particular with stable course leader.

No immediate plans to validate or more widely disseminate, though.

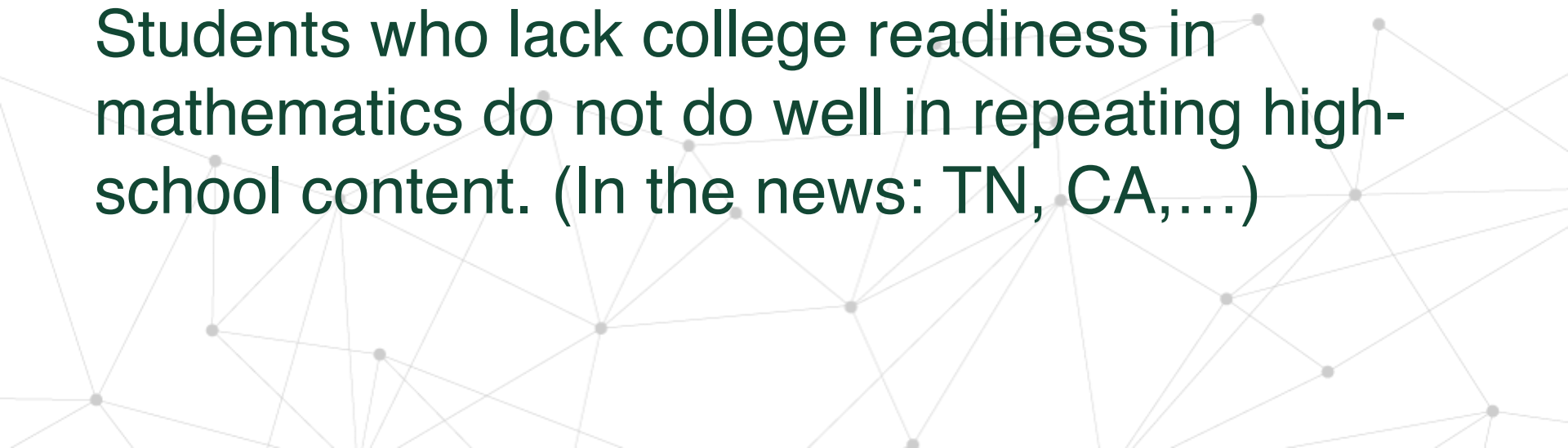


III – Mathematical modeling, as preparation for college algebra



Mathematical modeling is a neglected practice, not usually addressed by math or science classes.

Students who lack college readiness in mathematics do not do well in repeating high-school content. (In the news: TN, CA,...)



Mathematical Modeling



Student population of new students interested in STEM (Bio, Hphy) but tested into remedial math (“intermediate algebra”). Students taking same chemistry class as well.

UR minorities and first-generation college are over-represented.

All have taken algebra 2, many precalculus, some calculus(!).

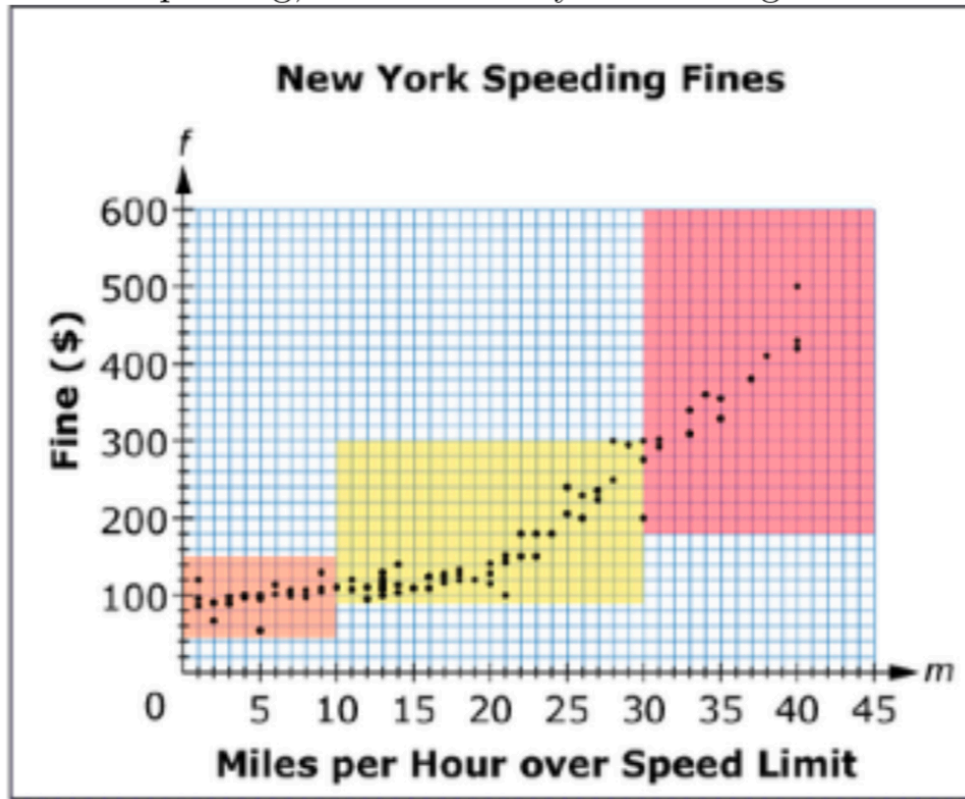
College credit earned on the basis of setting up and interpreting mathematical models, often using real data, including in projects.



