Li Anqi

On Density of Integers and the Sumset

under the direction of Hong Wang

Abstract

The Schnirelmann density of a set X of non-negative integers containing 0 is defined as $d(X) = \inf_{n \geq 1} \frac{X(n)}{n}$. Mann's theorem states that for sets A, B of non-negative integers, we have $d(A+B) \geq d(A)+d(B)$. We consider a modified density that accounts for the global average density $d_{\lim}(X) = \lim_{n\to\infty} \inf_{m \geq n} \frac{X(m)}{m}$ and establish an analogue to Mann's Theorem which holds for this modified density: $d_{\lim}(A+B) \geq \max\{d_{\lim}(A), d_{\lim}(B)\} + \frac{\min\{d_{\lim}(A), d_{\lim}(B)\}}{2}$. We also show that this bound is sharp.

Jordan Lee

Stability of Finite Difference Schemes on the Diffusion Equation with Discontinuous Coefficients

under the direction of Sungwoo Jeong

Abstract

The diffusion equation is one of the most fundamental PDEs, and is a standard introductory equation in numerical analysis texts. When the diffusivity coefficient k is constant, the stability condition is known for explicit finite difference schemes by Von Neumann's stability analysis. The stability condition for a variable diffusivity coefficient $k(x)$ can be extended by using $\max_x(k(x))$ as a constant coefficient k, by the method of frozen coefficients. However, the heuristic reasoning for this condition assumes the continuity of the diffusivity coefficient. We prove this stability condition still holds for the 1-D and 2-D conservative finite difference schemes with discontinuous coefficients, and provide numerical evidence for the 1-D and 2-D non-conservative finite difference schemes with discontinuous coefficients.

Tanya Otsetarova

Boundaries on the Number of Points in Acute Sets

under the direction of Zhulin Li

Abstract

A famous Erdős problem asks for the maximum cardinality of the sets of points determining only angles less than or equal to $\frac{\pi}{2}$. We examine similar problem but for sets in $\mathbb{R}^d$ determining only angles less than given angle θ. Our results hold for values of θ in different intervals. First, we prove that any set of points determining only angles less than or equal to $\frac{\pi}{3}$ has maximal cardinality $d+1$ and it is attained if and only if the polytope defined by all points in the set is a simplex. We also find a lower bound for the maximal cardinality of sets with angles $< \theta$, when $\theta \in (\frac{19\pi}{45}, \frac{\pi}{2})$. Lastly, we prove that every set determining angles less than or equal to θ is finite for $\theta < \pi$.

Alan Peng

The Lusztig-Vogan Bijection in the Case of the Trivial Representation

under the direction of Guangyi Yue

Abstract

Let G be a connected complex reductive algebraic group, and let $\mathfrak{g}$ be its Lie algebra. Then the Lusztig-Vogan bijection relates Λ^+, the set of dominant weights of G, to Ω, the set of pairs of the form $(\mathcal{O}, \mathcal{E})$, where $\mathcal{O}$ is a nilpotent orbit in $\mathfrak{g}$ and $\mathcal{E}$ is an irreducible representation of the centralizer G^e for $e \in \mathcal{O}$. Algorithms computing the Lusztig-Vogan bijection $\gamma : \Omega \to \Lambda^+$ when G is the complex general linear group $GL_n(\mathbb{C})$ were described by Achar and Rush. Here we study the algorithm given by Rush and give an explicit description in the case where $\mathcal{E}$ is the trivial representation.

Sílvia Casacuberta Puig

On the Divisibility of Binomial Coefficients

under the direction of Oscar Mickelin

Abstract

We analyze an open problem in number theory regarding the divisibility of binomial coefficients. It is conjectured that for every integer n there exist primes p and r such that if $1 \leq k \leq n-1$ then the binomial coefficient $\binom{n}{k}$ is divisible by at least one of p or r. We prove the validity of the conjecture in several cases and obtain inequalities under which the conjecture is satisfied. We relate the problem to Cramér's, Oppermann's and Riemann's conjectures on prime gaps and study cases in which the conjecture is true using three primes instead of two. Moreover, we establish four upper bounds on the minimum number of primes needed for the conjecture to be true.

Michelle Shen

Modeling Population Dynamics in Changing Environments

under the direction of YounHun Kim

Abstract

Discrete replicator dynamics view evolution as a coordination game played among genes. While previous models of discrete replicator dynamics do not consider environments that respond to the mixed strategy that a population plays, our model incorporates a feedback-based payoff matrix. From this model we construct, using Nash equilbria, situations in which the population dynamics do not converge, but rather oscillate and form a stable limit cycle. We also find that a noisy variation of our model behaves similarly to the original model. Finally, we explore a new mutation model, which incorporates horizontal gene transfer.

Grace M. Tian

The Ratio of Length to Width of Flat Knotted Ribbons

under the direction of Vishal Patil

Abstract

Knotted ribbons have applications in biology and chemistry. They have been used to model the cyclic duplex DNA in molecular biology. A flat knotted ribbon can be obtained by gently pulling a knotted ribbon tight so that it becomes flat and folded.

We study the minimal ratio of the length to width for a flat knotted ribbon. We find a way to calculate a local upper bound on the width of a flat knotted ribbon from its knot diagram. We also investigate a family of flat knotted ribbons for the twist knots when the number of half twists is an even integer. For each member in the family, we calculate the ratio of the length to width and thus obtain an upper bound on the minimal ratio. We finally investigate the possible relation between the minimal ratio and the number of crossings in the knot for each flat knotted ribbon in the family.

David Wu

Nonuniform Distributions of Patterns of Sequences of Primes in Prime Moduli

under the direction of Robert Burklund

Abstract

For positive integers q, Dirichlet's theorem states that there are infinitely many primes in each reduced residue class modulo the common difference q. Extending a proof of Dirichlet's theorem shows that the primes are equidistributed among the $\varphi(q)$ reduced residue classes modulo q. We consider patterns of sequences of consecutive primes $(p_n, p_{n+1}, \ldots, p_{n+k})$ modulo q. Numerical evidence suggests a preference for certain prime patterns. For example, computed frequencies of the pattern (a, a) modulo q up to x are much less than the expected frequency $\pi(x)/\varphi(q)^2$. We begin a project of rigorously connecting the Hardy-Littlewood prime k-tuple conjecture to a conjectured asymptotic formula for the frequencies of prime patterns modulo q. We extend a data gathering procedure to estimate prime patterns up to 10^18, an improvement of 8 orders of magnitude over previous methods. Using the extended range of data, we identify, via curve fitting, a possible lower order term in the conjectured formula. We begin to extend a numerical model to reduce the uncertainty in the predictions of these biases in prime patterns. The improved numerical could guide future progress towards understanding implications of the Hardy-Littlewood prime k-tuple conjecture.

Karthik Yegnesh

On the Relationship Between Braid Groups on Triangulated Surfaces and their Singular Homology

under the direction of Augustus Lonergan

Abstract

Let Σ_g denote the closed orientable 2-manifold of genus g and fix an arbitrary simplicial triangulation of Σ_g. We study a natural group homomorphism from the surface braid group on n strands on Σ_g to its first singular homology group with integral coefficients. In particular, we show that the kernel of this homomorphism is generated by canonical braids which arise from the triangulation of Σ_g.