Program for Research in Mathematics, Engineering and Science

Stata Center, MIT. Photo: Slava Gerovitch

Fourteenth Annual Fall-Term PRIMES Conference

October 12-13, 2024

Fourteenth Annual Fall-Term PRIMES Conference, October 12-13, 2024

Saturday, October 12

Mathematics

Room 4-370, MIT

9:00 am: Welcoming Remarks

- Prof. Michel Goemans, Head of the MIT Mathematics Department
- Prof. Pavel Etingof, PRIMES Chief Research Advisor
- Dr. Slava Gerovitch, PRIMES Program Director

9:15-10:20 am: Session 1

- Jerry Zhang, *Quantum-Sound Property Tests for Linear and Affine Linear Functions* (mentor David Cui)
- Rohan Dhillon, *Patterns in the Stable SL(N) Homologies of Torus Knots* (mentor Dr. Joshua Wang, Princeton/IAS)
- Qiao (Tiger) Zhang, *On Variants of Graph Neural Networks with Stronger Expressive Power* (mentor Dr. Ziang Chen)
- Lexing (Alex) Huang, Agniv Sarkar, and Kartik Ramachandrula, *Neural Network Pipeline for Systems Biology: Solving the Notch Signaling Pathway* (mentor Prof. Lu Lu, Yale)

10:40-11:40 am: Session 2

- Eric Wang, *Conformal Type Problem via Discrete Analysis* (mentor Prof. Sergiy Merenkov, CCNY)
- Henry Jiang, *Electric Potential of a Torus Knot Along the Axis* (mentor Dr. Max Lipton)
- Skyler Mao, *Investigating the Collapse and Convergence of Particle-Wave Statistics in Pilot-Wave Hydrodynamics* (mentor David Darrow)
- Michael Lu, *Special Bounds for Theta Sums* (mentor Dr. Tariq Osman, Brandeis University)

12:00-1:00 pm: Session 3

- Owen Zhang, *Tetrahedron-Intersecting Families of 3-Uniform Hypergraphs* (mentor Nitya Mani)
- Katelyn Gan, *The Ungar Games Played on Various Lattices* (mentor Yunseo Choi, Harvard University)
- Aarush Vailaya, *Chromatic Symmetric Function of Cycle Chains* (mentor Dr. Foster Tom)
- Andrew Brahms, Alan Duan, and Jacob Greene, *Saturation of 0-1 Matrices* (mentor Dr. Jesse Geneson, SJSU)

2:00-3:05 pm: Session 4 (in parallel with Computer Science sessions in the afternoon)

- Jason Mao, *Differentiating Point Cloud Distributions Using Persistence Homology* (mentor Jonathan Rodriguez Figueroa)
- Hannah Fox, *Monochromatic Components with Many Edges in Random Graphs* (mentor Dr. Sammy Luo)
- Neil Krishnan, *On the Connectivity of Friends-and-Strangers Graphs* (mentor Rupert Li)
- Christopher Bao, Joshua Wang and William Zhao, *Minimum and Approximate Minimum k-Cuts in Hypergraphs* (mentor Yuchong Pan)

3:25-4:25 pm: Session 5

- Anay Aggarwal, Ekan Kaur, and Susie Lu, *Bringing Reproducibility to Cancer Research: A Comparative Analysis of Machine Learning Models for Thyroid Cancer Recurrence Prediction* (mentor Dr. Marly Gotti, Apple)
- Arjun Agarwal, Rachel Chen, and Rohan Garg, *Automorphically Equivalent Elements in Finite Abelian Groups* (mentors Prof. Jim Coykendall and Jared Kettinger, Clemson University)
- Eddy Li, Advaith Mopuri, and Charles Zhang, *A Goldbach Theorem for Group Semidomains* (mentor Dr. Harold Polo, University of California, Irvine)

4:45-5:45 pm: Session 6

- Jonathan Du, Bryan Li, and Nick Zhang, *On the Internal Sum of Positive Monoids* (mentor Dr. Felix Gotti)

- Evin Liang, Alexander Wang, and Lerchen Zhong, *Maximal Common Divisors of Puiseux Monoids* (mentor Dr. Felix Gotti)
- Jiya Dani, Leo Hong, and Shimon Schlessinger, *Finitary Power Monoids: Atomicity, Divisibility, and Beyond* (mentors Dr. Felix Gotti and Benjamin Li)

Computer Science and Computational Biology (in parallel with Math sessions in the afternoon)

Room 4-270, MIT

1:30 pm: Welcoming Remarks

- Dr. Slava Gerovitch, PRIMES Program Director
- Prof. Srini Devadas, PRIMES Computer Science Section Coordinator