Speeding fines task

(thanks to Smarter Balanced.)

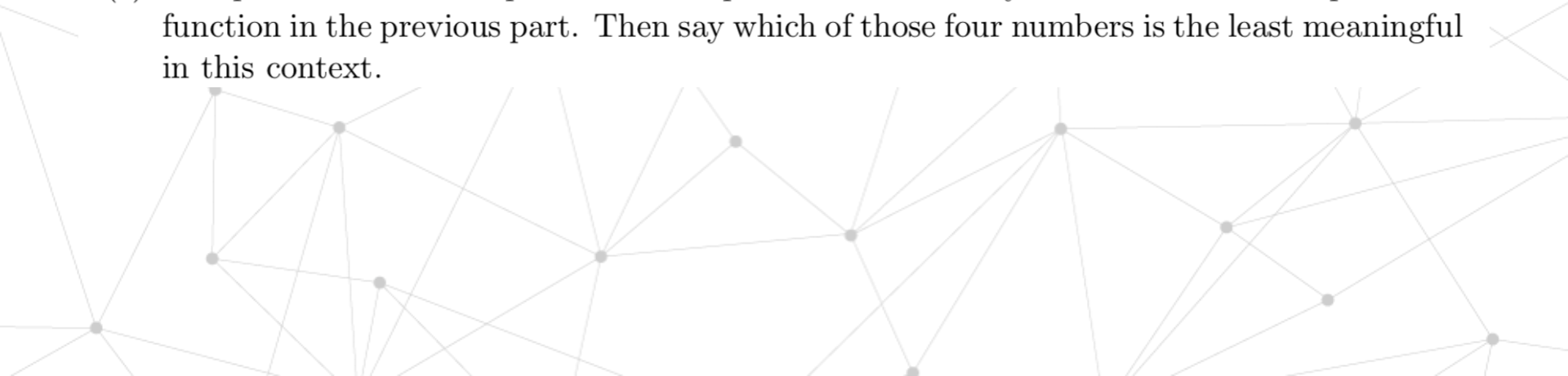
The state of New York has guidelines for fines for speeding based on the range of how much one was speeding, as indicated by the rectangles below. The data points are actual fines given.



Speeding fines task

(thanks to Smarter Balanced.)



- What do you notice? Name three features of this graph which catch your attention, and say what they mean in terms of the situation (speeding fines).
 - Say in words what the current guidelines for fines are, and then translate that to mathematical notation by naming variables and writing inequalities.
 - Referring to the data, argue why having a function which determines the amount of each speeding fine would be more fair than the current system.
 - Find a *piecewise linear* function which fits the data well, with one linear function for under 20mph over the speed limit and another for over 20mph over the speed limit.
 - Interpret each of the slopes and intercepts of the two lines you used to define the piecewise function in the previous part. Then say which of those four numbers is the least meaningful in this context.
- 

Speeding fines task

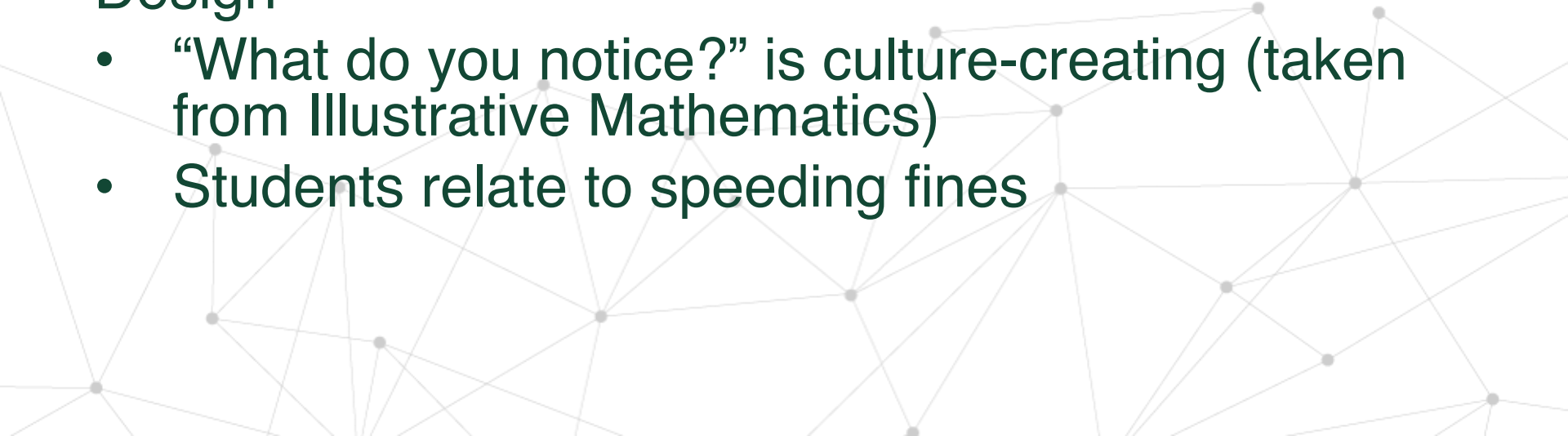
(thanks to Smarter Balanced.)



