2016 SPUR CONFERENCE

Room 2-449

9:30 am Conference Opening, by SPUR Faculty Advisors Prof. David Jerison and Prof. Ankur Moitra

9:40 am: Yonah Borns-Weil and Kavish Gandhi, Bounds on Moment-Based Learning of Mixture Distributions (mentor Will Perry)

10:10 am: YiYu Zhang, Correspondence between Generic Alcoved Polytopes and Subdivisions of the Root Polytope of Lie Type A (mentor Pavel Galashin)

10:40 am: Jonathan Tidor and Victor Wang, 1-color-avoiding Paths, Special Tournaments, and Incidence Geometry (mentor Ben Yang)

11:10-11:20 am: break

11:20 am: Tahsin Saffat, Linearizable Laurent Phenomenon Sequences (mentor Pavel Galashin)

11:50 am: Lingfu Zhang, Refinements of 2-dimensional Strichartz Estimate by the Maximum Wave Packet (mentor Hong Wang)

12:20 am - 1:20 pm: lunch break

1:20 pm: Joseph Zurier, Symplectic Representation Theory and the Weyl Algebra in Positive Characteristic (mentor Augustus Lonergan)

1:50 pm: James Rowan, S-unit Equations and Curves of Genus 2 with Good Reduction away from 3 (mentor Borys Kadets)

2:20 pm: Hyun Jong Kim, Local Conjugacy in $\mathrm{GL}_2(\mathbb{Z}/p^2\mathbb{Z})$ (mentor Atticus Christensen)

2:50-3:00 pm: break

3:00 pm: Colin Aitken, Generators for Maximal Finite Subgroups of the Morava Stabilizer Group (mentor Lyuboslav Panchev)

3:30 pm: Arkadiy Frasinich, An Investigation on an Extension of Mullineux Involution (mentor Augustus Lonergan)

4 pm: Conference Closing

Kavish Gandhi and Yonah Borns-Weil

Bounds on Moment-Based Learning of Mixture Distributions

Mentor: Will Perry

Project suggested by Prof. Ankur Moitra

We study the problem of learning the parameters of a mixture of members of a given distribution family. To do this, we apply the method of moments, dating to Pearson in the late 1800's: we directly solve for the parameters in terms of estimated sample moments. We prove upper and lower bounds on the number of moments that uniquely determine mixtures for various distribution families. In particular, we show that $2k-1$ moments are necessary and sufficient to determine a large class of mixtures of k one-parameter distributions, including Poissons and exponentials, and develop an efficient algorithm to learn the parameters of these mixtures. We also show that $4k-2$ moments are necessary for determining a mixture of k Gaussians matching a known upper bound shown by Moitra and Valiant, and with this we improve the existing lower bound on the sample complexity of learning such a mixture. We prove similarly that $4k-2$ moments are necessary and sufficient to learn a mixture of k uniform distributions, and conjecture that a similar result holds for general two-parameter distributions whose moments satisfy certain polynomial dependence conditions on the parameters. Finally, for a general family of Gaussian-like distributions of the form $p(x)e^{q(x)}$, we derive a bound on the number of moments necessary that is exponential in k.

YiYu Zhang

A Correspondence between Generic Alcoved Polytopes and Subdivisions of the Root Polytope of Lie Type A

Mentor: Pavel Galashin

Project suggested by Prof. Alexander Postnikov

Alcoved polytopes of Lie type A are polytopes whose facets are orthognal to the roots of root system A_n. An alcoved polytope of type A_n is generic if it has a facet orthogonal to each root in A_n. In this paper, we prove that there is one-to-one correspondence between equivalence classes of generic alcoved polytopes of type A_n and regular central subdivisions of the convex hull of the roots of A_n. We apply this result to give an explicit classification for small n.

Jonathan Tidor and Victor Wang

1-color-avoiding Paths, Special Tournaments, and Incidence Geometry

Mentor: Ben Yang

Project suggested by Prof. Po-Shen Loh (CMU)

We discuss two approaches to a recent question of Loh: must a 3-colored transitive tournament on N vertices have a 1-color-*avoiding* path of vertex-length at least $N^{2/3}$? This question generalizes the Erdős–Szekeres theorem on monotone subsequences.

First, we define three canonical transformations on these tournaments called Color, Record, and Dual. We use these to establish a reduction to certain special tournaments that have natural geometric and combinatorial properties. In many cases (including all known tight examples), these tournaments have recursive Gallai decompositions. Not all relevant tournaments have Gallai decompositions, but those that do satisfy the desired $N^{2/3}$ bound, proven inductively using Cauchy–Schwarz and a *weighted* version of Erdős–Szekeres. This last ingredient resembles a weighted Ramsey's theorem used by Fox, Grinshpun, and Pach for a similar *undirected* problem. It can also be used to partially address a problem of Erdős documented by Steele.

Second, we consider the related geometric problem of bounding *slice-increasing* sets $S \subseteq [n]^3$, which—under an additional ordering hypothesis on S—was shown by Loh to be equivalent to the original question. In particular, we prove various facts and partial results about such sets, establish a rigorous connection to the problem of Szabó and Tardos, and mention a surprising overlap with the joints problem.