1:35-2:45 pm: Session 7

- Eric Chen and Rohith Raghavan, *Comparing Methods of Opportunistic Risk-Limiting Audits* (mentor Mayuri Sridhar)
- Adam Ge and Aadya Goel, *Unlearning Mechanisms in Graph Models for Document Classification* (mentor Mayuri Sridhar)
- Coleman DuPlessie, *Sparse Autoencoders for Interpretability in Reinforcement Learning Models* (mentor Andrew Gritsevskiy, University of Wisconsin–Madison)
- Sophia Yan, *A Multi-Omic Approach to Uncover Enhancer-Gene Interactions in the Human Brain* (mentor Dr. Nicole Rockweiler, Broad Institute)

3:05-4:05 pm: Session 8

- Michael Han and Ashley Yu, *Introducing Multi-Stage Multiplicative-Weights Update and An Empirical Evaluation of Convergence to Correlated Equilibria* (mentor Noah Golowich)
- Eric Archerman and Celine Zhang, *Using Ideas from Hardware to Accelerate Zero-Knowledge Virtual Machines* (mentor Simon Langowski)
- Adrita Samanta and Govind Velamoor, *Adaptive Timeout Strategies for Microservice Applications* (mentors Prof. Raja Sambasivan, Max Liu, and Zhaoqi Zhang, Tufts University)

4:25-5:15 pm: Session 9

- Shreyas Ekanathan, *Adaptive Order Radau Methods* (mentor Dr. Christopher Rackauckas)
- Albert Lu, *Alcatraz: Secure Remote Computation via Sequestered Encryption in Hardware Security Module* (mentors Jules Drean and Sacha Servan-Schreiber)
- Maya Kalai and Ella Kim, *Inner-Product Predicate Encryption from Weaker Assumptions* (mentor Sacha Servan-Schreiber)

5:35-6:35 pm: Session 10

- Raj Saha, *Figurative Language as a Mobilizer to Act: A Multi-method Approach* (mentors Prof. Ann Kronrod, UMass Lowell, and Prof. Ivan Gordeliy, EDHEC Business School)
- Stephanie Wan, *Transparent Authorship Verification with Machine Learning Models* (mentor Dr. Gil Alterovitz, Harvard Medical School)
- Siddharth Nirgudkar, *Contextualized Transfer Learning: Transforming Heterogeneity into Predictive Power with Generative Latent Structures in Resource-Limited Settings* (mentor Dr. Ben Lengerich)
- Rajarshi Mandal, *Epigenetic Clocks and Aging Biomarkers: A Multi-Omics Exploration of DNA Methylation, SURF1 Mutation, and Regenerative Therapies* (mentor Dr. Gil Alterovitz, Harvard Medical School)

Sunday, October 13

Mathematics

Room 4-370, MIT

9:00-9:45 am: Session 11

- Marina Lin, *CAACS: A Carbon-Aware Ant Colony System* (mentor Prof. Laura Schaposnik, University of Illinois at Chicago)
- Weian (Andrew) Xie, *Worst-Case Error Bounds on Online Learning of Smooth Functions* (mentor Dr. Jesse Geneson, SJSU)
- Aidan Gao, *Spatial Clustering and Classification with Graph Neural Networks* (mentor Junhong Lin)

10:05-11:05 am: Session 12

- Rohan Das, *New Multiplicative Structures on Frobenius Algebras* (mentors Prof. Julia Plavnik, Indiana University Bloomington, and Dr. Pablo Ocal, UCLA)
- Enmei (Emma) Yang, *Examples of the Reflective Algebra for Various Hopf Algebras* (mentors Prof. Julia Plavnik and Dr. Héctor Peña Pollastri, Indiana University Bloomington)
- Sargam Mondal, *Exact Factorizations of G-crossed Braided Fusion Categories* (mentors Prof. Julia Plavnik, Indiana University Bloomington, and Prof. Monique Müller, Universidade Federal de São João del-Rei, Brazil)
- Jiwu Jang, *Vertex Functions of Type D Nakajima Quiver Varieties* (mentor Dr. Hunter Dinkins)

11:25 am - 12:30 pm: Session 13

- Hwisoo (Harry) Kim, *Subregular Affine Kazhdan-Lusztig Polynomials in Type D* (mentor Kenta Suzuki)
- Sophia Liao, *Transitivity of Bender–Knuth Moves on Standard and Semistandard Young Tableaux* (Prof. Leonid Rybnikov, Université de Montréal)
- Eric Yee, *Hilbert Series of Quasi-Invariant Polynomials in Characteristics* $p \leq n$ (mentor Frank Wang)
- Sidarth Erat and Shihan Kanungo, *Mixed Tensor Products for Lie Superalgebras* (mentor Arun Kannan)

Saturday, October 12

Session 1

Jerry Zhang

Quantum-Sound Property Tests for Linear and Affine Linear Functions

Mentor: David Cui

Ito and Vidick (FOCS 2012) show that the linearity test of Blum, Luby, and Rubinfeld (STOC 1990) certifies the presence of a linear function over $\mathbb{F}_2$ even when the provers are allowed to share entanglement, a property we call *quantum-soundness*. The existence of quantum-sound protocols allows us to quantize existing classical interactive protocols and prove containment between quantum multi-prover interactive proof systems. In the case of Ito and Vidick, this was used to show that MIP $\subseteq$ MIP* and in the work of Ji et al., it was used to show that MIP* = RE. In this talk, we generalize the property testing result of Ito and Vidick and show that the linearity test over $\mathbb{F}_p$ is also quantum-sound. Additionally, we show that even without the consistency test, the presence of linear functions can be certified.