Purposes

- Opportunities to read graphs with real data.
- Mixing math with other thinking!
- See usefulness of different forms

Design

- “What do you notice?” is culture-creating (taken from Illustrative Mathematics)
 - Students relate to speeding fines
- 

Barbie Bungee task, college variant



Popular high-school task

<http://fawnnguyen.com/barbie-bungee-revisited-better-class-lists/>

College version – account for different characters, which requires a second regression (for spring constant as a function of weight) and a multivariable function. Use Google sheets; write a scientific report.

Thanks, #MTBoS (aka #math-teacher-twitter)!




Barbie Bungee task, college variant



Purpose:

- Engage in full modeling process, early in class.

Design:

- Only start with materials and question.
 - Ask for scientific report (with sample file provided).
- 

Low-oxygen paper reading task

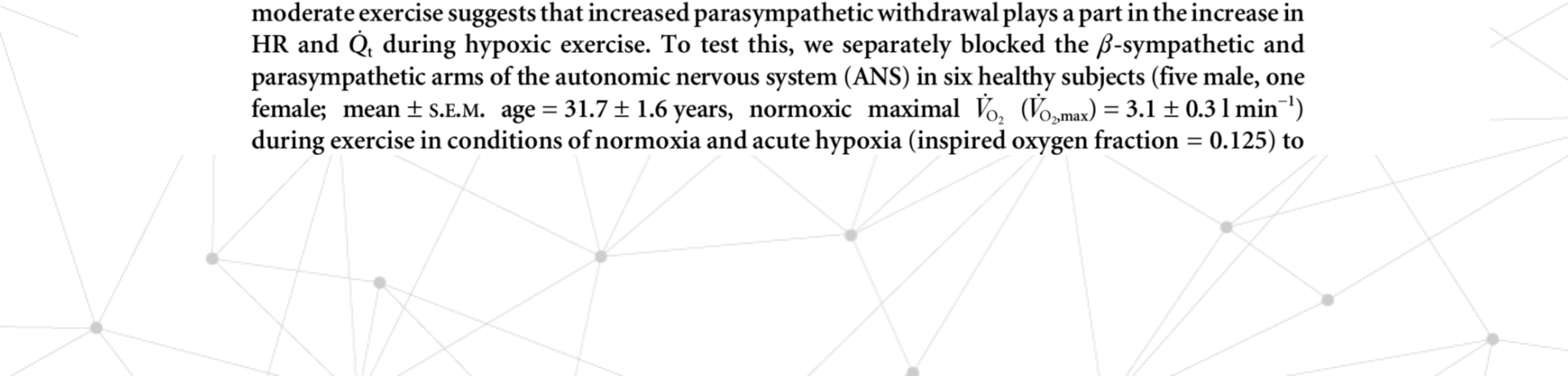


β -Adrenergic or parasympathetic inhibition, heart rate and cardiac output during normoxic and acute hypoxic exercise in humans

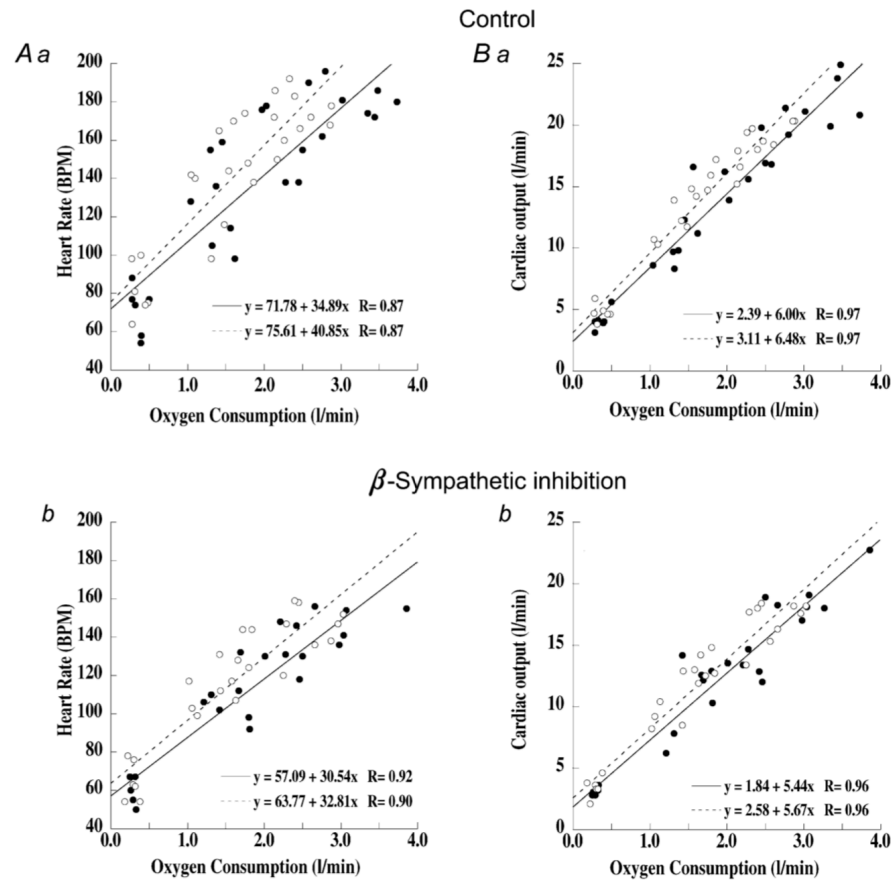
Susan R. Hopkins, Harm J. Bogaard, Kyuichi Niizeki, Yoshiki Yamaya, Michael G. Ziegler and Peter D. Wagner

