Grace

(1) Which sample exam question from the Study Guide for the Oral Final Exam would you like to answer? Please give the question's number and letter; e.g., Q9(a) and then describe your solution.

2. Let $P(t) = 100e^{0.1t}$ be the population of bacteria in a culture at time t (measured in hours). What is the rate of change in the the culture's population at time $t = 0$? Include units.

Problem 2A is to be completed only if the preceding question was already answered in response to the prompt on slide 1.

2.A If $f(x) = \dfrac{e^{2x}}{x^2+1}$, what is $f'(x)$?

3. State the definition of horizontal asymptote and use it to find the horizontal asymptotes of the graph of

$$f(x) = \begin{cases} \dfrac{3x^2+1}{x^2+2} & \text{if } x < 0 \\[2ex] \dfrac{1}{x^2+2} & \text{if } x > 0. \end{cases}$$

Problem 3A is to be completed only if the preceding question was answered in response to the prompt on slide 1.

3A. Explain how to use an appropriate sign line to locate points of inflection on the graph of a function f.

4. Pictured below right is the graph of a function f. How would you fill in the blank **using the information provided by the graph**? If the answer doesn't exist, enter DNE.

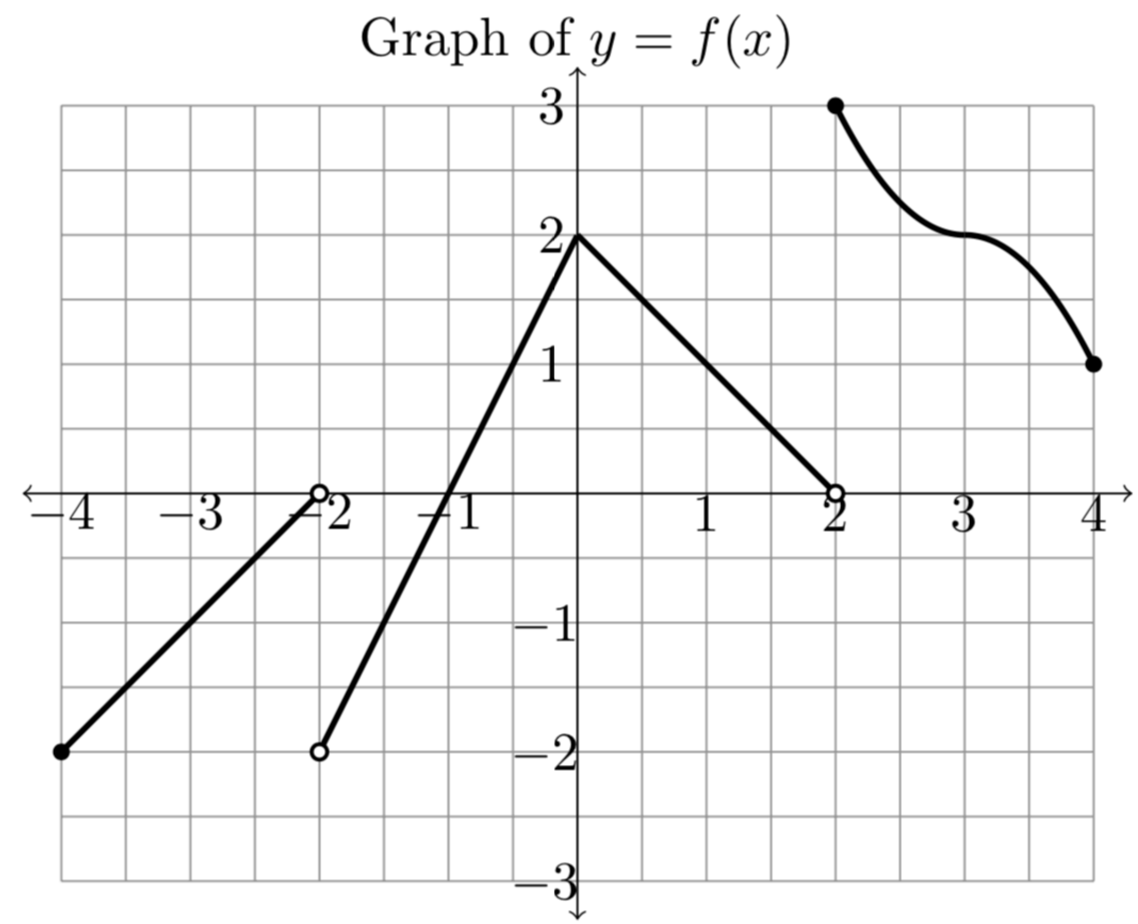
(a) The domain of f is ____.

