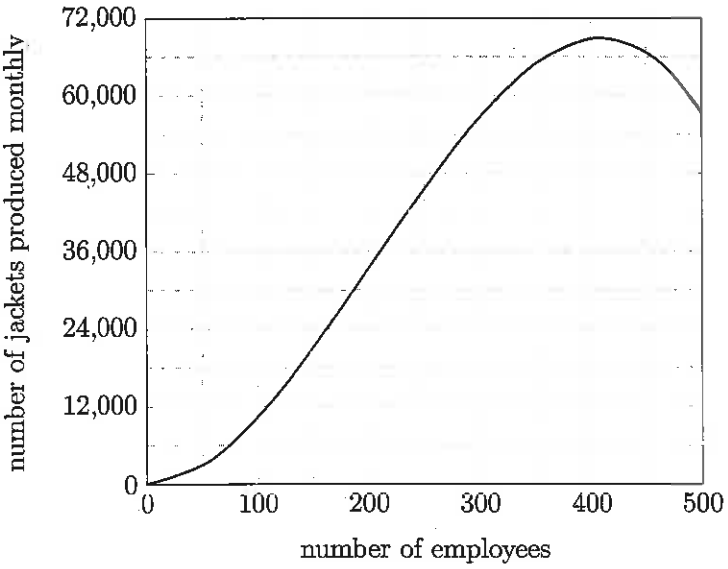
Not for Students

Name: _____ Section: _____

Collaborators: _____

Lab 9: The Law of Diminishing Returns

A company producing hand-detailed jackets finds that the number of jackets produced each month depends on the number of employees working in production according to the function $f(x)$ graphed below, where x is the number of employees, and $f(x)$ is the number of jackets produced each month.



1. From the graph, approximate the value of each of the following derivatives.

   For these they should sketch tangent lines on the graph & approximate their slopes. It's hard to do this with great accuracy.

   - $f'(150)$
   - $f'(350)$

2. For each of the derivatives you estimated in problem 1, interpret the meaning of that value in terms of employees and monthly production:

   Something like:

   - Interpret $f'(150)$: When there are 150 employees working in production, the addition of one more employee increases monthly production by approximately _____ jackets.

   1

   [answer from 1.]

- Interpret $f'(350)$: Similar to $f'(150)$

3. Using the graph, fill in the table below indicating whether $f'(x)$ and $f''(x)$ are *positive* or *negative* at the indicated values of x:

Value of x	$f'(x)$: positive or negative?	$f''(x)$: positive or negative?
100	+	+
300	+	−
500	−	−

4. Using the graph, approximate the following:

- Critical point(s) of $f(x)$: There is actually a critical point at $x=0$ but that isn't very clear from the picture, so it is fine if they leave that off of their list of critical points.
- Interval(s) where $f'(x)$ is positive:
- Interval(s) where $f'(x)$ is negative:

5. Interpret what it means for $f'(x)$ to be positive, in terms of employees and monthly production.

If $f'(x)$ is positive, the addition of one more employee increases the total monthly production of jackets.

6. Interpret what it means for $f'(x)$ to be negative, in terms of employees and monthly production.

If $f'(x)$ is negative, the addition of one more employee decreases the total monthly production of jackets.

7. Does it make sense for this company to hire 500 employees to work in production? Why or why not? In order to maximize monthly production, approximately how many employees should they hire?

No. They would have higher production with fewer employees. Max production is around 410 employees.

8. Using the graph, approximate the following:

   - Interval(s) where $f'(x)$ is increasing:
   - Interval(s) where $f'(x)$ is decreasing:
   - Interval(s) where $f(x)$ is concave up:
   - Interval(s) where $f(x)$ is concave down:
   - Inflection point(s) of $f(x)$:

We hope students will see the connection between the concavity of f and the increasing / decreasing behavior of f'.

9. Interpret what it means for $f'(x)$ to be increasing, in terms of employees and monthly production.

As the number of employees gets bigger, the change in monthly production per additional worker is increasing.

10. Interpret what it means for $f'(x)$ to be decreasing, in terms of employees and monthly production.

As the number of employees gets bigger, the change in monthly production per additional worker is decreasing.

In a graph such as this, economists call the inflection point the ***point of diminishing returns***. Starting at the point of diminishing returns, if the number of workers is increased, while total monthly production may continue to rise, the increase in production per additional worker is decreasing.

In economics the ***law of diminishing returns*** says that anytime you increase one factor of production (e.g. employees, machinery, fertilizer, etc) while keeping all other factors of production constant, eventually you will hit a point of diminishing returns, where the incremental per-unit returns begin to drop.

If increasing the factor of production actually decreases total production, that is known as ***negative returns***. The law of diminishing returns does not say that you must eventually have negative returns, but it does frequently happen that way.

11. Using the graph, estimate where the point of diminishing returns is.

Around 200 employees.

12. Might a company choose to hire a worker that exhibits diminishing returns? Why or why not? Potentially. A worker may exhibit diminishing returns but still increase total monthly production. So it depends on the production goals of the company.

13. For what values of x does the graph show negative returns? Should a company choose to hire a worker that exhibits negative returns? Why or why not?

Approx. $410 < x \leq 500$. It does not make sense to hire a worker that exhibits negative returns. The additional worker decreases total monthly production so you would be paying more to produce less.

14. Give some reasons that negative returns could occur at this company. In other words, why would increasing the number of workers decrease total production?

- Shortage of equipment decreasing productivity of workers
- People getting in each other's way.

etc..

The company determines that the function graphed above modeling monthly production is given by

$$f(x) = \frac{1}{500}(615x^2 - x^3) \qquad \text{for } 0 \leq x \leq 500$$

where x is the number of employees working in production.

15. Find the derivative $f'(x)$.

Easy

16. Using the derivative, find the number of employees that maximizes monthly production.

They should find critical points, then use the first or second derivative test.

17. Find the second derivative $f''(x)$.

Easy.

18. Find all possible points of inflection for f by setting $f''(x) = 0$ and solving for x.

Easy.

19. Using the second derivative, find the following:

- Interval(s) where $f(x)$ is concave up:
- Interval(s) where $f(x)$ is concave down:
- Inflection point(s) of $f(x)$:

Straightforward.

20. Where is the point of diminishing returns for $f(x)$?

21. A company that grows and sells arugula is interested in how fertilizer use affects their arugula production. They observed the following data:

Pounds of fertilizer per acre	40	80	120	160	200	240	280	320
Yield of arugula per acre	6000	6500	7200	8000	8600	9000	8750	8400

- Using the table, approximate the pounds of fertilizer per acre that the company should use to maximize arugula production.

240

- Using the table, approximate the point of diminishing returns.

160

- Interpret what the point of diminishing returns means in this situation.

If you increase fertilizer use to more than 160 lbs. per acre, then the change in arugula yield per additional pound of fertilizer decreases.

22. For a function $g(x)$, fill in the following table illustrating the relationship between $g(x)$, $g'(x)$, and $g''(x)$. For each box in the table fill it with the appropriate selection from the following choices: *negative, positive, zero, decreasing, increasing, level, concave down, concave up, neither concave up nor concave down.*

If $g''(x)$ is:	Then $g(x)$ is:	Then $g'(x)$ is:
Positive	Concave up	increasing
Negative	Concave down	decreasing
Zero	neither concave up nor concave down	level

Participation	/5
Correctness	/5
Total	/10