Rohan Dhillon

Patterns in the Stable SL(N) Homologies of Torus Knots

Mentor: Dr. Joshua Wang, Princeton/IAS

Gorsky, Oblomkov, and Rasmussen conjectured that the stable Khovanov homology of $T(n, \infty)$ — which is the limit of $Kh(T(n,m))$ as $m \to \infty$ — is isomorphic to the homology of a certain Koszul complex W_n. In this talk, we conjecture that the q-homogeneous summands of the homology of W_n satisfy a recursive relationship, reminiscent of the Principle of Inclusion-Exclusion, that would imply the homology of W_n is determined by finitely many bidegrees. We present theoretical and computational evidence for this relationship, and discuss an analogous formulation of the conjecture for $SL(N)$ and Lee homologies. We present evidence for our conjectural limits on the orders of torsion in regular and deformed $SL(N)$ homology.

Qiao (Tiger) Zhang

On Variants of Graph Neural Networks with Stronger Expressive Power

Mentor: Dr. Ziang Chen

Graph neural networks (GNNs) have been widely used in graph-related contexts. It is known that the separation power of GNNs is equivalent to that of the Weisfeiler-Lehman (WL) test; hence, GNNs are imperfect at identifying all non-isomorphic graphs, which severely limits their approximation power. We prove that two modified WL tests using information from neighbors with distances up to k can identify all non-isomorphic graphs with all cycles of size $O(k)$ with an additional assumption. We also give examples that show the effectiveness of these modified WL tests.

Lexing (Alex) Huang, Agniv Sarkar, and Kartik Ramachandrula

Neural Network Pipeline for Systems Biology: Solving the Notch Signaling Pathway

Mentor: Prof. Lu Lu, Yale

The process of neurogenesis in the mammalian brain is controlled by the Notch signaling pathway, which can be modeled with a system of ordinary differential equations relating the concentrations of species. However, this system contains a relatively large number of state variables (species) and parameters; as such, it is computationally costly to model the system, even with current techniques. In this talk, we describe a pipeline to elucidate properties of the system as well as forecast species. First, we extensively discuss the use of identifiability analysis in systems biology problems to offer guidance in modelling. We show the utilization Systems Biology Informed Neural Networks (SBINNs) architecture to extract values of ODE parameters as well as model the dynamics of the chemical species. In addition, we describe the implementation of additions to SBINNs that enhance the learning of the model. Our results should provide accurate predictions of the biochemical dynamics in the Notch signaling pathway and help neuroscientists in the field better understand how neurons form. We also describe how we can further this technique and evaluate other modern architectures such as PINNformers and KANs to enhance predictions.

Session 2

Eric Wang

Conformal Type Problem via Discrete Analysis

Mentor: Prof. Sergiy Merenkov, CCNY

The conformal type (parabolic or hyperbolic) of a covering surface of the Riemann sphere with n singular values is consistent with the type (reccurent or transient respectively) of the corresponding extended Speiser graph. In our PRIMES project we look at the extended Speiser graphs of some surfaces whose conformal type is known via analytic methods. We use various discrete techniques such as shorting, cutting, discrete extremal length, and flows to verify that the type of the extended Speiser graph does indeed match the type of the surface.

Henry Jiang

Electric Potential of a Torus Knot Along the Axis

Mentor: Dr. Max Lipton

We focus on the torus knot, specifically a parametrization that embeds it on a torus centered at the origin with rotational symmetry about the z-axis. Particularly, in this project, we analyze the electric field along the z-axis to take advantage of symmetry. We show that the electric field is zero only at the origin, and investigate the extreme points of the electric field and electric potential. We find our results by employing both elementary methods and methods in complex analysis for contour integration.

Skyler Mao

Investigating the Collapse and Convergence of Particle-Wave Statistics in Pilot-Wave Hydrodynamics

Mentor: David Darrow

The "walking droplet" system of Couder and Fort has inspired a number of macroscopic analogues to quantum mechanics, as well as new approaches to classical pilot-wave theory. In particular, authors have observed a correspondence between the probability density function (PDF) of the walking droplet and the mean wave field (MWF) of the underlying bath, analogizing the relationship between quantum probabilities and the wavefunction. Although this correspondence has been proved to hold in simple system geometries with particular droplet models, the principle is believed to hold more generally. In this direction, we derive the same PDF-MWF correspondence in the general case of droplet (and droplet-like) models, and for arbitrary system geometry. We demonstrate these results in 1D numerical examples, and looking forward, we discuss how they might apply to hydrodynamic analogues of quantum measurement.