(b) $f'(1) =$ ____.

(c) $\lim_{x \to 0} f\big(f(x)\big) =$ ____.

(d) The absolute maximum value of f on $[-1, 1]$ is ____. The absolute minimum value of f on $[-1, 1]$ is ____.

(e) What is a point of inflection on the graph of f? ______.



5 The population of bacteria in a culture at time t is modeled by a function $p(t)$, where t is measured in hours. Suppose that the rate of change of p with respect to time is modeled by

$$p'(t) = 300\sqrt{t} + 50$$

and the initial population is given by $p(0) = 10$. Find a formula for $p(t)$.

6. Suppose f is a continuous function defined on a closed interval $[a, b]$.

(a) What theorem guarantees the existence of an absolute maximum value and an absolute minimum value for f?

(b) What steps would you take to find those absolute maximum and minimum values?

Make your choice! Sharing of thinking optional!

7. (a) Circle the one correct choice for the definite integral $\int_{-1}^{1} \frac{1}{x^2}\, dx$

(a) $= -2$ (b) $= 2$ (c) DNE (c) $= 0$

(b) **True** **False** If $f''(a) = 0$, then $(a, f(a))$ is an inflection point of the graph of f.

(c) **True** **False** If f' is a decreasing function on the interval $(-1, 3)$ and $f'(2) = 0$, then f has a relative maximum value at $x = 2$.

8. As a spherical balloon is inflated, its volume V is changing with respect to time t. If the radius r of the balloon is measured in inches and t is measured in seconds, what are the units of

$$\frac{dV}{dt}?$$

9. . A particle travels along the x-axis such that its position at t seconds is given by the function $s(t)$ (where $s(t)$ is measured in centimeters). Which of the following expressions gives the average velocity of the particle over the time interval from $t = 0$ to $t = 2$ seconds? **More than one expression may work; select all that apply.**

(A) $s'(2)$

(B) $\dfrac{s(2) - s(0)}{2 - 0}$

(C) $\dfrac{1}{2} \displaystyle\int_0^2 s'(t)\, dt$

(D) $\dfrac{1}{2} \displaystyle\int_0^2 s(t)\, dt$

(E) $\dfrac{s'(2) - s'(0)}{2 - 0}$

THE END!

Rubrics and instructions. Problems 1-7 are 10 points each; 8 and 9 are 4 each.

2. **Allow 1 minute of silence before prompting to share work/thinking If student Seems clueless (unlikely), then ask for P(0) [if correct 3/10]**

$P'(t) = 10e^{0.1t}$. The bacteria population at time $t = 0$ is increasing at $P(0) = 10$ bacteria/hr.

Rubric: 4/10 for attempting to differentiate with respect to time. 5/10 if student gets $P'(t) = 100e^{0.1t}$ (so is saying $\frac{d}{dt}[e^{0.1t}] = e^{0.1t}$), 8/10 for $P'(t) = 10e^{0.1t}$, 1 point for correct evaluation at 0 of whatever derivative found. 1 point for units.

Student should work 2(A) if and only if student's selected problem (Slide 1) matches problem 2 or 3, or student did not select a problem to present (Slide 1), have the student complete 3(A).

2(A) If $f(x) = \dfrac{e^{2x}}{x^2+1}$, what is $f'(x)$?