Department of Medicine, University of California, San Diego, La Jolla, CA 92093, USA

Acute hypoxia increases heart rate (HR) and cardiac output (\dot{Q}_t) at a given oxygen consumption (\dot{V}_{O_2}) during submaximal exercise. It is widely believed that the underlying mechanism involves increased sympathetic activation and circulating catecholamines acting on cardiac β receptors. Recent evidence indicating a continued role for parasympathetic modulation of HR during moderate exercise suggests that increased parasympathetic withdrawal plays a part in the increase in HR and \dot{Q}_t during hypoxic exercise. To test this, we separately blocked the β -sympathetic and parasympathetic arms of the autonomic nervous system (ANS) in six healthy subjects (five male, one female; mean \pm S.E.M. age = 31.7 ± 1.6 years, normoxic maximal \dot{V}_{O_2} ($\dot{V}_{O_{2,max}}$) = 3.1 ± 0.3 l min⁻¹) during exercise in conditions of normoxia and acute hypoxia (inspired oxygen fraction = 0.125) to



Low-oxygen paper reading task



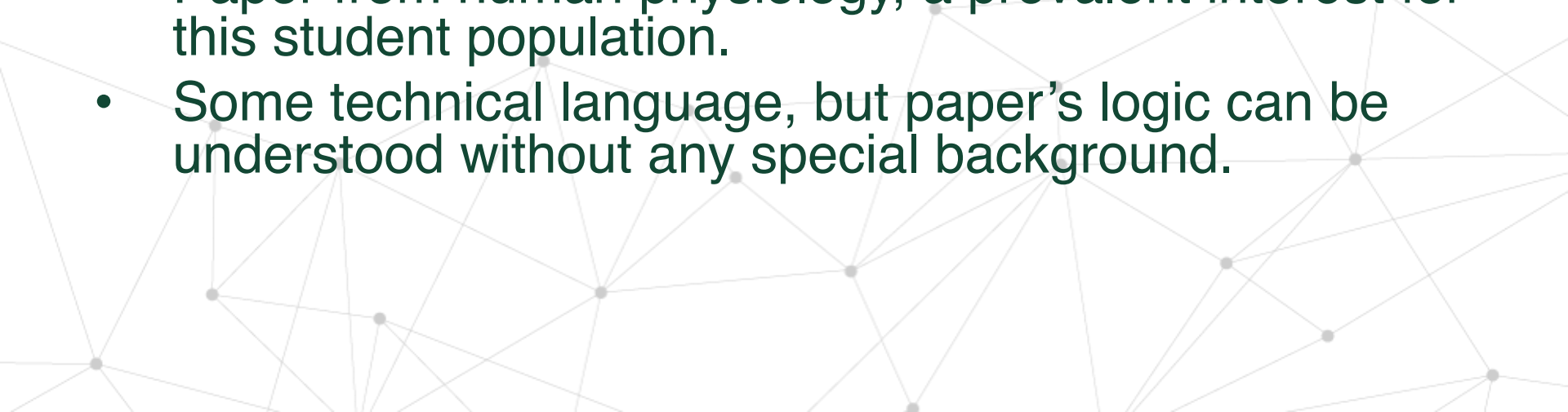
Low-oxygen paper reading task



Purposes:

- Engage students in authentic university-based work
- Engage in mathematical interpretation as part of reading.


Design:

- Paper from human physiology, a prevalent interest for this student population.
 - Some technical language, but paper's logic can be understood without any special background.
- 

Coffee cooling task



Students run linear, exponential and then exponential-with-constant fits (in Google Sheets) with data provided.



Coffee cooling task


Purpose and design



Purposes:

- Evaluate quality of fits through expected long-term behavior
- Discussion of purpose of modeling to choose between models.
- Transform an exponential expression to interpret it.

Design:

- Google sheets gives exponential fit with a base of e .
 - Residuals are better for exponential model, with no constant!
- 

Assessment



Most “difficult” item (quiz):


A company had 1.2 million dollars in sales in 2012 and 1.8 million dollars in sales in 2016. Find a linear model for their sales as a function of time which fits these two data points, and interpret your model. Make sure to define your variables and name your function.

What did students do? Why was this “difficult”? Answers in chat box.

