# RSI 2009 ABSTRACTS

## Anirudha Balasubramanian

### On the Lower Central Series Quotients of a Graded Associative Algebra

Martina Balagović

Let A be a noncommutative associative algebra, viewed as a Lie algebra via definition of the Lie bracket $[x, y] = xy - yx$. Define the *lower central series filtration* inductively by $L_1(A) = A$, and $L_{i+1}(A) = [A, L_i(A)]$. Denote the components of its associated graded space by $B_i(A) = L_i(A)/L_{i+1}(A)$.

The components of the associated graded space are well understood in the cases when $A = A_n =$ the free algebra over $\mathbb{C}$ with $n$ generators, and when $A$ is a quotient of $A_n$ by several relations with some additional smoothness conditions on the abelianization of $A$. In both those cases, the $B_i(A)$ have polynomial growth for $i > 1$, and $B_2(A) \cong \Omega^{odd}(A_{ab})/d\Omega^{even}(A_{ab})$. Here $\Omega(A_{ab})$ denotes differential forms on the abelianization of $A$, and $d$ is a diferentiation map.

We study the case when $A = A_n/\langle P\rangle$, for $P$ a homogeneous polynomial of any degree. This does not satisfy the aforementioned smoothness conditions. In this case, $A$ and all the $B_i(A)$ are graded by degree of polynomial.

We find a basis for $B_2(A_n/\langle x^m+y^m\rangle)$ for $n = 2, 3, m > 1$. Using its Hilbert series and the previous results in the smooth case, we can then conclude that for a generic homogeneous polynomial $P$ and $A = A_n/\langle P\rangle$, we have the isomorphism

$$B_2(A) \cong \Omega^1(A_{ab})/d\Omega^0(A_{ab}).$$

We also conjecture a Hilbert series for $B_2(A_4/\langle P\rangle)$ and the existence of a similar isomorphism.

Martin Camacho


# Lattice Representations and Linear Extensions of Series-Parallel and $(m+n)$-free Posets


Yan Zhang



We analyze three statistics of particular families of posets. First, we investigate the sets of comparable and incomparable elements of a poset and their relationship to the number of linear extensions of that poset. We then extend results of Tenner to particular families of posets, and specifically look at the number of linear extensions of (3+1)-free posets. We then relate our study of linear extensions to lattice representations of posets through connecting theorems. We end with a theorem which proves an equivalence between posets lacking a particular subposet and lattice representations lacking a corresponding sublattice. Our work has possible applications in cryptography, event processing, and computing theory.

Jonathan Hung

# The Kummer congruence for Hurwitz numbers

Jennifer French

We begin with the Kummer congruence for Bernoulli numbers and discuss its relevance towards number theory. We introduce how one obtains a lattice from an elliptic curve, and define the Hurwitz numbers as the Eisenstein series of a lattice in $\mathbb{C}$. The goal is to give elementary proofs of the Kummer congruence for Hurwitz numbers, which is done for certain lattices associated to particular elliptic curves at the primes 5 and 7. Some work towards a generalization to all primes and all lattices of good reduction for such a prime is shown.

# Jacob Hurwitz


## Decycling Density of Tessellations


Nan Li



We investigate decycling numbers of graphs and, in particular, decycling densities of tessellations. First, we find an equivalent description for the greedily-drawn tree in the two-dimensional integer lattice, and we use this description to find an upper bound for the decycling number of the finite grid.

We also provide a lower bound for the decycling density of any infinite graph where all vertices have the same degree, such as for certain tessellations. For each of the three regular and eight semiregular tessellations in two dimensions, we bound the decycling density (as well as the densities of the largest induced tree and forest) and then prove exact values for these densities, showing when the bound is and is not achievable.

Finally, we briefly consider a few extensions. For example, we tile root systems of Lie algebras, and we also use a clever mapping from $\mathbb{Z}^d \to \mathbb{Z}$ to form a Cayley graph. From this graph, we find the decycling density of the $d$-dimensional hypercubical lattice. In the future, we hope to extend our work to tessellations in higher dimensions; we would also like to prove some conjectures regarding the decycling densities of Cayley graphs in general.