Michael Lu

Special Bounds for Theta Sums

Mentor: Dr. Tariq Osman, Brandeis University

In this talk we define special functions on the upper half of the complex plane known as Siegel theta series and show some pictures of the function evaluated on special curves near the boundary of the upper half plane. We then describe our recent results concerning bounds for such functions near the boundary of the upper half plane.

Session 3

Owen Zhang

Tetrahedron-Intersecting Families of 3-Uniform Hypergraphs

Mentor: Nitya Mani

Intersection problems are of significant interest in extremal combinatorics and theoretical computer science. In this presentation, we begin by discussing the maximal size of triangle-intersecting families of graphs. Then, we explore generalizations, including a conjecture on the maximal size of K_t-intersecting families of graphs and an analogous conjecture for 3-uniform hypergraphs. We conclude with our findings on the maximal size of tetrahedron-intersecting families of 3-uniform hypergraphs.

Katelyn Gan

The Ungar Games Played on Various Lattices

Mentor: Yunseo Choi, Harvard University

In 2023, Defant and Li introduced the Ungar move, which sends an element v of a finite meet-semilattice L to the meet of some subset of the elements covered by v. More recently, in 2024, Defant, Kravitz, and Williams introduced the Ungar game on L, in which two players Atniss and Eeta take turns making Ungar moves starting from an element of L until the player that cannot make a nontrivial Ungar move loses. We settle two conjectures by Defant, Kravitz, and Williams on the Ungar games on the Young-Fibonacci lattice and the lattices of the order ideals of shifted staircases. In addition, we provide a partial characterization of Atniss wins in the lattice of the weak order on B_n, a full characterization of the Eeta wins of the lattice of order ideals of $\mathbb{N}^3$, and a partial characterization of the Eeta wins in the lattice of order ideals of $\mathbb{N}^k$.

Aarush Vailaya

Chromatic Symmetric Function of Cycle Chains

Mentor: Dr. Foster Tom

We provide an e-positive formula for the chromatic symmetric function for adjacent cycle chains, which are graphs of cycles connected at adjacent vertices. We extend these results to graphs formed by connecting a sequence of cycles and cliques at adjacent vertices, as well as other more general graphs.

Andrew Brahms, Alan Duan, and Jacob Greene

Saturation of 0-1 Matrices

Mentor: Dr. Jesse Geneson, SJSU

A 0-1 matrix M *contains* a 0-1 matrix P if M has a submatrix P' which can be changed to P by changing some of the 1 entries to 0. Matrix M is *P-saturated* if M does not contain P, but any matrix M' derived from M by changing a 0 entry to a 1 does contain P. The *saturation function* $\mathrm{sat}(n, P)$ is defined as the minimum number of 1s of an $n \times n$ P-saturated 0-1 matrix. Fulek and Keszegh showed that each pattern P has $\mathrm{sat}(n, P) = O(1)$ or $\mathrm{sat}(n, P) = \Theta(n)$. This leads to the natural problem of classifying patterns according to linear and bounded saturation functions. Some progress has been made on this problem: multiple infinite families of matrices both with bounded saturation function and with linear saturation function have been identified. We resolve all patterns with fewer than 5 ones, as well as several others, including multiple new infinite families. We also consider the effects of certain matrix operations, including the Kronecker product and insertion of empty rows and columns. Additionally, we consider the simpler case of fixing one dimension, answering several past questions here.

Session 4

Jason Mao

Differentiating Point Cloud Distributions Using Persistence Homology

Mentor:Jonathan Rodriguez Figueroa

The field of topological data analysis aims to interpret datasets by their topological structure. In particular, representative tools such as persistent homology and persistence landscapes are used to condense the information provided by a shape or a point set into more condensed forms that highlight the properties of the data. Historically, these have been used to analyze global properties of a shape, e.g. number of components or number of holes, but recent work has shown its potential in capturing local geometry. In this talk, we apply these tools and related results to evaluate the similarities and differences in the shapes of varying point clouds. More precisely, we use persistence diagrams to differentiate between (possibly noisy) point sets and determine whether or not they were produced from similar distributions.

Hannah Fox

Monochromatic Components with Many Edges in Random Graphs

Mentor: Dr. Sammy Luo

In an r-coloring of the complete graph on n vertices, how many edges are there in the largest monochromatic component? A construction of Gyárfás shows that for infinitely many values of r, there exist colorings where all monochromatic components have at most $\left(\frac{1}{r^2-r} + o(1)\right)\binom{n}{2}$ edges. Conlon, Luo, and Tyomkyn conjectured that components with at least this many edges are attainable for all $r \geq 3$. Subsets of these authors proved this conjecture for $r = 3$ and $r = 4$, along with a lower bound of $\frac{1}{r^2-r+\frac{5}{4}}\binom{n}{2}$ for all $r \geq 2$ and n.