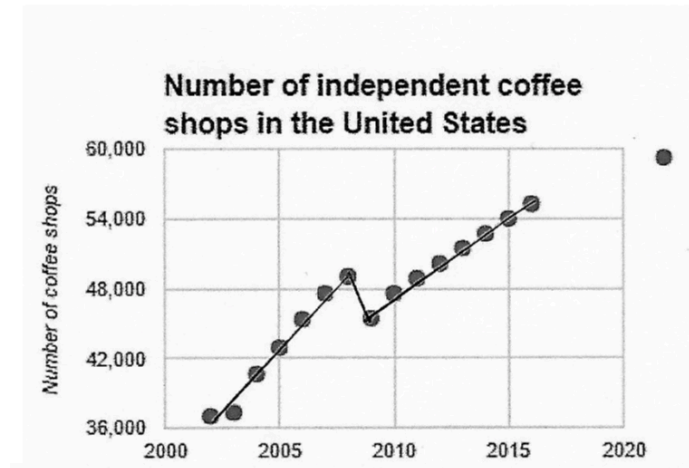
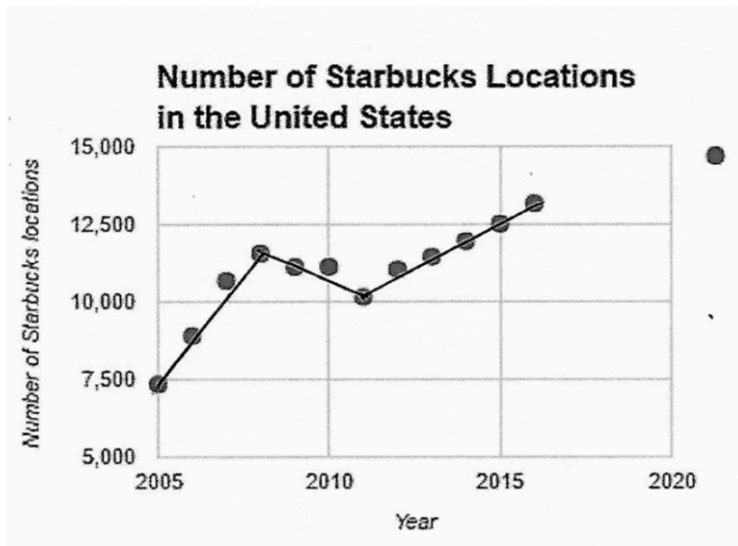


Assessment – final exam



- Give and interpret (piecewise) linear fits, with data.
 - Set up, solve, and interpret equations describing desired ratios.
 - Give a spreadsheet call to calculate residuals.
 - Produce an exponential model from a verbal description.
- 

Assessment - projects



Starbucks locations in future years using the formula:

$$S = 603(y - 2011) + 10,157$$

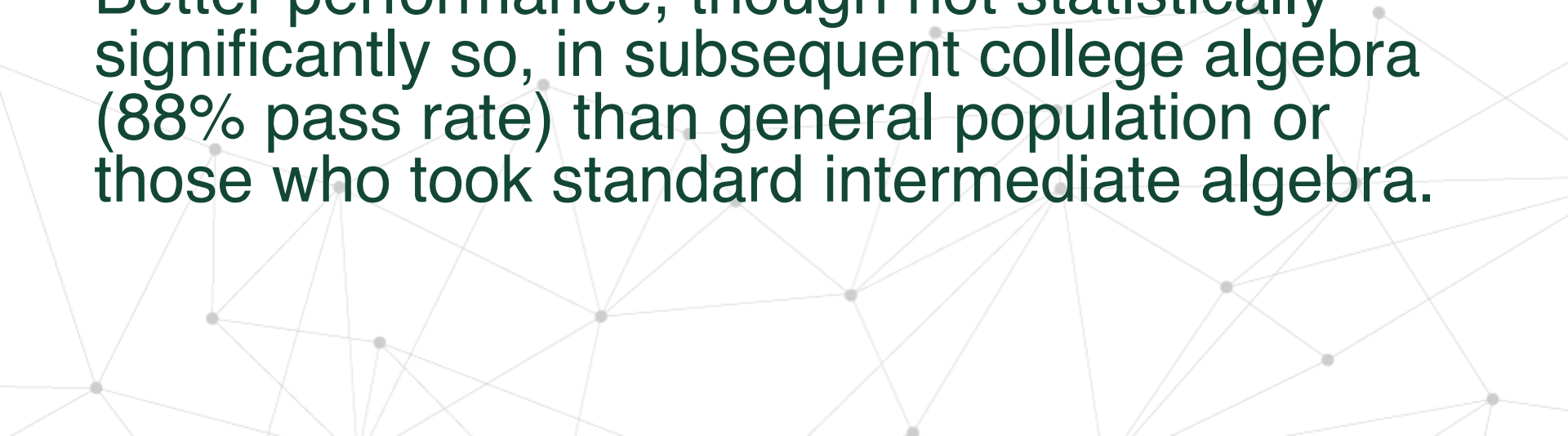
In this formula, S represents the number of Starbucks stores and y represents the year that is being predicted. This can be interpreted as, the number of Starbucks locations increases by 603 for every year since 2011, starting at 10,157 locations.