Allow the student to scribble for 30 seconds. Then encourage him/her to share thinking/answer—perhaps holing it up to the screen..

$$f'(x) = \frac{(x^2+1)e^{2x} \cdot 2 - e^{2x}(2x)}{(x^2+1)^2}.$$

- 5 points for quotient rule structure—6 points if pattern is completely correct
- 3 points (which can be earned independently of quotient-rule points) for $\frac{d}{dx}[e^{2x}] = e^{2x}(2)$, 1 point for e^{2x} and 2 points for the factor of 2.
- Then the final point is for $|\frac{d}{dx}[x^2+1] = 2x$,

3. Again, allow 30 seconds of silence on this one—then prompt for the definition. If student clueless, the partial-credit question is: "Would you sketch a graph that has a horizontal asymptote?" If they can, then 4/10 earned. Cheerfully move on!

State the definition of horizontal asymptote and use it to find the horizontal asymptotes of the graph of

$$f(x) = \begin{cases} \dfrac{3x^2+1}{x^2+2} & \text{if } x < 0 \\[2ex] \dfrac{1}{x^2+2} & \text{if } x > 0. \end{cases}$$

$y = b$ is a HA of f if either $b = \lim_{x\to\infty} f(x)$ or $b = \lim_{x\to-\infty} f(x)$. $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 3$, so that $y = 0$ and $y = 3$ are horizontal asymptotes.

Rubric:

- 4 points for definition. 2.5/4 if only one of the limits is given.
- 2 points for each correct limit computation. No partial credit.No work required.
- 1 additional point for "0 and 3" are horizontal asymptotes.
- Final point for $y = 0$ and $y = 3$ are horizontal asymptotes.

3A.

Work ONLY if 3 already worked in resonse to prompt on slide 1

Explain how to use an appropriate sign line to locate points of inflection on the graph of a function f.

Allow 30 seconds of silence on this one. Then ask student to start sharing his/her thinking. If student appears clueless. Ask: Can you define "Point of Inflection".? If so 5/10 and cheerfully move on.

Rubric:

- 5/10 if student starts talking about the 2nd derivative sign line or if student can state the definition of point of inflection.
- 8/10 if student says they are looking for sign changes on the f'' sign line (or looking for sign changes of f'').
- 10/10 if they indicate that the sign must change at a and $f(a)$ must exist in order to have a point of inflection.

MOVE STUDENT ALONG ON THIS—EXCEPT FOR (C),
NO MORE THAN 15 SECONDS OF SILENCE ON ANY PART.

4. Pictured below right is the graph of a function f. How would you fill in the blank **using the information provided by the graph**? If the answer doesn't exist, enter DNE.

2 pts, 1 if they break it up
But have wrong brackets

(a) The domain of f is [-4,-2) U (-2, 4].

(b) $f'(1) =$ -1. 2 pts NPC

(c) $\lim_{x \to 0} f\big(f(x)\big) =$ 0.