In this talk, we look at extensions of this problem where the graph being r-colored is a sparse random graph or a graph of high minimum degree. By extending several intermediate technical results from previous work in the complete graph setting, we prove analogues in both the sparse random setting and the high minimum degree setting of the bounds for $r = 3$ and general r.

Neil Krishnan

On the Connectivity of Friends-and-Strangers Graphs

Mentor: Rupert Li

Friends-and-strangers graphs, coined by Defant and Kravitz, are denoted by $\mathsf{FS}(X, Y)$ where X and Y are both graphs on n vertices. Their vertex set consists of all bijections between $V(X)$ and $V(Y)$, and two bijections σ and σ' are adjacent if $\sigma' = \sigma \circ (i', j')$ for $i', j' \in V(X)$ such that $(i', j') \in E(X)$ and $(\sigma(i'), \sigma(j')) \in E(Y)$. Previous papers have studied when $\mathsf{FS}(X, Y)$ is connected. In this talk, we consider the question of when $\mathsf{FS}(X, Y)$ is k-connected where a graph is k-connected if it remains connected after removing any $k-1$ vertices. We first consider $\mathsf{FS}(X, Y)$ when Y is a complete graph or star graph. We find tight bounds on their connectivity, proving both graphs are maximally connected, i.e., their connectivity equals their minimum degree. We further consider the size of the connected components of $\mathsf{FS}(X, \mathsf{Star}_n)$ where X is connected. We modify Bangachev's original argument for sufficient bounds on the minimum degree to show that slightly modified conditions are sufficient for $\mathsf{FS}(X, Y)$ to be k-connected. Finally, we consider the case when X and Y are independently Erdős–Rényi random graphs on n vertices and edge probability p, modifying Alon, Defant, and Kravitz's work to show that $p = n^{-1/2+o(1)}$ is a threshold probability above which $\mathsf{FS}(X, Y)$ is k-connected with high probability. This is asymptotically tight as Alon, Defant, and Kravitz also showed that below an asymptotically similar threshold $p' = n^{-1/2+o(1)}$, the graph $\mathsf{FS}(X, Y)$ is disconnected with high probability.

Christopher Bao, Joshua Wang and William Zhao

Minimum and Approximate Minimum k-Cuts in Hypergraphs

Mentor: Yuchong Pan

The minimum cut problem and its generalizations are important to combinatorial optimization and have numerous applications in network reliability, circuit design, and clustering. Our work considers the minimum k-way cut problem in hypergraphs, which asks for a k-way partition of the vertex set that minimizes the number of crossing hyperedges. We begin by extending the work of Kogan and Krauthgamer (2014), using the random contraction technique introduced by Karger and Stein (1995), to bound the number of approximate minimum k-way cuts in low-rank hypergraphs. Next, we consider the running time of the branching contraction algorithm of Fox et al. (2019) as applied to the minimum k-way cut problem in unweighted hypergraphs. We obtain improvements by utilizing bounds on the number of small hyperedges as constraints in a linear program. Finally, we generalize the near-linear time $(2+\varepsilon)$-approximation algorithm of Quanrud (2019) for the graph k-way cut problem, achieving an approximation ratio of $r(1+\varepsilon)$ for hypergraphs of rank r. As a component, we provide an algorithm for finding a minimum hypertree with improved runtime compared to the prior result of Baïou and Barahona (2023).

Session 5

Anay Aggarwal, Ekan Kaur, and Susie Lu

Bringing Reproducibility to Cancer Research: A Comparative Analysis of Machine Learning Models for Thyroid Cancer Recurrence Prediction

Mentor: Dr. Marly Gotti, Apple

In this presentation, we will explore the effectiveness of various machine learning algorithms—such as Artificial Neural Networks, Logistic Regression, and Random Forests—in predicting recurrence of Differentiated Thyroid Cancer (DTC), using a robust dataset from the UCI Machine Learning Repository. We identify the best algorithms for predicting DTC recurrence based on key performance metrics and Bayesian model comparison. We further improve algorithm performance using two feature selection techniques, aiming to advance personalized treatment strategies and improve outcomes in thyroid cancer care.

Arjun Agarwal, Rachel Chen, and Rohan Garg

Automorphically Equivalent Elements in Finite Abelian Groups

Mentors: Prof. Jim Coykendall and Jared Kettinger, Clemson University

Given a finite abelian group G and elements $x, y \in G$, we prove that there exists $\phi \in \mathrm{Aut}(G)$ such that $\phi(x) = y$ if and only if $G/\langle x\rangle \cong G/\langle y\rangle$. This result leads to the development of the two fastest known algorithms to determine if two elements of a finite abelian group are automorphic images of one another. The second algorithm also computes $G/\langle x\rangle$ in near-linear time for groups with exponent less than 10^{20}. We finish with an algorithm that computes the automorphism orbits of finite abelian groups with the fastest known time complexity.