Results (non-scientific)

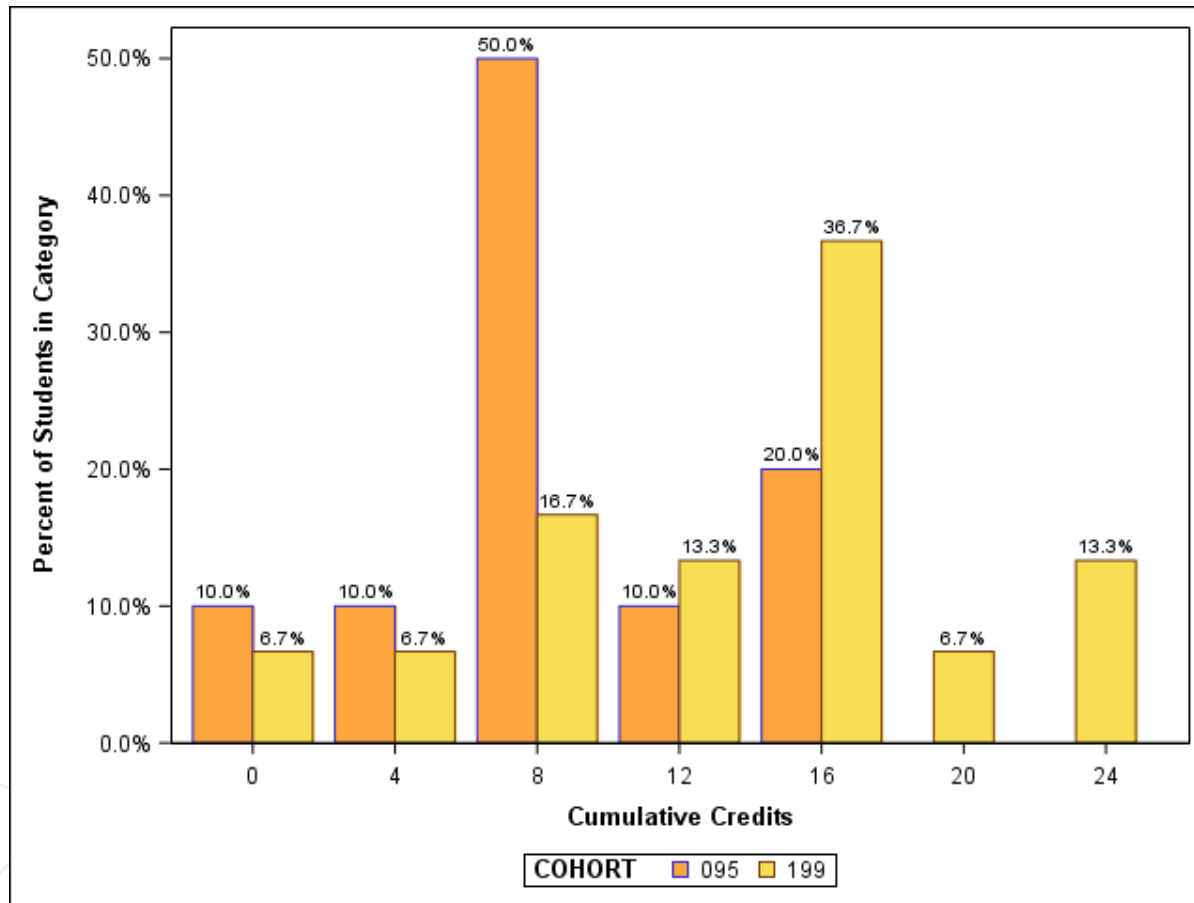


Remarkably high pass-rates (28/30; 24/28), likely because of relatively high weight given to worksheets and projects (22.5% weight for midterm and final combined).

Better performance, though not statistically significantly so, in subsequent college algebra (88% pass rate) than general population or those who took standard intermediate algebra.