GIVE 30 seonds
of silence then encourage
thought-sharing

2 pts NPC

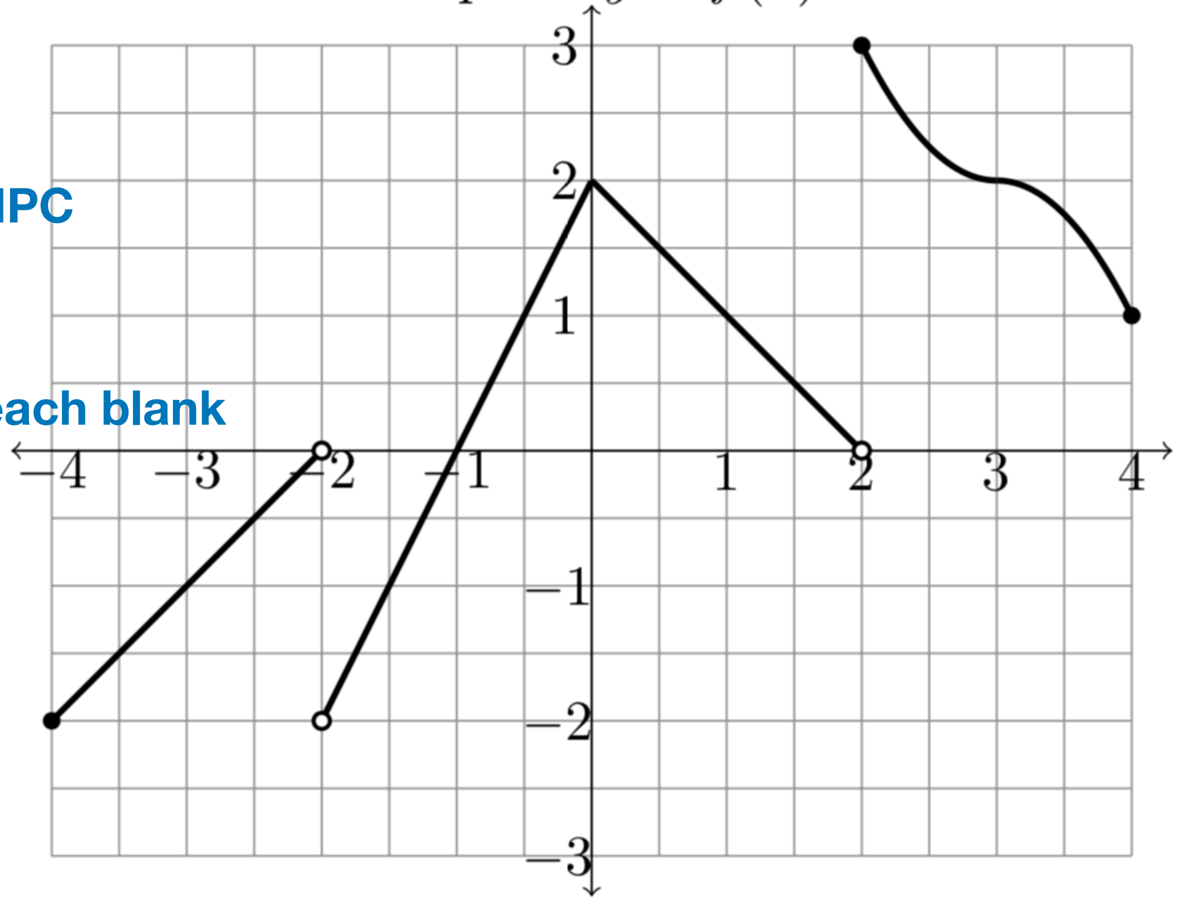
(d) The absolute maximum value of f on $[-1, 1]$ is 2. The absolute minimum value of f on $[-1, 1]$ is 0.

1 for each blank

(e) What is a point of inflection on the graph of f? (3, 2).

2 pts only patial credit: 1 for "3" or "x = 3"

Graph of $y = f(x)$



6. Allow 30 seconds of silence before prompting student to share thoughts. If student is clueless ask for definition of critical number. If critical number perfectly defined, 4/10 and move on. Rubric for critical number c definition: 1.5 for $f'(c) = 0.$, 1.5 for $f'(c)$ DNE and 1 for in domain, Be sure you ask student for definition of critical number at some point during their discussion.

Suppose f is a continuous function defined on a closed interval $[a, b]$.

(a) What theorem guarantees the existence of an absolute maximum value and an absolute minimum value for f?

(b) What steps would you take to find those absolute maximum and minimum values?

