NOAH ARBESFELD

On the lower central series for the free algebra with two generators

DAVID JORDAN

Let A_n be the free associative algebra on n letters, viewed as a Lie algebra with $[a,b]=ab-ba$, and let $L_i=L_i(A_n)$ be the lower central series filtration, with associated graded component $B_i=L_i/L_{i+1}$. In [FS], an action of the Lie algebra W_n of polynomial vector fields was constructed on $\oplus_{i>1}B_i$. In [DE], each B_i was shown to have a finite-length Jordan-Hölder series.

We establish a linear bound on the degree of W_2 representations appearing in the Jordan- Hölder series of $B_i(A_2)$, thus strengthening the quadratic bound from [DE] to a conjecturally sharp bound, and we show that $B_{m+1}(A_2)$ is spanned by

$$[x, B_m(A_2)], [y, B_m(A_2)], \text{ and } [xy, B_m(A_2)],$$

thus establishing that the density a_m defined in [FS] is bounded by 3^{m-2}. As applications of these results, we compute the complete Jordan-Hölder series for $B_i(A_2)$ for $i=2,\ldots,7$. We also present partial series for $B_i(A_2)$ for $i=8,\ldots,12$ and $B_i(A_3)$ for $i=2,\ldots,7$.

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[FS] B. Feigin, B. Shoikhet. On $[A,A]/[A,[A,A]]$ and on a W_n-action on the consecutive commutators of free associative algebras. Math. Res. Lett. 14 (2007), no. 5, 781795.

KRISTIN CORDWELL

On G-Difference: A Property of Permutations and Words

JOEL LEWIS

Consider a simple graph G labeled with distinct positive integers so that a vertex v is labeled $n_v \in \mathbb{N}$. Let $x = (x_1, \ldots, x_n)$, $y = (y_1, \ldots, y_n)$ be permutations of $1, \ldots, n$. If for some $1 \leq i \leq n$, $x_i = n_v$ and $y_i = n_w$ so that v and w are two vertices of G joined by an edge, we say (following Körner, Malvenuto and Simonyi [1]) that x and y are *G-different*. The maximum number of pairwise G-different permutations of length n is denoted $\kappa(G, n)$. Various bounds of $\kappa(G, n)$ on permutations for general and specific G were described in [1]. In this paper we continue the construction of bounds on permutations of $\kappa(G, n)$ for specific graphs G. We also look at $\kappa_w(G, n)$, the maximum number of pairwise G-different *words* of length n for a graph G. We construct general bounds for $\kappa_w(G, n)$ as well as evaluate $\kappa_w(G, n)$ for specific G. Finally, we relate $\kappa_w(G, n)$ to $\chi_f(G)$, the fractional chromatic number of G.

Miles Edwards

2-Sylow subgroups of ideal class groups of imaginary quadratic fields

Liang Xiao

If $K = \mathbb{Q}(\sqrt{-D})$ is an imaginary quadratic number field, we use $Cl(-D)$ to denote its ideal class group, which is a finite abelian group. Cohen-Lenstra heuristic suggests that if we pick a random square-free D, the probability of p-Sylow subgroup of $Cl(-D)$ to be a fixed group G is inverse proportional to $\#\mathrm{Aut}(G)$. This heuristic was conjectured for odd prime p's and was known to fail for $p = 2$ because the genus theory can compute the number of 2-torsion elements which apparently does not match the conjecture. We computed the 2-Sylow subgroups of $Cl(-D)$ up to $D = 10^6$, using the theory of binary quadratic forms. Then, we sorted the class group according to the number of 2-torsions. The experimental data showed that if we fix the number of 2-torsion elements, the distribution of some 2-Sylow subgroups matches the prediction by Cohen-Lenstra heuristic. However, there are also some mysterious numbers which cannot be explained by Cohen-Lenstra heuristic. We plan to work on this after the RSI.

Katrina Evtimova

Representations of Rational Cherednik Algebras of Rank 1 and 2

Emanuel Stoica

This paper deals with the classification of finite dimensional irreducible representations of the rational Cherednik algebra $H_c(W)$ over a field k depending on some parameters $c \in k$ and an indecomposable reflection group W. We analyze the case when W is any group of rank 1 or a certain group of rank 2. In rank 1, namely $W = \mathbb{Z}/r\mathbb{Z}$, we complete the classification that was started by Latour, analyzing the case when *char* k divides r. In rank 2, we study the case when $W = S_2 \ltimes (\mathbb{Z}/r\mathbb{Z})^2$ and *char* $k = 0$.