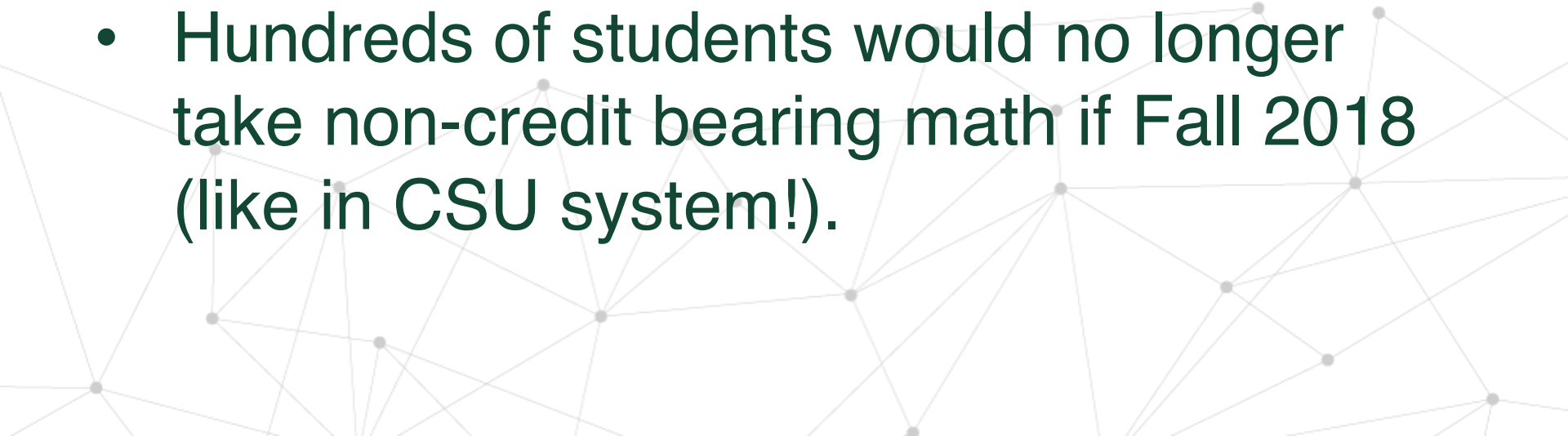


Results (non-scientific)



Plans



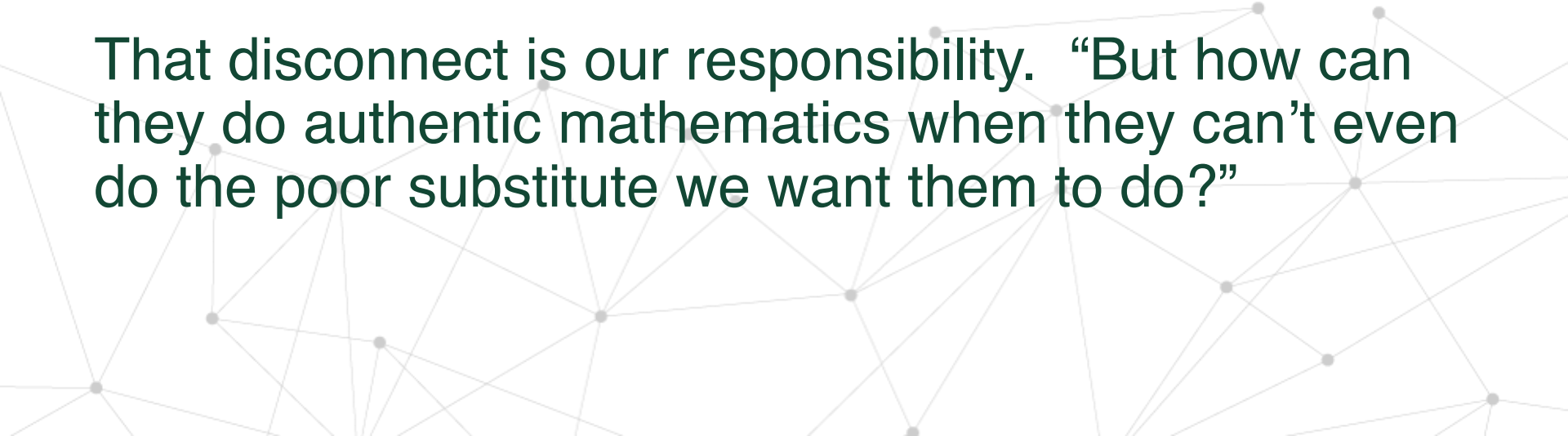
- Third pilot, first by another instructor, now.
 - Develop through summer, scale up in some form (less “radical”?) for roughly ten instructors. (Need more material on function notation.)
 - Hundreds of students would no longer take non-credit bearing math if Fall 2018 (like in CSU system!).
- 

Commonalities



All these populations – prospective math majors, potential elementary teachers, unprepared STEM-track students – struggle with misunderstandings of what mathematics is and what math classes should entail.

That disconnect is our responsibility. “But how can they do authentic mathematics when they can’t even do the poor substitute we want them to do?”



Commonalities



Persistent asking for authentic practice (show me what you think!), with support, is the main tool to address this disconnect.

Requires shift in instructional practices. The activity-debrief cycle based classroom (as implemented by MIT probability & IM MS &...) is where I've been headed here.

Classes developed to address such a roadblock are “interventions” in edu-speak.




Soapbox



One person's coherent vision...

