2020 SPUR Conference

July 31. 2020

Zoom webinar

9:30 am: Conference Opening by SPUR faculty advisors Prof. David Jerison and Prof. Ankur Moitra

9:35 am: Christina Meng, "Betti Numbers of Configuration Spaces of Manifolds" (mentor Araminta Gwynne)

10:05 am: Hung-Hsun Yu, "Joints of Varieties" (mentor Jonathan Tidor)

10:35 am: Korina Digalaki, "Deformed Airy Kernels in Height Function Fluctuations of the Stochastic Six-Vertex Model" (mentor Roger Van Peski)

11:05 am: Kerri Lu and Aparna Ajit Gupte, "Fine-Grained Complexity of Sparse Linear Regression" (mentor Frederic Koehler)

11:35-12:20 pm: lunch break

12:20 pm: Calvin Yost-Wolff, "A Geometric RSK Correspondence and Relative Positions Within Springer Fibers" (mentor Oron Propp)

12:50 pm: Dylan Pentland, "Filtrations on Block Subalgebras of Reduced Universal Enveloping Algebras" (mentor Andrei Ionov)

1:20 pm: Carl Schildkraut and William Zhao, "Relating Counts in Different Characteristics for Steiner's Conic Problem" (mentor Chun Hong Lo)

2:00 pm: Kevin Chang, "Towards the v_1-periodic Adams Spectral Sequence for the mod 2 Moore Spectrum" (mentor Robert Burklund)

2:30 pm: Kevin Ren, "Incidence Estimates for α-dimensional δ-tubes" (mentor Yuqiu Fu)

3:00 pm: Elisabeth Bullock and Katie Gravel, "Higher Secondary Polytopes for Zonotopal Tilings" (mentor Alexey Balitsky)

3:30 pm: Conference Closing

Christina Meng

Betti Numbers of Configuration Spaces of Manifolds

Mentor: Araminta Gwynne

Project suggested by Araminta Gwynne

Configuration spaces are studied due to their connections to numerous areas outside of algebraic topology, as well as being interesting in their own right. They are defined in relation to a given topological space, and in this talk we are concerned with when the given space is a manifold. I will present an introduction to configuration spaces and how we can compute their Betti numbers. In particular, I will give a partial description of how we would compute the Betti numbers when the background manifold is a higher-dimensional analogue of a genus g surface.

Hung-Hsun Hans Yu

Joints of Varieties

Mentor: Jonathan Tidor

Project suggested by Prof. Yufei Zhao

In 2010, Guth and Katz proved that N lines in $\mathbb{R}^3$ can form at most $O(N^{3/2})$ joints, where a joint is an intersection of three non-coplanar lines. The technique they used is the polynomial method, which was inspired by Dvir and found many other applications in incidence geometry. Many generalizations of the joints problem for lines have been solved afterwards, but little is known when lines are replaced with higher dimensional flats or varieties over arbitrary fields. In this talk, I will talk about our new results resolving this case along with many other generalizations.

Korina Digalaki

Deformed Airy Kernels in Height Function Fluctuations of the Stochastic Six-Vertex Model

Mentor: Roger Van Peski

Project suggested by Prof. Alexei Borodin

In this paper, we consider the Stochastic Six-Vertex Model for a special class of initial data, known as generalized step Bernoulli initial data, that were first introduced by Aggarwal and Borodin. In this setting, we study the asymptotic behavior of the height function, whose fluctuations are known to exhibit a phase transition along a critical line. In particular, we exploit the asymptotic equivalence between the height function and a sequence of Schur measures, to show that on the critical line, the height function has fluctuations of order $N^{\frac{1}{3}}$, whose distribution is a generalization of the Tracy-Widom distribution previously found in random matrix and percolation models by Borodin and Peche.

Aparna Ajit Gupte and Kerri Lu

Fine-Grained Complexity of Sparse Linear Regression

Mentor: Frederic Koehler

Project suggested by Frederic Koehler

We study the problem of sparse linear regression over the rationals from the perspective of fine-grained complexity. More precisely, we consider the noiseless/realizable setting where $Y = \langle X, w \rangle$, w is a k-sparse vector, $X \sim N(0, \Sigma)$ and the goal of the algorithm is to exactly reconstruct w from samples (X, Y). Under the Exponential Time Hypothesis, we show that solving this problem requires time $n^{\Omega(k)}$, give more precise bounds under stronger hypotheses, and also prove a nontrivial upper bound of the form n^{k-1} beating the trivial brute force approach.