Damien Jiang

# On the Period Lengths of the Parallel Chip-Firing Game

Yan Zhang

We give a proof, using and extending methods similar to those of Levine, that the set of all periods of the parallel chip-firing game on the complete regular bipartite graph $K_{a,b}$ is

$$\{i, 1 \leq i \leq \min(a,b)\} \cup \{2i, 1 \leq i \leq \min(a,b)\}.$$

We also prove for general connected graphs $G$ that if the chip-firing game on $G$ starting from a position $\sigma$ has nontrivial period, then all vertices have at most $2\deg(v) - 1$ chips, and that every vertex has between $\Phi(v)$ and $\Phi(v) + n - 1$ chips, where $\Phi(v)$ is the number of firing neighbors of $v$. Furthermore, we show that the complement $\sigma_c$ of a position $\sigma$ behaves exactly as $\sigma$ does, and that a period of $\sigma$ occurs if and only if every vertex has fired the same number of times.

George Kerchev

# On the Filtration of the Free Algebra by Ideals Generated by its Lower Central Series

Bhairav Singh

This project concerns the lower central series $L_i$ of a free associative algebra $A_n$. In this paper we study the sequence of $N_i(A_n) = AL_i/AL_{i+1}$, which are the quotients of two-sided ideals generated by the lower central series. We prove a recent conjecture of Arbesfeld and Jordan for a upper bound of $|\lambda|$ for the decomposition of $N_i$ into $\mathcal{F}_\lambda$. This allows us to compute these decompositions for small $n$ and $i$.

Akhil Mathew

# A Classification of Finite-Dimensional Simple Objects in the Generic Deligne Category of the Degenerate Affine Hecke Algebra

Dustin Clausen

In this paper, we study the "finite-dimensional" objects in Etingof's category $\mathrm{Rep}(\mathscr{H}_T, \mathbb{C}(T))$ of representations of the degenerate affine Hecke algebra for generic $T$ that project to objects in $\mathrm{Rep}(S_T)$. We construct combinatorial interpolations of the evaluation and induction functors for the dAHA. Then, we use twisted versions of Zelevinsky's classification of simple $\mathscr{H}_n$-representations to construct a family of simple objects in $\mathrm{Rep}(\mathscr{H}_T, \mathbb{C}(T))$. Using algebraic techniques, we prove this family is complete. In doing so, we parametrize the simple objects using a scheme of finite type over $\mathbb{C}$. The techniques developed suggest generalizations to the study of suitable families of categories parametrized by schemes.

Dimitrios Pagonakis

# An Improved Lower Bound on Mosers Worm Problem

Tirasan Khandhawit

Mosers Worm problem, a deceivingly simple problem, poses the question: What is the minimum area of a convex region that can accommodate all curves of length one? Because of its great difficulty, mathematicians have tried to bound it, with the latest lower bound to be established on 2007 by Khandhawit and Sriswasdi. Their bound was 0.227498. In this paper we use four special unit arcs and determine that the convex hull of these curves has area greater than 0.23199. Consequently we improve the lower bound of this open problem, from 0.227498 to 0.231999.

# Arjun Ranganath Puranik

## Finite-dimensional Irreducible Representations of Rational Cherednik Algebras Associated to the Coxeter Group H3

Martina Balagović

We study representation theory of rational Cherednik algebras $H_c(H_3, \mathfrak{h})$ associated to the finite Coxeter group $H_3$ (the group of symmetries of the icosahedron), its complexified reflection representation $\mathfrak{h}$, and a complex parameter $c$. For such an algebra and any irreducible representation $\tau$ of $H_3$, one can define a standard (Verma) lowest weight module $M_c(\tau)$. This module has a unique maximal proper submodule $J_c(\tau)$, and the quotient $L_c(\tau) = M_c(\tau)/J_c(\tau)$ is irreducible. Here we determine which values of $c$ and $\tau$ yield finite-dimensional $L_c(\tau)$, and its character in those cases. It is known that all finite-dimensional irreducible representations of these algebras occur as $L_c(\tau)$ for some $c$ and some $\tau$, so we in fact determine all finite-dimensional irreducible representations .