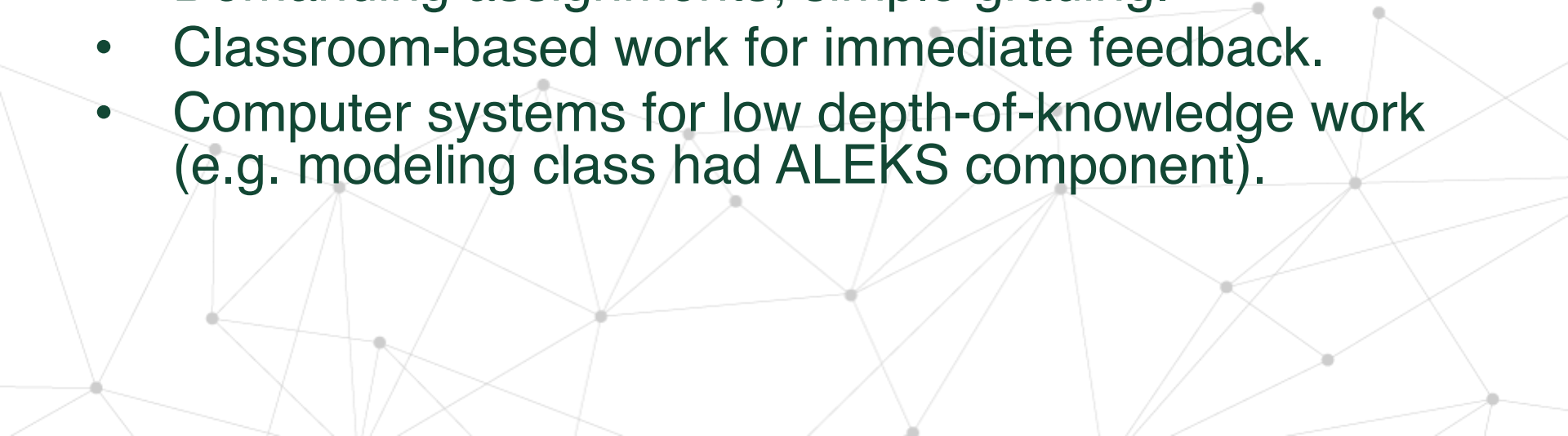


Deepening student practices throughout curriculum




Every undergraduate course should attend to authentic practices at some level (e.g. spreadsheet projects for statistics class of 500+), choosing between primarily pure or applied practices.

To make this manageable (as seen in other EMES):

- Demanding assignments; simple grading.
 - Classroom-based work for immediate feedback.
 - Computer systems for low depth-of-knowledge work (e.g. modeling class had ALEKS component).
- 

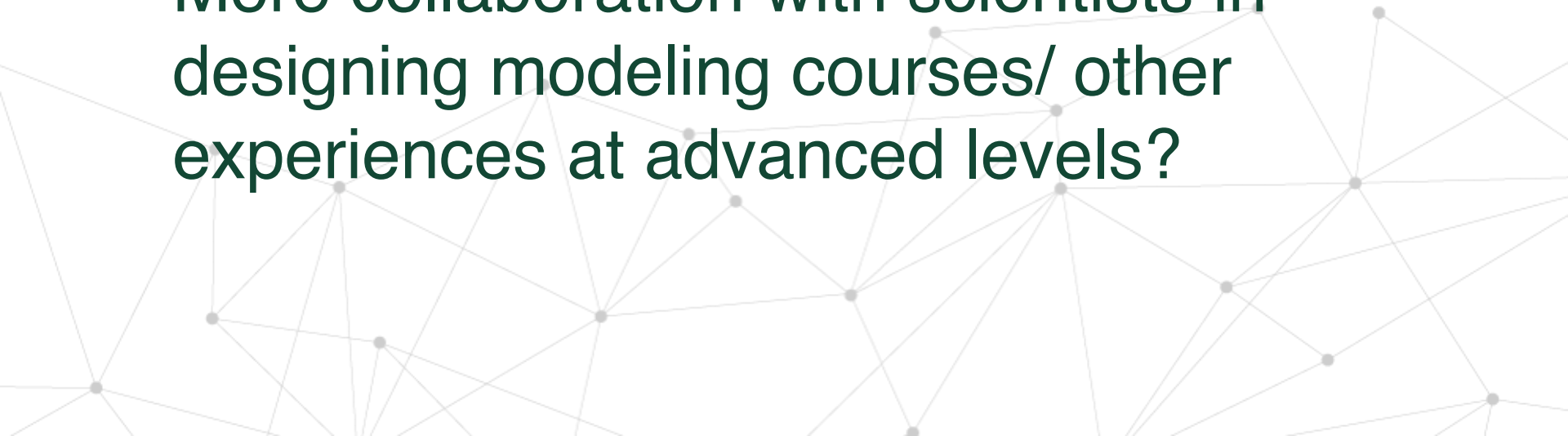
Deep practice experiences sprinkled through pure track



- Entry level “labs” as an intervention, as just created by other faculty at UO
 - Introduction to proofs courses/ sequences
 - Experimental mathematics labs and REUs
 - (Honors) theses
- 

Deep practice experiences sprinkled through applied track



- Mathematical modeling intervention
 - Calculus-based modeling courses?
 - COMAP contests
 - More collaboration with scientists in designing modeling courses/ other experiences at advanced levels?
- 

Soapbox



Amazing time to be working on college mathematics pedagogy!

- This seminar
- CBMS statement
- MAA Instructional Practices Guide
- CCSSM and other reform at K-12

But substantial challenges still ahead.



Challenge: Better dissemination!



Scale up Curated Courses (see first MIT EMES) and have functionalities and usage(!) comparable to ArXiv + ArXiv Overlay Journals + MathReviews + MathOverflow + #MTBoS (+...?), for pedagogical materials. Fold in Webwork.

Financial models to better capture materials revenue (as currently practiced at Kansas State) to promote and sustain development.




Challenge: Culture and Incentives



Challenge: Validation and Refinement



Need more (resources for) partnerships between innovative practitioners, RUME community, institutional researchers, professional developers and others to support refinement – especially usability – and dissemination.





**Thank you, especially
our hosts at MIT!**

