2022 SPUR (Summer Program in Undergraduate Research) Conference

Friday, August 05, 2022

1) Milan Haiman, "Irreducibility of Generalized Permutohedra, Supermnodular Functions, and Balanced Multisets." (mentor: Yuan Yao)

2) Shu Ge, "Cohomology of heavy/light moduli spaces of curves" (mentor: Nitya Mani)

3) Sanjay Raman, "Multiplication Kernels for the Analytic Langlands Program in Genus Zero." (mentor: Danill Kliuev)

4) Evgeniya Artemova and Christina Yu, "Lower Bounds on the Discrete Newton's Algorithm for the Submodular Line Search Problem." (mentor: Yuchong Pan)

5) Merrick Cai, "Decomposition of Frobenius Pushforwards of Line Bundles on Wonderful Compactifications." (mentors: Vasily Krylov)

6) Paige Dote, "Exceptional Set Estimates For Orthogonal and Radial Projections In $\mathbb{R}^n$" (mentor: Shengwen Gan)

7) Honglin Zhu, "Maximum Overlap Area of a Convex Polyhedron and a Convex Polygon Under Translation." (mentor: Hyuk Jun Kweon)

8) Carlos Alvarado and Ayodeji Linblad, "Dynamical Stability of Translators Under Mean Curvature Flow." (mentor: Tang-Kai Lee)

9) Kenta Suzuki, "Gelfand-Kirillov Dimension of Representations of GL_n Over a Non-Archimedean Local Field" (mentor: Hao Peng)

10) Andrew Yao, "Densities for Elliptic Curves over Global Function Fields." (mentor: Hao Peng)

Milan Haiman with Yuan Yao
(Project suggested by Prof. Yufei Zhao)

"Irreducibility of Generalized Permutohedra, Supermnodular Functions, and Balanced Multisets."

Abstract

We study generalized permutohedra and supermodular functions. Specifically we analyze decomposability and irreducibility for these objects and establish some asymptotic behavior. We also study a related problem on irreducibility for multisets.

Shu Ge with Nitya Mani
(Project suggested by Yufei Zhao)

"Cohomology of heavy/light moduli spaces of curves."

Abstract

The problem on the largest triangle-intersecting family of graphs on n labeled vertices was proposed by Simonovits and Sós in 1976. They conjectured the upper bound to be $2^{\binom{n}{2}-3}$, which is obtained by taking all graphs containing some fixed triangle. This was proven by Ellis, Filmus, and Friedgut, who also conjectured that their results extend to the K_t -intersecting families. Berger and Zhao recently proved the case for t = 4. We present current progress to prove the case for t = 5, including reductions and optimizations in the computational process. Moreover, we extend these conjectures to hypergraph $K_4^{(3)}$-intersecting families of graphs.

Sanjay Raman with Danill Kliuev
(Project suggested by Prof. Pavel Etingof)

"Multiplication Kernels for the Analytic Langlands Program in Genus Zero."

Abstract

We provide an explicit proof of a recent result of Gaiotto [5] which gives an explicit formula for a so-called "multiplication kernel" $K_3(x, y, z; t)$ intertwining the action of Hecke operators and Gaudin operators in three sets of variables. This function K3 arises naturally in the context of the analytic formulation of the geometric Langlands program in the genus-zero case [2, 3, 4]. We also discuss how the kernel K3 relates to other objects typically considered in the analytic Langlands program.

Evgeniya Artemova and Christina Yu with Yuchong Pan
(Project suggested by Prof. Michel Goemans)

"Lower Bounds on the Discrete Newton's Algorithm for the Submodular Line Search Problem."

Abstract

Submodular functions arise naturally from the principle of diminishing marginal returns in economics. We are interested in the line search problem in the submodular polytope of a submodular function, which asks for the maximum step length within the polytope in a given search direction from a given starting point. This problem can be solved by the discrete Newton's algorithm. While linear and quadratic upper bounds are known on the number of iterations of the algorithm in the cases where the search direction is nonnegative and arbitrary, respectively, we have little knowledge on whether there are matching lower bounds in these two cases. In this paper, we provide several initial attempts towards lower bound constructions in these problems.

