2018 SPUR Conference

Friday, August 3

Room 2-449

SCHEDULE

9:05 am	Conference Opening by SPUR faculty advisors Prof. Davesh Maulik and Prof. Ankur Moitra
9:10 am	Julian Wellman, "Flips in reduced plabic graphs" (mentor Alexey Balitskiy)
9:40 am	Junyao Peng, "Counting shellings of complete bipartite graphs and trees" (mentor Yibo Gao)
10:10 am	Yunkun Zhou, "The Sidorenko problem for directed graphs in tournaments" (mentor Jonathan Tidor)
10:35 am	BREAK
10:50 am	Dhruv Rohatgi, "Biclique partitions and off-diagonal ordered Ramsey numbers" (mentor Jake Wellens)
11:20 am	Juan Gil and Joshua Amaniampong, "Uniform generation of w-free strings and H-free subgraphs with partial rejection sampling" (mentor Jake Wellens)
11:45 am	LUNCH BREAK
1:15 pm	Laura Koemmpel and Olga Medrano, "How graph structure and connectivity affects the propagation of cooperation in a network" (mentor Ashwin Narayan)
1:45 pm	Nikhil Reddy, "Generalizing auxeticity to aperiodic networks" (mentor Vishal Patil)
2:15 pm	Deven Jay Lahoti, "Torsion subspaces of complex Lie groups" (mentor Gurbir Dhillon)
2:40 pm	BREAK
3:10 pm	Julius Baldauf-Lenschen, "Sharp entropy bounds for plane curves and dynamics of the curve shortening flow" (mentor Ao Sun)
3:40 pm	Sanath Devalapurkar, "Uncompleting E-theory and Johnson-Wilson theory at height 2" (mentor Robert Burklund)
4:10 pm	Calvin Hsu, "Quantitative decay for nonlinear wave equations" (mentor Ruoxuan Yang)
4:35 pm	Conference Closing

Sharp Entropy Bounds for Plane Curves and Dynamics of the Curve Shortening Flow

Julius Baldauf-Lenschen
Mentor: Ao Sun
Project suggested by William Minicozzi

We prove that a closed immersed plane curve with total curvature $2\pi m$ has entropy at least m times the entropy of the embedded circle, as long as it generates a type I singularity under the curve shortening flow. Furthermore, we show that there exist closed immersed plane curves of total curvature $2\pi m$ whose entropy is less than m times the entropy of the embedded circle. Consequently, such curves must generate type II singularities under the curve shortening flow. Our method allows us to calculate the entropy index of all Abresch-Langer curves. Using this, we construct a generic curve shortening flow whose only singularities are embedded circles and grim reapers.

Uncompleting E-Theory and Johnson-Wilson Theory at Height 2

Sanath Devalapurkar
Mentor: Robert Burklund
Project suggested by Sanath Devalapurkar

We prove that there is an $\mathbf{E}_\infty$-ring structure on a particular form of Johnson-Wilson theory $E(2)$ at height 2 at any prime, which provides a realization in spectral algebraic geometry of a flat cover of the moduli stack of formal groups of height at most 2. We show this by using techniques developed by Lawson and Naumann and with Zhu's computations of power operations for Morava E-theory E_2 at height 2 to construct an $\mathbf{E}_\infty$-ring spectrum R whose $K(2)$-localization is E_2, such that the homotopy fixed points of R with respect to the action of a certain finite group is an $\mathbf{E}_\infty$-form of $E(2)$.

Uniform Generation of w-Free Strings and H-Free Subgraphs with Partial Rejection Sampling

Juan Gil and Joshua Amaniampong
Mentor: Jake Wellens
Project suggested by Jake Wellens

Recently, Guo, Jerrum, and Liu introduced a simple and powerful general purpose algorithm for exact uniform sampling, which they called *partial rejection sampling* (PRS). The technique has applications to a variety of sampling problems, including random spanning trees, sink-free orientations of a graph, satisfying assignments of certain CNFs, and point configurations from the so-called Hard Disks model. It was also used to give an FPRAS for the all-terminal network reliability problem. PRS always produces uniform samples by design, but in general, it can be quite inefficient – in many cases, it simply degenerates to *rejection sampling*. In this paper, we prove that PRS can be used to efficiently sample from two types of spaces. The first space is the set of strings $s \in \Sigma^n$ avoiding some contiguous substring w. The second is the set of (non-induced) subgraphs of various grids avoiding a certain subgraph H. In each case, we exploit the geometry of the underlying dependency graphs to prove the efficiency of PRS in parameter regimes that are much larger than those obtained by simply applying the original analysis of Guo et al. to these problems.