Hyun-Sub Hwang

Permutations with a Special Property and their Extension to Abelian Groups

Joel Lewis

A permutation of a finite set is an ordered list containing each element of the set exactly once. Inspired by a question from the 1995 Russian Mathematical Olympiad, we call a permutation $a_1, a_2, \ldots, a_n$ of $1, 2, \ldots, n$ a "good permutation" if it has the property that $\{a_2 - a_1, a_3 - a_2, \ldots, a_n - a_{n-1}\}$ equals the set $\{1, 2, \ldots, n-1\}$ when taken modulo n. We prove that the number of good permutations of length n is divisible by $2n\phi(n)$, where $\phi(n)$ is the Euler totient function of n (i.e., the number of positive integers less than and relatively prime to n) and provide some bounds on the number of good permutations of length n for each n. We also extend the notion of a good permutation to any finite abelian group and prove the following simple condition for the existence of good permutations of an abelian group:

Theorem. *An abelian group G admits a good permutation if and only if, when G is written as a product of cyclic groups of prime-power order, exactly one of the factors is of even order.*

Eric Larson

Fusion categories of dimension qp^2

David Jordan

Let G be a group with square order. We consider the fusion ring R_G obtained by adjoining to $\mathbb{Z}G$ two additional elements $\mathbf{1}, \mathbf{2}$, with multiplication table

	g	$\mathbf{1}$	$\mathbf{2}$
h	gh	$\mathbf{1}$	$\mathbf{2}$
$\mathbf{1}$	$\mathbf{1}$	$\sqrt{\lvert G\rvert}\mathbf{2}$	$\sum_{g\in G}$
$\mathbf{2}$	$\mathbf{2}$	$\sum_{g\in G}$	$\sqrt{\lvert G\rvert}\mathbf{1}$

R_G is a $\mathbb{Z}/3\mathbb{Z}$-graded fusion ring with trivial component $\mathbb{Z}G$. We classify the categorifications of the fusion rings R_G when $|G|$ is not divisible by 3, and achieve an explicit construction when $G = \mathbb{F}_p^n$ for $p > 3$. Our results generalize those of Tambara and Yamagami and provide a key step toward the classification of fusion categories of Frobenius–Perron dimension qp^2.

Paul Lee

Modelling Salt Effect on DNA Conformations

Xia (Carol) Hua

We represent circular DNA in the simple cubic lattice in order to model the effect of salt concentration on its topological and geometric properties. Chemically, a high salt concentration weakens the DNA's electrostatic bonds allowing for DNA to adopt a highly compact state, with the bending and writhing energy dominating over other energies. A short-range attractive force between non-adjacent monomers, DNA's nucleotides, and a screened Coulomb potential to gauge the forces between the monomers with a salt variable are in the potential energy model. The radius of gyration, contact number, and writhe were considered to test for overall compactness and complexity of the conformation of DNA. All results verified that an increased salt concentration increases compactness, thus contributing to higher knotting probabilities.

Patricia Li

On the number of permutations with a given number of cycles and left-to-right maxima

Matjaž Konvalinka

Foata's transformation gives a bijection between the permutations with p cycles and permutations with p left-to-right maxima. R. Cori recently found a bijection between the permutations with p cycles and q left-to-right maxima and permutations with q cycles and p left-to-right maxima. We implement his bijection in Java, and study its properties. Our results are about the fixed points of this bijection. Our main theorem states that the number of fixed points of the bijection for n is equal to the number of involutions of length n.

Zane Li

On the intersection of quadric and cubic surfaces

Ryan Reich

Projective space, $\mathbb{RP}^3$, is obtained from ordinary space by including as a "plane at infinite distance" the points on the horizon of each flat surface; thus, every direction (a line) terminates at a unique point on the plane at infinity. Complex projective space, $\mathbb{CP}^3$, is the space in which every *complex* "line" (a copy of the plane of complex numbers) in $\mathbb{C}^3$ terminates at a complex "plane" at infinity. One defines surfaces in $\mathbb{CP}^3$ using polynomial equations such as $xy = zw$ with *four* variables, in which all terms of the same degree; for a polynomial of degree 2, the surface is called "quadric" or "quadratic", and for one of degree 3, it is "cubic". The intersection curve between two quadric surfaces in $\mathbb{CP}^3$ is of fundamental importance in computer graphics and solid modeling. Its features have been studied extensively in [2] and [4]. These features include the "pencil" of quadric surfaces which surrounds the intersection curve, and the projection of the intersection curve onto the plane at infinity. In this paper, the corresponding properties are studied for the intersection curve between a quadric and a cubic surface, and results are obtained for a family of cubic surfaces, the analogue of the quadric pencil.