AFTERNOON SESSION

Calvin Yost-Wolff

A Geometric RSK Correspondence and Relative Positions Within Springer Fibers

Mentor: Oron Propp

Project suggested by Prof. Roman Bezrukavnikov

Springer fibers are algebraic varieties associated to unipotent elements of an algebraic group that have deep representation-theoretic significance. A classical result of Spaltenstein–Tits shows that any two points in relative position σ in a Springer fiber can be connected by a sequence of projective lines determined by the sequence of simple reflections in a reduced expression for σ. In this talk, I will explain how these paths realize a geometric version of the RSK correspondence. I will then introduce certain subvarieties of orbital varieties, demonstrating how they determine the paths produced by this geometric RSK correspondence and hinting at a bijection between their irreducible components and classes of paths associated to a particular relative position. Time permitting, I will share results on the dimensions of these subvarieties and a characterization of the irreducible components of Springer fibers satisfying a certain "convexity" property with respect to the paths in the geometric RSK correspondence.

Dylan Pentland

Filtrations on Block Subalgebras of Reduced Universal Enveloping Algebras

Mentor: Andrei Ionov

Project suggested by Prof. Roman Bezrukavnikov

We study the interaction between the block decompositions of reduced universal enveloping algebras in positive characteristic, the PBW filtration, and the nilpotent cone. We provide two natural versions of the PBW filtration on the block A_λ and show these are dual to each other, and provide a general method for understanding the associated graded algebra $\mathrm{gr}_{\mathrm{sh}} A_\lambda$ for a shifted PBW filtration. In the case of $\mathfrak{sl}_2(k)$ for k of characteristic $p > 2$ and an arbitrary character χ, using these general results we determine the associated graded algebras of these filtrations on blocks of $\mathcal{U}_\chi(\mathfrak{sl}_2)$ and their structure as SL_2-representations.

Carl Schildkraut and William Zhao

Relating Counts in Different Characteristics for Steiner's Conic Problem

Mentor: Chun Hong Lo

Project suggested by Chun Hong Lo

A standard problem in enumerative geometry is to calculate the number of smooth conics tangent to a general set of 5 conics in a projective plane. This count is known in all characteristics, and differs from characteristic 2 (where it is 51) to characteristic 0 (where it is 3264) by a factor of 2^6. Following in the lines of Pacini and Testa, we give a new proof of the characteristic 2 count, dependent on the count in characteristic 0, that explains this factor. In particular, we show that half of the 3264 conics, when taken modulo 2, merge into 51 groups of 2^5, while the other half degenerate. By considering the flat limit of the scheme of complete conics into characteristic 2, we interpret these degenerate conics to be inside a dual space, which explains this split.

Kevin Chang

Towards the v_1-periodic Adams Spectral Sequence for the mod 2 Moore Spectrum

Mentor: Robert Burklund

Project suggested by Robert Burklund

The mod 2 Moore spectrum $C(2)$ is the cofiber of the self-map $2 : \mathbb{S} \to \mathbb{S}$. Building on work of Burklund, Hahn, and Senger, we prove that the Adams spectral sequence for $C(2)$ collapses at its E_5-page above a line of slope $\frac{1}{5}$ and characterize the surviving classes above this line. This completes the proof of a result of Mahowald, announced in 1970 but never proven. Notably, however, we prove this without computing the whole E_2-page above this line. In addition to proving Mahowald's result, we discuss progress towards computing the E_2-page in this range.

Kevin Ren

Incidence Estimates for α-dimensional δ-tubes

Mentor: Yuqiu Fu

Project suggested by Prof. Larry Guth

Given a collection of α-dimensional δ-tubes and β-dimensional δ-balls in the plane, what is the maximum number of incidences, or pairs (t, b) of tubes and balls such that b lies in t? We find an essentially sharp estimate for this maximum number in terms of α and β and present a construction for the special case $\alpha = \beta = 1$. We will also outline the proof of the upper bound for $\alpha, \beta > 1$.

Elisabeth Bullock and Katie Gravel

Higher Secondary Polytopes for Zonotopal Tilings

Mentor: Alexey Balitskiy

Project suggested by Prof. Pavel Galashin

Very recently, Galashin, Postnikov, and Williams introduced the notion of *higher secondary polytopes*, generalizing the *secondary polytope* of Gelfand, Kapranov, and Zelevinsky. Given an n-point configuration $\mathcal{A}$ in $\mathbb{R}^{d-1}$, they define a family of convex $(n-d)$-dimensional polytopes $\widehat{\Sigma}_1, \ldots, \widehat{\Sigma}_{n-d}$. The 1-skeletons of this family of polytopes are the flip graphs of certain combinatorial configurations which generalize triangulations of $\operatorname{conv} \mathcal{A}$. In our project we restrict our attention to $d = 2$. First, we show that $\widehat{\Sigma}_1$ is the product of simplices. Second, we relate the 1-skeleton of the Minkowski sum $\widehat{\Sigma}_k + \widehat{\Sigma}_{k-1}$ to the flip graph of "hypertriangulations" of the deleted k-sum of $\mathcal{A}$ when $\mathcal{A}$ consists of distinct points. Finally, we compute of the diameter of $\widehat{\Sigma}_k$ and $\widehat{\Sigma}_k + \widehat{\Sigma}_{k-1}$ for all k.