Eddy Li, Advaith Mopuri, and Charles Zhang

A Goldbach Theorem for Group Semidomains

Mentor: Dr. Harold Polo, University of California, Irvine

A semidomain is an integral domain that does not require additive inverses. Given a semidomain S and a torsion-free abelian group G, we let $S[G]$ denote the semidomain consisting of all polynomial expressions with coefficients in S and exponents in G. Similarly, $S[[G]]$ is the semidomain consisting of all power series with coefficients in S and exponents in G. In our talk, we will discuss an analogue of the statement of the Goldbach's conjecture (resp., weak Goldbach's conjecture) in $S[G]$ (resp., in $S[[G]]$) when S and G satisfy certain conditions.

Session 6

Jonathan Du, Bryan Li, and Nick Zhang

On the Internal Sum of Positive Monoids

Mentor: Dr. Felix Gotti

A positive monoid is a submonoid of the nonnegative cone of a linearly ordered abelian group. In this talk, we discuss the behavior of some classical factorization properties under the internal (finite) sum of positive monoids. These properties include both the existence and uniqueness in the statement of the Fundamental Theorem of Arithmetic (considered separately), and both the bounded and the finite factorization properties, which are two relaxations of the unique factorization property.

Evin Liang, Alexander Wang, and Lerchen Zhong

Maximal Common Divisors in Puiseux Monoids

Mentor: Dr. Felix Gotti

Let M be a commutative monoid. An element $d \in M$ is called a maximal common divisor (MCD) of a nonempty subset S of M if d is a common divisor of S in M and the only common divisors in M of the set $\{\frac{s}{d} : s \in S\}$ are the units of M. In this talk, we discuss the existence of MCDs in rank-1 torsion-free commutative monoids, also known as Puiseux monoids. For the same monoids, we also establish some connections between the existence of MCDs and the ascending chain condition on principal ideals.

Jiya Dani, Leo Hong, and Shimon Schlessinger

Finitary Power Monoids: Atomicity, Divisibility, and Beyond

Mentors: Dr. Felix Gotti and Benjamin Li

A commutative monoid M is called a linearly orderable monoid if there exists a total order relation on M that is compatible with its operation. The finitary power monoid of a commutative monoid M is the monoid consisting of all nonempty finite subsets of M under the so-called sumset. In our talk, we plan to discuss whether certain atomic and divisibility properties ascend from linearly orderable monoids to their corresponding finitary power monoids.

Session 7

Eric Chen and Rohith Raghavan

Comparing Methods of Opportunistic Risk-Limiting Audits

Mentor: Mayuri Sridhar

Auditing elections is an important part of preserving faith in the electoral system and verifying the accuracy of the reported results of an election. Conventional election audits involve taking a set number or percentage of ballots and checking if the samples match the reported winner. However, these methods are unreliable for close races and excessive for races with a wide margin. Risk-limiting audits use statistical tests in order to assign a certain risk limit, the maximum probability that the results are incorrect, by sampling ballots one at a time until the risk limit is achieved. Our research focuses on opportunistic auditing, the ability to audit multiple races simultaneously, and attempts to determine what strategies are most effective for opportunistic auditing.

Adam Ge and Aadya Goel

Unlearning Mechanisms in Graph Models for Document Classification

Mentor: Mayuri Sridhar

We look at the idea of Machine Unlearning, the concept of making AI models "forget" about a particular section of data. In our research, we look at how the use of graphs helps in the convergence of two problems: unlearning a document classification label and unlearning the edges of a graph. We consider a graph containing nodes of words and documents, with edges indicating whether there is a relationship between a word and a document or between two documents. Current state of the art algorithms randomly reclassify the documents, but we argue that this decreases model utility. Instead, we use similarity scores to reclassify the documents into the next best class. Additionally, we introduce edge unlearning to refine this process. The current state of the art method for edge unlearning, called GNNDelete, decreases the predicted probability of an unlearned edge to very close to 0 and assumes there is no latent relationship, which we argue decreases model utility. Instead, we refine this assumption and forget only what we want to forget.

Coleman DuPlessie

Sparse Autoencoders for Interpretability in Reinforcement Learning Models

Mentor: Andrew Gritsevskiy, University of Wisconsin–Madison

Recent work has shown that sparse autoencoders (SAEs) are able to effectively discover human-interpretable features in language models, at scales ranging from toy models to state-of-the-art Large Language Models (LLMs). This work explores whether the use of SAEs can be generalized to other varieties of machine learning, specifically, reinforcement learning, and what, if any, modifications are necessary to adapt SAEs to this substantially different task. This research investigates both qualitative and quantitative measures of SAEs' ability to represent reinforcement learning models' activations as interpretable features, as well as the problem of dead neurons in SAEs.