[2] R.T. Farouki, C.A. Neff, and M.A. O'Connor. Automatic parsing of degenerate quadric- surface intersections. ACM Transactions on Graphics 8 (1989), no. 3, 174-203.

[4] W. Wang, B. Joe, and R. Goldman. Computing quadric surface intersections based on an analysis of plane cubic curves. Graphical Models 64 (2003), 335-367.

Xiao Tian Liew

Predominant intersection vertices in spanning trees

Maxim Maydanskiy

Given a graph $G = (V, E)$ and vertices $a, b, e \in V$, we find out how a, b, e are connected in G using the spanning tree approach. For a spanning tree $t \subset G$, we find the earliest intersection vertex j between the unique paths from a to e and from b to e, and say that t "votes" for j. We seek to determine the number of votes a given vertex gets, and in particular to find the vertex with the most votes in G. We obtain some results for complete graphs and complete bipartite graphs. This problem can be generalized to directed weighted graphs, and has applications to drug delivery.

Young Wook Lyoo

On The Linear Extensions and Interval Extensions of Poset

Yulan Qing

Let $(P, \leq_P)$ be a partially ordered set(denoted *poset*). An extension of $(P, \leq_P)$ is a poset $(Q, \leq_Q)$ such that

1. $Q = P$, and
2. if $a \leq_P b$, then $a \leq_Q b$.

The underlying set of an extension of a poset does not change. An extension Q of P is said to be linear if Q is a linearly ordered set. An extension Q of P is said to be an *interval extension* if each element of Q is a closed interval on the real line, and $x = (a_1, b_1) \leq y = (a_2, b_2)$ iff $b_1 \leq a_2$. An intersection of two extensions is a poset such that $x \leq y$ if and only if $x \leq y$ in each extension. A set of extensions *realizes* a poset if and of if the intersection of extensions is isomorphic to the poset. We define S_m, n to be the set of posets that can be minimally realized by a set of m interval extensions and n linear extensions. In this research we study the containment of S_m, n with different values of m and n. We prove that $S_x, y \subseteq S_z, w$ if and only if $2x+y \leq 2z+w$.

Benjamin Mirabelli

Finding non-degenerate critical points of the superpotential associated to a smooth Fano plytope

Maxim Maydanskiy

Toric varieties are studied extensively in algebraic geometry and symplectic topology. It is known that such a variety is encoded in a polytope, with Fano varieties - a particularly nice class - corresponding to reflexive lattice polytopes. Information about symplectic topology of the variety can be retrieved from the superpotential - a Laurent polynomial associated to the corresponding polytope. Critical points of the superpotential, and non-degenerate critical points in particular, are of special importance. We prove that the superpotential of any polytope that is either the convex-hull product or the Cartesian product of two polytopes whose superpotentials only have non-degenerate critical points also only has non-degenerate critical points. From this we show that every critical point of every facet-symmetric smooth Fano polytopes superpotential is non-degenerate.

Dimitrios Papadimitriou

Factorization in terms of Cyclotomic Polynomials and Algorithms for their Coefficients

Tathagata Sengupta

In this study, we find two algorithmic methods to improve on an existing description of the coefficients of the cyclotomic polynomial $\Phi_n(x)$ when n is the product of two primes. We use Beiter's description which refers to the representability of n as $n = ap + bq$ and $n = ap + bq + 1$ as a criterion to find the coefficients of $\Phi_{pq}(x)$, where p, q are 2 primes. In our first method we use inequalities, while in the second devise an algorithm using congruences. In essence, with our second algorithm, we can read off the coefficients as soon as we have written out the $ap \pmod q$'s as $n \pmod q$'s for $0 \leq n < q$. This method, in particular, gives a very easy description for the coefficients when $p \equiv \pm 1 \pmod q$.

Furthermore, we investigate which polynomials can be written as a product of cyclotomic factors. In particular, we find a characterization of all polynomials of a given degree with odd coefficients, which can be written as such a product. These are also the only polynomials that have Mahler measure 1. We make iterated use of Graeffe's root powering method, check that we always have a fixed point for these, and use these fixed points to get a very precise characterization for these polynomials. This also describes an algorithm for finding all these polynomials explicitly.