Quantitative Decay for Nonlinear Wave Equations

Calvin Hsu
Mentor: Ruoxuan Yang
Project suggested by Andrew Lawrie

In this paper we discuss the decay rate for semilinear energy-critical wave equations that behaves like the free wave. It is known that free waves in $\mathbb{R}^n$ have a decay rate of $(1+t)^{-\frac{n-1}{2}}$. Using the Klainerman-Sobolev inequality and Strichartz estimates, we are able to prove non-linear waves that scatter in $\mathbb{R}^3$ have a decay rate of $t^{-\frac{1}{2}}$. Moreover, we generalize the results to $\mathbb{R}^n$ to obtain a $(1+t)^{-\frac{n-2}{2}}$ decay rate, and also discuss some results that might be helpful to improve our bound.

How Graph Structure and Connectivity Affects the Propagation of Cooperation in a Network

Laura Koemmpel and Olga Medrano Martin del Campo
Mentor: Ashwin Narayan
Project suggested by Bonnie Berger

The notion of cooperation in biological systems has recently motivated the formation of *evolutionary game theory*, the study of evolving populations through game theoretic methods. Through modeling individuals as nodes in a graph and connections between them as edges, evolutionary game theory focuses on the dynamics of *node strategy updating*. This consists of nodes changing their strategies or behaviors according to a given process in either a discrete or continuous time basis.

In this paper we study the long-term behavior of graphs with highly interconnected and highly sparse regions (such as the *two-island*, *two-dumbell*, and *rich-club* graphs) under the *birth-death* and *death-birth* reproduction models given a *prisoner's dilemma* payoff matrix.

We discuss empirical evidence and observe that the *degree-weighted frequency difference between two cliques* tends to zero after long periods of time through observing the expectation of its derivative.

Torsion Subspaces of Complex Lie Groups

Deven Lahoti
Mentor: Gurbir Dhillon
Project suggested by Gurbir Dhillon

Let G be a simply-connected semisimple complex Lie group. For a positive integer ℓ, we study the space $G[\ell]$ of ℓ-torsion elements of G. We establish a canonical bijection between the components of $G[\ell]$ and the elements of the long root lattice lying the in the fundamental alcove Δ_ℓ. Under this bijection, the face of Δ_ℓ on which a given lattice point lies determines the topology of the corresponding component. Applications include quasipolynomial growth in ℓ of components of a given topology, and we include several worked examples.

Counting Shellings of Complete Bipartite Graphs and Trees

Junyao Peng
Mentor: Yibo Gao
Project suggested by Richard Stanley

A shelling of a graph, viewed as an abstract simplicial complex that is pure of dimension 1, is an ordering of its edges such that every edge is adjacent to some other edges appeared previously. In this paper, we focus on complete bipartite graphs and trees. For complete bipartite graphs, we obtain an exact formula for their shelling numbers. And for trees, we propose a simple method to count shellings and bound shelling numbers using vertex degrees and diameter.

Generalizing Auxeticity to Aperiodic Networks

Nikhil Reddy
Mentor: Vishal Patil
Project suggested by Vishal Patil

Materials with a negative Poisson's ratio are defined to be auxetic, causing them to contract along every axis or stretch along every axis when deformed. This property can be represented mathematically as an auxetic deformation of an underlying network. Traditionally, auxeticity defined in terms of a lattice structure has been used to classify deformations of a network. In this paper, we construct an equivalent lattice-independent definition of auxeticity, which generalizes well to arbitrary networks. Using this, we are able to methodically classify the auxeticity of aperiodic networks constructed by the cut-and-project method. Notably, in 2 dimensions, the set of infinitesimal auxetic deformations at the identity is equivalent to the nullspace of a linear combination of complex numbers. We investigate the implications of our results on the Penrose tiling, constructing multiple auxetic deformations.