Sophia Yan

A Multi-Omic Approach to Uncover Enhancer-Gene Interactions in the Human Brain

Mentor: Dr. Nicole Rockweiler, Broad Institute

Gene regulation is a complicated and essential process for maintaining and differentiating cell type-specific functions by controlling gene expression. Diseases such as cancer, neurological disorders, and autoimmunity conditions stem from the misregulation of gene expression, so we explored one of the primary regulators of gene expression, enhancers. Mutations in an enhancer can have significant consequences on the target gene's expression; however, enhancer-gene interactions have not been comprehensively studied on a cell-type level. Here, we propose a supervised approach to identify cell-type specific enhancer-gene pairs using commercial 3' single nucleus RNA-seq (snRNA-seq) kits to detect enhancer RNAs (eRNA), a canonical marker of active enhancers. To validate these putative active enhancers, we created a scoring system to measure how bidirectional the transcription is coming from the locus — a key property of eRNA. We found a significant score margin between that of our putative enhancer locations compared to nearby regions. We then utilized the unique power of our single cell sequencing data to explore the cell type-specificity of these putative active enhancers. Cell type-specific enhancers were three times as common as enhancers ubiquitously expressed across cell types. We also observed an enrichment of cell-type related pathways in the genes closest to the cell-type specific enhancers, providing further confidence for a specific subset of our enhancers. In addition, we mapped these active enhancers to their target genes by testing the association between putative eRNA expression and gene expression within the same topologically associating domain using publically available high-throughput chromosome conformation capture data. We found 4,523 putative enhancer-gene pairs across six cell types, 1,609 of which (35%) likely interact with each other in 3D space through DNA looping. The putative enhancer-gene pairs provide insight into the complex gene regulatory networks in the brain at a cell-type specific level, and our framework for identifying putative active enhancers and cell-type specific enhancer-gene pairs is widely applicable to analyzing snRNA-seq data in all tissues across species.

Session 8

Michael Han and Ashley Yu

Introducing Multi-Stage Multiplicative-Weights Update and An Empirical Evaluation of Convergence to Correlated Equilibria

Mentor: Noah Golowich

No-regret learning algorithms are an important component of advances in solving large-scale games. These algorithms are commonly used to solve games such as Diplomacy, an AI benchmark with a large action space where agents compete to dominate a map of Europe. We introduce Multi-Stage Multiplicative-Weights Update (MS-MWU), which shows an improvement upon existing external-regret minimizing algorithms such as MWU across all our experiments. We also perform an empirical evaluation of classic no-regret algorithms such as Multiplicative-Weights Update (MWU) and Optimistic Multiplicative-Weights Update (OMWU). Furthermore, we test swap regret minimization algorithms such as the no swap-regret algorithm of Blum & Mansour (2007) and the TreeSwap algorithm of Dagan et al (2024). We play these algorithms against each other and randomized adversaries on hundreds of subgames of Diplomacy along with Kuhn Poker and random games. Across all these games, our experiments show that MS-MWU converges significantly faster than MWU/OMWU. We experimentally show that swap regret and external regret remain very similar at all iterations. In other words, external regret minimization algorithms such as MWU outperform swap regret minimization algorithms such as BM in terms of rate of convergence and time complexity, even for very large time horizons.

Eric Archerman and Celine Zhang

Using Ideas from Hardware to Accelerate Zero-Knowledge Virtual Machines

Mentor: Simon Langowski

Succinct Non-interactive Arguments of Knowledge (SNARKs) are often used to verify the correct execution of computer programs, providing the verifier cryptographic trust in the prover's honesty. Traditional SNARKs often involve work with niche computational objects such as circuits and constraint systems, and this creates a large barrier to entry around SNARKs. Thus, there have been many attempts to democratize access to verifiable computation, most notably through Zero-Knowledge Virtual Machines (zkVMs), which can generate SNARKs for any computation trace in addition to running the computer program itself. These zkVMs enable new applications and increase usability with the drawback of being slower than traditional SNARKs in some settings. One major component of this slowdown is offline memory checking, a protocol which ensures that memory accesses are valid, usually with a computationally expensive permutation checking design. In computer hardware, various optimizations such as memory batching, multiple in-flight instructions, and caching can be used to speed up memory accesses. We apply these hardware ideas to optimize zkVMs.

Adrita Samanta and Govind Velamoor

Adaptive Timeout Strategies for Microservice Applications

Mentors: Prof. Raja Sambasivan, Max Liu, and Zhaoqi Zhang, Tufts University

Timeouts are critical in determining whether a request has succeeded or failed. Developers face several challenges when setting timeout values in distributed systems; the specific challenge we investigate being the systems' propensity to change, both over the short- and long-term. We propose three timeout-optimized policies targeting change over different time scales, assuming APIs that are both idempotent and atomic. We evaluate our approaches on a home-grown microservices testbed and on DeathStarBench's socialNetwork application. We will compare the median and tail latency requests when each approach is used in simulated environments with changing system performance.