SESSION 2

Tahsin Saffat

Linearizable Laurent Phenomenon Sequences

Mentor: Pavel Galashin

Project suggested by Prof. Pavlo Pylyavskyy (UMN)

Laurent phenomenon sequences are sequences $(x_i)_{i\in\mathbb{N}}$ such that there is a polynomial P with integer coefficients and an integer N such that $x_{n+N}x_n = P(x_{n+1}, ..., x_{n+N-1})$ and all of whose terms are Laurent polynomials in $x_1, ..., x_N$. We study Laurent phenomenon sequences that additionally satisfy a linear recurrence whose coefficients depend on the initial values of the sequence. We study several Laurent phenomenon sequences that were constructed by Alman, Cuenca, Huang, using period 1 seeds in Laurent phenomenon algebras introduced by Lam and Pylyavskyy. We conjecture which of the sequences arising from period 1 seeds are linearizable. For a particular sequence $x_{n+p+q}x_n = x_{n+p}x_{n+q} + 1$, the cluster algebra generated by the seed has an associated marked surface. By studying triangulations of this surface, we give a combinatorial formula in terms of almost perfect matchings for the linear recurrence coefficients.

Lingfu Zhang

Refinements of 2-dimensional Strichartz Estimate by the Maximum Wave Packet

Mentor: Hong Wang

Project suggested by Prof. Larry Guth

For the linear Schrödinger equation in $1+1$ dimension, the solution for initial data u_o is given by $e^{it\Delta/2}u_o$. The Strichartz inequality is used to prove the existence of the solution when there is non-linear perturbation. When p less than 6, the Strichartz inequality is not sharp when the initial data is "spread out" (both in physical space and frequency space). We aim to improve the Strichartz estimate in this case, using different approaches including polynomial partition and ideas from the proof of l^2-decoupling.

SESSION 3

Joseph Zurier

Symplectic Representation Theory and the Weyl Algebra in Positive Characteristic

Mentor: Augustus Lonergan

Project suggested by Prof. Roman Bezrukavnikov

Given a symplectic representation $V \oplus V^*$ of a finite group G over a field k with characteristic $p > 0$, we can extend the G-action in a natural way to an action on the Weyl algebra W on $\dim(V)$ variables. This allows us to form the smash algebra $W\#G$ as a product of the Weyl algebra and the group algebra $\mathrm{k}G$. In this paper, we explore the problem of whether the ideal generated by the trivial idempotent of the group algebra contains 1. We are able to give an explicit condition in the case where G is abelian, and extend these techniques to tackle the case where G is solvable. In addition, we shed some insight on the problem for a general group G.

James Rowan

S-unit equations and curves of genus 2 with good reduction away from 3

Mentor: Borys Kadets

Project suggested by Dr. Andrew Sutherland

The Shafarevich conjecture (now a theorem of Faltings) guarantees that for any genus $g \geq 1$, there are only finitely many isomorphism classes of curves over $\mathbb{Q}$ with good reduction outside any given finite set of primes. For hyperelliptic curves, an effective bound is known, but it is too large to enable an explicit enumeration even for single primes. N. P. Smart has produced an explicit list of all genus 2 curves with good reduction outside 2 by transforming the problem into the problem of solving S-unit equations over a specific set of number fields. We adapt these methods to the prime 3 and produce tighter bounds on the number of possible hyperelliptic curves of genus 2 with good reduction outside 3, subject to the restriction that the curve must have a rational Weierstrass point. We also give a partial list of such curves.

Hyun Jong Kim

Local Conjugacy in $\mathrm{GL}_2(\mathbb{Z}/p^2\mathbb{Z})$

Mentor: Atticus Christensen

Project suggested by Dr. Andrew Sutherland

Subgroups H_1 and H_2 of a group G are said to be locally conjugate if there is a bijection $\varphi : H_1 \to H_2$ such that h and $\varphi(h)$ are conjugate in G. We study local conjugacy among subgroups of $\mathrm{GL}_2(\mathbb{Z}/p^2\mathbb{Z})$, where p is an odd prime, building on Andrew Sutherland's categorizations of subgroups of $\mathrm{GL}_2(\mathbb{Z}/p\mathbb{Z})$ and local conjugacy among them. We obtain a classification of locally conjugate subgroups of $\mathrm{GL}_2(\mathbb{Z}/p^2\mathbb{Z})$ in the kernel of the natural map $\varphi : \mathrm{GL}_2(\mathbb{Z}/p^2\mathbb{Z}) \to \mathrm{GL}_2(\mathbb{Z}/p\mathbb{Z})$. We further inspect local conjugacy among subgroups of $\mathrm{GL}_2(\mathbb{Z}/p^2\mathbb{Z})$ using this classification.

SESSION 4

Colin Aitken

Generators for Maximal Finite Subgroups of the Morava Stabilizer Group

Mentor: Lyuboslav Panchev

Project suggested by Prof. Haynes Miller

The Morava stabilizer group $\mathbb{S}_n$ at a prime p is the automorphism group of the Honda formal group law F_n. The maximal finite subgroups of $\mathbb{S}_n$ are central to the construction of higher real K-theories, and were first classified by Hewett. In this paper, we use techniques inspired by Kummer theory to produce explicit expressions for the generators of the largest nonabelian finite subgroup when p is odd.

Arkadiy Frasinich

An Investigation on an Extension of Mullineux Involution

Mentor: Augustus Lonergan

Project suggested by Prof. Roman Bezrukavnikov

In this paper, the action of Mullineux Involution on the b-core and b-quotient of a partition is studied. We then offer a new perspective on a conjecture regarding Mullineux Involution by considering the implicit extension of the involution present in the conjecture, which is then approached via its action on the b-quotient. When the b-quotient has rank at most 2, we give a description of this action and a conjecture on the distribution of lengths of cycles it creates among partitions with size n. The original conjecture is then revisited, where future approaches for work on the conjecture and the extension of Mullineux Involution are given.