Eliyahu Putterman

Determination of the Rate of Convergence of the Equi-Energy Sampler

Xia (Carol) Hua

The equi-energy sampler of Kou et. al. is a new sampling algorithm that aims to circumvent the problem that high energy barriers present to Markov chain Monte Carlo methods for sampling from complex Boltzmann distributions. To test this claim, we simulate the equi-energy sampler and the standard heat-bath algorithm on the Potts model and compare their correlation times. Our data suggest that the equi-energy sampler does not offer a significant performance improvement over the heat-bath algorithm in the case of the Potts model.

Maxim Rabinovich

On the Scaling Limit of a Generalized Divisible Sandpile Model

Emanuel Stoica

Internal DLA is a way to generate a region from a discrete random walk. Levine and Peres have shown that these regions converge, with probability one, to the same limit as those produced by a deterministic process known as the divisible sandpile and that the limiting shape is given by the solution to the obstacle problem. In this paper, we consider a more general version of the divisible sandpile and prove that it converges to the solution to a different obstacle problem. This research is a first step toward determining what happens in the most general kind of divisible sandpile model. It is also likely that our methods can be extended to determine the scaling limit of certain generalized internal DLA models.

David Harry Richman

Counting diagonal matrices over finite fields

Tonghoon Suk

Matrix diagonalization is a useful technique in studying linear transformations, discrete dynamical systems, and systems of differential equations. As a result, diagonalizable matrices have important applications in many areas of engineering and applied sciences. In this paper, we study the set of diagonalizable matrices over a finite field. We use the group theoretic concepts of orbits and stabilizers to derive a formula for the number of $n \times n$ matrices over a finite field $\mathbb{F}_q$. We then investigate asymptotically the behavior of this function for large values of n and q.

Adam Sealfron

Hypergraph Property Testers: The Role of Adaptivity

Victor Chen

A *property tester* is a probabilistic algorithm that makes queries into its input (instead of reading the input in its entirety). Such a tester determines with high probability whether its input exhibits a certain property or its fractional difference with any input having such property is at least ϵ, for some $\epsilon > 0$. A tester may be *adaptive* if its queries may depend on the answers to its previous queries and *nonadaptive* otherwise.

In this work we investigate the role of adaptivity in property testing of hypergraphs. In particular we focus our attention on 3-uniform hypergraphs. As a first step, we observe that any adaptive hypergraph tester making q queries may be converted into an nonadaptive tester making $O(q^3)$ queries, generalizing an argument of Goldreich and Trevisan (2000). We seek properties of hypergraphs such that the separation between adaptive and nonadaptive query complexity approaches cubic. Towards this goal, we present an adaptive algorithm making $\tilde{O}(\epsilon^{-1})$ queries that determines if a given hypergraph consists of isolated hypercliques. We provide a heuristic analysis for this algorithm and furthermore conjecture that any nonadaptive algorithm testing for this property must make at least $\Omega(\epsilon^{-9/4})$ queries. Our technique is based on the recent work of Goldreich and Ron (2008) exploring the adaptivity gap in graph property testing.

Jean Shiao

A study on finite subgroups of multiplicative non-zero Quaternions and $SO(3)$ groups

Tonghoon Suk

The properties of the finite subgroups of multiplicative non-zero quaternions and $SO(3)$ are studied. Elements in $SO(3)$ can be regarded as rotations in $\mathbb{R}^3$ and vice versa. So, we can introduce a group actions of $SO(3)$ on S^2. Studying poles – fixed points on S^2 – enables us to classify finite subgroups in $SO(3)$. As a result we have a specific finite subgroups which can be expressed as Euclidean solids – tetrahedron, octahedron and icosahedron. Also, we find finite subgroups in quaternion group using surjective homomorphism from quaternion groups to $SO(3)$.

Sang-Hun Song

Imaginary quadratic fields with class groups exponent a power of 2

Liang Xiao

If $K = \mathbb{Q}(\sqrt{-D})$ is an imaginary quadratic number field, we use $Cl(-D)$ to denote its ideal class group, which is a finite abelian group. (Generalized) Gauss conjecture states that there are only finitely many D such that $\#Cl(-D) = n$ for a fixed n. We worked on a variant of Gauss number problem, which is to find all possible D such that $Cl(-D)$ has exponents 2^k for some fixed k, where the exponent of a group is the minimal number that kills all the element in the group. It is already proven that there are only finitely many of them. We used the correspondence between ideal class groups and equivalent classes of binary quadratic forms to turn the question into computation on binary quadratic forms, which is much easier to implement on computer. Also, we developed a fast algorithm to testing if the class group has exponents 2^k (faster than computing the class number). This should allow us to easily go beyond $D = 10^6$.