Merrick Cai with Vasily Krylov
(Project suggested by Prof Roman Bezrukavnikov)

"Decomposition of Frobenius Pushforwards of Line Bundles on Wonderful Compactifications."

Abstract

De Concini-Procesi introduced varieties known as wonderful compactifications, which are smooth projective compactifications of semisimple adjoint groups G. We study the Frobenius pushforwards of invertible sheaves on the wonderful compactifications, and in particular its decomposition into locally free subsheaves. We give necessary and sufficient conditions for a specific line bundle to be a direct summand of the Frobenius pushforward of another line bundle, formulated in terms of the weight lattice of $\bar{G}$, the universal cover of G (identified with the Picard group of the wonderful compactification). In the case of $G = PSL_n$, we offer lower bounds on the multiplicities (as direct summands) for those line bundles satisfying the sufficient conditions. We also decompose Frobenius pushforwards of line bundles into a direct sum of vector subbundles, whose ranks are determined by invariants on the weight lattice of G. We study a particular block which decomposes as a direct sum of line bundles, and identify the line bundles which appear in this block. Finally, we present two approaches to compute the class of the Frobenius pushforward of line bundles on wonderful compactifications in the rational Grothendieck group and in the rational Chow group.

Paige Dote with Shengwen Gan
(Project suggested by)

"Exceptional Set Estimates For Orthogonal and Radial Projections In $\mathbb{R}^n$"

Abstract

We give different proofs of classic Falconer-type and Kaufmantype exceptional estimates for orthogonal projections using the high-low method. With the new techniques, we resolve Liu's conjecture on radial projections:

given a Borel set A Rn, we have

dim({x $\in \mathbb{R}^n \setminus A \mid dim(\pi_x(A)) < dimA\}) \leq \lceil dimA \rceil$.

Honglin Zhu with Hyuk Jun Kweon
(Project suggested by Prof. Bjorn Poonen)

"Maximum Overlap Area of a Convex Polyhedron and a Convex Polygon Under Translation."

Abstract

Let P be a convex polyhedron and Q be a convex polygon with n vertices in total in three-dimensional space. We present a deterministic algorithm that finds a translation vector $v \in \mathbb{R}^3$ maximizing the overlap area $|P \cap (Q + v)|$ in $O(n\ log^2\ n)$ time. We then apply our algorithm to solve two related problems. We give an $O(n\ log^3\ n)$ time algorithm that finds the maximum overlap area of three convex polygons with n vertices in total. We also give an $O(n\ log^2\ n)$ time algorithm that minimizes the symmetric difference of two convex polygons under scaling and translation.

Carlos Alvarado and Ayodeji Linblad with Tang-Kai Lee
(Project suggested by Prof. Bill Minicozzi)

"Dynamical Stability of Translators Under Mean Curvature Flow."

Abstract

Convergence of a class of perturbations of the line in $\mathbb{R}^3$ to a line under the curve shortening flow is proven. Progress is presented towards conditions on perturbations of the Grim Reaper cylinder which guarantee convergence under mean curvature flow to the Grim Reaper cylinder.

Kenta Suzuki with Hao Peng
(Project suggested by Prof. Bjorn Poonen)

"Gelfand-Kirillov Dimension of Representations of GL_n Over a Non-Archimedean Local Field."

Abstract

We calculate the asymptotic behavior of the dimension of the fixed vectors of π with respect to compact open subgroups $1 + Mn(\mathfrak{p}^N) \subset GL_n(F)$ for an admissible representation of $GL_n(F)$, and F a nonarchimedean local field. Such dimensions can be calculated by germs of the character of π. We also make some observations on how those dimensions behave under instances of Langlands functoriality, such as the Jacquet-Langlands correspondence and cyclic base change, where relations between characters are known.

Andrew Yao with Hao Peng
(Project suggested by Prof. Bjorn Poonen)

"Densities for Elliptic Curves over Global Function Fields."

Abstract

Let K be a global function field. Using Haar measures, we compute the densities of the Kodiara types and Tamagawa numbers of elliptic curves over a completion of K. Also, we prove results about the number of iterations of Tate's algorithm that are completed when the algorithm is used on an elliptic curve over a completion of K.