Rubric:

- 2 pts for (a) [Extreme Value Theorem]
- 8 pts for (b)
  - 4 pts if student knows to test f at critical numbers in (a, b) (or $[a, b]$ AND KNOWS WHAT A CRITICAL NUMBER IS. (1/4 if student says something about $f'(x)$ but doesn't mention critical numbers, 3/4 if student doesn't mention anywhere in the explanation that the critical numbers should be in (a, b)) If student says to test f at critical number in (a, b) and givess both conditions $f'(c) = 0$ and $f'(c)$ DNE give them full credit here–4 points.
  - 3 pts if student knows to test f at the endpoints
  - 1 pt if student describes the need to choose the largest/smallest value of f to find the absolute max. and min.

5. **Bacteria IVP** The population of bacteria in a culture at time t is modeled by a function $p(t)$, where t is measured in hours. Suppose that the rate of change of p with respect to time is modeled by

$$p'(t) = 300\sqrt{t} + 50$$

and the initial population is given by $p(0) = 10$. Find a formula for $p(t)$.

$$p(t) = 200t^{3/2} + 50t + 10.$$

Allow student 1 minute of silence/scribbling before prompting to share thinking. If student is clueless ask him/her to find the indefinite integral of $\sqrt{t} + 1$. if perfectly worked out 6/10: 5/10 for correct antideriative–2.5 each summand and 1 for +C.

RUBRIC
For a full credit solution:
Student recognizes the need to find antiderivative / indefinite integral.
Student recognizes the answer is of the form "f(x)+C" with constant of integration.
Student computes general antiderivative correctly.
Student recognizes need to set "f(a)=b" to solve for C.
Student finds correct value of C and writes particular solution.

STUDENT-RESPONSE POINT VALUES:
If student recognizes need to find antiderivative -- 4/10
If student recognizes need for constant of integration -- 5/10
If student conveys need to plug in "f(a)=b" to solve for C -- 6/10
Distribution of the remaining 4 points for Calculation
2 pts - antiderivative of f ; (1 pt for each summand)
1.5 pts - correctly solve for C
0.5 pts - write out correct final solution

7. (a) Circle the one correct choice for the definite integral $\int_{-1}^{1} \frac{1}{x^2}\, dx$

(a) $= -2$ (b) $= 2$ (c) DNE (c) $= 0$

(b) **True** **False** If $f''(a) = 0$, then $(a, f(a))$ is an inflection point of the graph of f.

(c) **True** **False** If f' is a decreasing function on the interval $(-1, 3)$ and $f'(2) = 0$, then f has a relative maximum value at $x = 2$.

7. I'd give the student 1 minute on each before asking for a decision. These are ALL quest problems. **[Quests were essentially take-home, low-stakes quiz-tests (hence "qu-ests"); students were told they would be given some problems on the oral final related to Quest problems.]**

(a) (c) DNE

(b) False **Rubric: 1 right 4/10; 2 right 7/10; all right 10/10**

(c) True

8. As a spherical balloon is inflated, its volume, V, is changing with respect to time, t. If the radius, r of the balloon is measured in inches and t in seconds, what are the units of

$$\frac{dV}{dt}?$$

Allow 30 seconds of silence on this one. Then prompt to share thinking.

4 points for $\frac{\text{in}^3}{\text{sec}}$; 2 points for $\frac{\text{in}^n}{\text{sec}}$ for some $n \neq 3$; 1 point for anything/sec.

9. . A particle travels along the x-axis such that its position at t seconds is given by the function $s(t)$ (where $s(t)$ is measured in centimeters). Which of the following expressions gives the average velocity of the particle over the time interval from $t = 0$ to $t = 2$ seconds? **More than one expression may work; select all that apply.**

(A) $s'(2)$

(B) $\dfrac{s(2) - s(0)}{2 - 0}$

(C) $\dfrac{1}{2}\displaystyle\int_0^2 s'(t)\, dt$

(D) $\dfrac{1}{2}\displaystyle\int_0^2 s(t)\, dt$

(E) $\dfrac{s'(2) - s'(0)}{2 - 0}$

Allow 1.5 minutes before prompting student to make choices. **Rubric**: 4 points total. 2 points for (B) and 2 points for (C); subtract 1 for each incorrect choice.