Using strong previous results from Bezrukavnikov-Etingof and Rouquier, the MAGMA computational algebra system, and algebraic and representation theoretic methods, we get the following list:

**Theorem 1.** *$L_c(\tau)$ is finite-dimensional for these pairs of $(c, \tau)$ :*

$$(\frac{r}{2}, \mathbb{C}_+), (\frac{r}{2}, \mathbb{C}^3_-), (\frac{r}{2}, \tilde{\mathbb{C}}^3_-), (\frac{r}{6}, \mathbb{C}_+), (\frac{r}{10}, \mathbb{C}_+)$$
$$(\frac{-r}{2}, \mathbb{C}_-), (\frac{-r}{2}, \mathbb{C}^3_+), (\frac{-r}{2}, \tilde{\mathbb{C}}^3_+), (\frac{-r}{6}, \mathbb{C}_-), (\frac{-r}{10}, \mathbb{C}_-)$$

*Here, $r \in \mathbb{Z}_+$, and all fractions are reduced.*
For each of these representations we also compute the character of $L_c(\tau)$.

Lauren Stephens

# Bounds on the Relative Sizes of Sumsets

Bhairav Singh

An additive set is a subset of an abelian group, where sets can be added element-wise. We investigate the conjecture that for any additive set $A$, the sumset inequality $\frac{|A+2A|}{|A|} \leq \left(\frac{|A+A|}{|A|}\right)^p$ holds for some $p < 3$. We give a proof of this for the cases when $|A+A| \geq |A|^{1+\frac{1}{p}}$. Then, we conjecture that $\frac{|A+2A|}{|A|} \leq \left(\frac{|A+A|}{|A|}\right)^p$ holds for all $p > \frac{log2}{log\frac{3}{2}}$ . To support our new conjecture, we provide examples of the sets for which the sumset inequality with $p = \frac{log2}{log\frac{3}{2}}$ holds. We also give computational evidence for the conjecture.

# Dennis Tseng

## Generalized Nonaveraging Integer Sequences

Nan Li

Let the sequence $S_m$ of nonnegative integers $0 = a_0 < a_1 < a_2 \ldots$ be generated by the greedy algorithm such that for all $k \geq 0$, $a_{k+1}$ is the minimum nonnegative integer such there are no *distinct* terms $x_1, x_2, \ldots, x_m$ in the sequence such that $x_1 + x_2 + \cdots + x_{m-1} = (m-1)x_m$. Let the sequence $A_m$ of nonnegative integers be also generated using the greedy algorithm such that there are no terms $x_1, x_2, \ldots, x_m$, *not all the same*, such that $x_1 + x_2 + \cdots + x_{m-1} = (m-1)x_m$.

We prove that the terms in $A_m$ are the integers with only 0's and 1's in base $m$. For $S_m$, Szekeres gave a closed-form description of $S_3$ in 1936, and Layman provided a similar description for $S_4$ in 1999. We extend the closed-form description of $S_m$ to any integer $m \geq 3$. Let $\mathfrak{A}_m$ be the set of integers that are terms of $A_m$. For every integer $m \geq 3$, there is a positive integer $c_m$ and set of nonnegative integers $R_m$, with all its elements less than $c_m$, such the set of integers that are terms of $S_m$ is $\{c_m a + r : a \in \mathfrak{A}_m, r \in R_m\}$.

Lynnelle Lin Ye

# Chomp on Graphs and Subsets

Tirasan Khandhawit

The game subset take-away begins with a simplicial complex $\triangle$. Two players take turns removing any element of $\triangle$ as well as all other elements which contain it, and the last player able to move wins. Graph Chomp is a special case of subset take-away played on a simplicial complex with only vertices and edges. The game has previously only been analyzed for complete graphs, forest graphs, and very small special cases of higher-dimensional simplicial complexes. We generalize a common method of reducing some game positions to simpler ones by symmetry and provide a complete analysis of complete $n$-partite graphs for arbitrary $n$ and all bipartite graphs. Finally, we give partial results for odd-cycle pseudotrees, which are non-bipartite graphs with a single cycle.