Galin Statev

Fermat-Euler Dynamics

Tathagata Sengupta

This paper studies the Fermat-Euler dynamics of traces of integer matrices modulo power of a prime integer p. We generalize some previous results about an analogue of the Fermat-Euler theorem for the trace function, and also provide counterexamples to the theorem for non-prime powers. We also provide a sharp upper bound for the period of the trace function that depends only on prime number p and the order of the matrix.

For every square integer matrix A, we prove that the congruence $\mathrm{tr}(A^n) \equiv \mathrm{tr}(A^{n-\phi(n)}) \pmod{n}$ holds if and only if $n = p^\alpha$ where p is a prime number and α is a nonnegative integer. Furthermore, if $n = p^\alpha$ and k is a positive integer coprime to p, then we show that the congruence $\mathrm{tr}(A^{kp^\alpha}) \equiv \mathrm{tr}(A^{kp^{\alpha-1}}) \pmod{p^\alpha}$ holds. We also prove that $\mathrm{tr}(A^{p^f}) = \mathrm{tr}(A)$ in $\mathbb{F}_q$, where $q = p^f$. We also show that the period T of the sequence $\mathrm{tr}(A^k) (\mathrm{mod}\ \mathrm{p})$ is always less than or equal to $p^n - 1$, when the determinant of the matrix is non-zero mod p. Furthermore, for every prime p and every integer n, we find an integer matrix A of order n such that this sequence has period exactly $p^n - 1$. Thus we show that the above upper bound is sharp for every p and n. We also talk about realizable sequences, which play a very important role in these proofs. In particular, we prove the realizability of the sequence $\{\sigma(n)\}$, which are the sums of divisors of n, for n a natural number.

Daniel Vitek

Hamiltonicity of Configuration Spaces

Yulan Qing

Let G be a graph. In this research we study the 1-skeleton of configuration space of a graph G. Given two distinct tokens x_1, x_2 on a simple graph G, let $\delta(x_1)$ be closed cell x_1 is on. A *configuration space* is a cube complex that give a 1-1 continous map of all possible locations of x_1 and x_2 such that $\delta(x_1) \cap \delta(x_2) = \phi$. Let $C(G)$ denote the 1-skeleton of the such a configuration space of G. We give conditions that completely determine the connectivity of $C(G)$. We also study the Hamiltonicity of $C(G)$. If there is a cycle visiting all vertices exactly once, we say that the cycle is a *Hamiltonian cycle.* In general, a useful condition both necessary and sufficient for a graph to be Hamiltonian is not known. We give complete description of Hamiltonicity of complete graphs and acyclic graphs, We also explore some enumerative aspect of this topic.

Brent Woodhouse

Characters of Induced Representations in Coxeter Groups

Matjaž Konvalinka

A parabolic subgroup of a Coxeter group (W, S) is a subgroup generated by a subset of S. There is a well-known formula for calculating characters induced from trivial representations of parabolic subgroups of the symmetric group. We present a bijective proof of this result, and then present and prove formulas for the characters induced from the trivial representation of parabolic subgroups of groups in other infinite classes of Coxeter groups: B_n, D_n, and $I_2(m)$. In addition, we use computer algorithms to find tables of the same characters for exceptional finite irreducible Coxeter groups. Finally, we introduce the Merris-Watkins formula to motivate future research in this area.

PETER ZHANG

The integrability of $\int e^{x^2} dx$ in fields of characteristic p

RYAN REICH

In the early 19th century, Liouville made a study of the symbolic properties of differentiation and showed that $\int e^{x^2} dx$ is not an "elementary function": one obtainable by combining the exponential, polynomial, and trigonometric functions and their inverses using the four arithmetic operations and composition. In this paper, we investigate the issues involved in adapting his proof when differentiation is performed in a context where $1+1+\cdots$ (p times) $= 0$, for some prime number p. This requires the notion of a "differential field": a set with the four arithmetic operations and a "symbolic derivative" with some of the properties of ordinary differentiation; it is said to have "characteristic p" if the above equation holds. We show how to define the elementary "functions" in such fields and how they resemble and differ from the usual ones. We then adapt Liouville's proof and show that it still holds.