Biclique Partitions and Off-diagonal Ordered Ramsey Numbers

Dhruv Rohatgi
Mentor: Jake Wellens
Project suggested by Asaf Ferber

We study two separate combinatorial problems. First, we look at the problem of partitioning the edges of the complete graph K_n into the minimum possible number of complete bipartite graphs (bicliques) such that every edge is covered exactly k times. It is known that at least $n-1$ bicliques are required. The previously best-known upper bound was $O(kn)$, but de Caen, Gregory, and Pritikin showed that for $k \leq 18$, merely $n-1$ bicliques are sufficient for all sufficiently large n, and conjectured that the same holds for every k. We make progress on this conjecture by showing that $n + o(n)$ bicliques suffice for every fixed k.

We then turn to the Ramsey theory of ordered graphs. For ordered graphs G and H, the ordered Ramsey number $r_<(G, H)$ is the smallest N such that every bicoloring of the complete graph on $[N]$ contains either a blue copy of G or a red copy of H, where the embedding must preserve the relative order of vertices. One number of interest, first studied by Conlon, Fox, Lee, and Sudakov, is the "off-diagonal" ordered Ramsey number $r_<(M, K_3)$, where M is an ordered matching on N vertices. The best-known upper bound is a trivial bound $O(N^2/\log N)$, but there is no known family of arbitrarily large matchings with $r_<(M, K_3) = \omega(N^{4/3})$, and Conlon et al. hypothesize that $r_<(M, K_3) = O(N^{2-\epsilon})$

for every ordered matching M. We resolve two special cases of this conjecture. We show that the off-diagonal ordered Ramsey numbers for matchings in which edges do not cross are nearly linear. We also prove a slightly sub-quadratic bound for random matchings with interval chromatic number 2.

Flips in Reduced Plabic Graphs

Julian Wellman
Mentor: Alexey Balitskiy
Project suggested by Alexander Postnikov

Planar bicolored (plabic) graphs are combinatorial objects introduced by Postnikov to give parameterizations of the positroid cells of the totally nonnegative Grassmannian $\mathrm{Gr}^{\geq 0}(n,k)$. Any two plabic graphs for the same positroid cell can be related by a sequence of certain moves. The *flip graph* has (trivalent) plabic graphs as vertices and has edges connecting the pairs of plabic graphs which can be related by a single move. A recent result of Galashin shows that plabic graphs can be seen as cross-sections of zonotopal tilings for the cyclic zonotope $Z(n,3)$. From this perspective, we show that the fundamental group of the flip graph is generated by cycles of length 4, 5, and 10, and use this result to prove a conjecture of Dylan Thurston about triple crossing diagrams. We also investigate the diameter of the flip graph for the top cell of $\mathrm{Gr}^{\geq 0}(2k,k)$ and of a particular subgraph corresponding to double wiring diagrams. In both cases we prove that the diameter is at least $\frac{k(k-1)^2}{2}$, and conjecture that this is exact.

The Sidorenko Problem for Directed Graphs in Tournaments

Yunkun Zhou
Mentor: Jonathan Tidor
Project suggested by Yufei Zhao

Sidorenko's problem asks to characterize the family of undirected graphs H for which the pseudorandom graph with edge density p has asymptotically the minimum number of copies of H over all graphs on the same number of vertices and edge density. In this paper, we study the directed analogue of Sidorenko's problem, namely to determine the family of directed graphs $\vec{H}$ for which the pseudorandom tournament has asymptotically the minimum number of copies of $\vec{H}$ over all tournaments on the same number of vertices. Here we show several ways to construct directed Sidorenko graphs out of other directed Sidorenko graphs, and give all Sidorenko graphs whose underlying undirected structure is a star. We also show a few other techniques we used to study the Sidorenko graphs whose underlying structure is a path. It is known that transitive tournaments are the only tournaments with the Sidorenko property. We characterize when a transitive tournament minus an edge has the Sidorenko property in most cases.