Session 9

Shreyas Ekanathan

Adaptive Order Radau Methods

Mentor: Dr. Christopher Rackauckas

Solving stiff ordinary differential equations, which arise in chemical and physical systems, poses several challenges for typical numerical integrators. Most Runge-Kutta methods struggle to model the perturbations in the system accurately. However, Radau methods, a class of implicit Runge-Kutta methods, are a tried-and-tested way to handle these systems, as their L-stability and computational efficiency allow them to tackle such equations effectively. However, merely choosing the method is insufficient for optimal solutions, as accurately selecting the step size and order for a given method is crucial to streamlining solver efficiency. In this talk, we describe the general properties of Runge-Kutta methods before exploring the implementation of the Radau methods, as we translate mathematics into functional code using a complex transformation for optimization. We further describe adaptivity in the step size and order of the method we implemented, improving the convergence rate.

Albert Lu

Alcatraz: Secure Remote Computation via Sequestered Encryption in Hardware Security Module

Mentors: Jules Drean and Sacha Servan-Schreiber

This talk introduces "Alcatraz", a new architecture that enables secure remote computation with minimal trust in the hardware. In Alcatraz, sensitive data is always encrypted, except when it is inside a small, trusted circuit, which is composed of an Arithmetic Logic Unit (ALU) gated by a decryption and encryption engine. Alcatraz has two new features: (1) It implements the sequestered encryption method, along with a formal proof of its security at the wire-level, which is stronger than the register-transfer-level (RTL) security achieved by prior architectures. Wire-level verification has the benefit that it's much closer to the physical reality, i.e., the timing and level of signals on the wire, that can be observed by attackers. (2) Alcatraz extends existing instruction set architecture (ISA) while all its modifications to hardware are constrained within the microarchitecture level. Alcatraz can be applied to a wide range of secure computation tasks.

Maya Kalai and Ella Kim

Inner-Product Predicate Encryption from Weaker Assumptions

Mentor: Sacha Servan-Schreiber

Consider a scenario where Alice would like to encrypt a message such that Bob and Charlie can decrypt it if and only if they have keys satisfying a predicate P. This problem is solved by predicate encryption. We consider the setting where the predicate is represented by an inner product between a descriptive attribute vector $\mathbf{x}$ and the predicate vector $\mathbf{y}$. A secret key associated with the predicate can be used the recover the original message if and only if $\langle \mathbf{x}, \mathbf{y} \rangle = 0$, meaning that the predicate is satisfied. This specific predicate has applications in searching over encrypted data and access control based on attributes.

We construct an inner product predicate encryption scheme from the Learning Parity with Noise (LPN) problem, which is an assumption that was not known to be suitable for inner product predicate encryption before. We take inspiration from the work of Gorbunov et al. (Crypto '15) and the recent work of Servan-Schreiber (Asiacrypt '24) to realize our construction.

Session 10

Raj Saha

Figurative Language as a Mobilizer to Act: A Multi-method Approach

Mentors: Prof. Ann Kronrod, UMass Lowell, and Prof. Ivan Gordeliy, EDHEC Business School

We seek to show that figurative language has a causative effect on mobilization to action and empowerment to act. The approach toward this hypothesis is multimodal; on the one hand, we will computationally analyze peer advice datasets, and on the other hand, we will experimentally discern the influence of non-literal language. The first computational approach necessitates an accurate figurative language detection tool. Following the success of previous literature, we plan to leverage Large Language Models for this task; BERT has already been effective for preliminary tests on metaphor detection. Upon measuring additional variables such as imagery, concreteness, and empowerment to act, we will note the influence of a response's figurative language on the follow-up. Experimentally, we have two planned studies. The first hopes to show that figurative language in advice and directives increases compliance due to trust, and the second would indicate that these mental responses affect the participant's resultant language.

Stephanie Wan

Transparent Authorship Verification with Machine Learning Models

Mentor: Dr. Gil Alterovitz, Harvard Medical School

Authorship Verification (AV) is the task of determining if two given documents were written by the same person. AV is critical in addressing issues such as misinformation and impersonation, though it holds risks in violating privacy rights. This talk presents a publicly accessible website hosting transparent AV machine learning models. We aggregate and pre-process diverse datasets to train a lexical model based on embeddings and a stylometric model leveraging feature vectors. To enhance model transparency, we incorporate attention-based highlighting and output important features. The code and website for this talk are available at GitHub and Streamlit.

Siddharth Nirgudkar

Contextualized Transfer Learning: Transforming Heterogeneity into Predictive Power with Generative Latent Structures in Resource-Limited Settings

Mentor: Dr. Ben Lengerich

Predicting biomedical outcomes in resource-limited settings is challenging due to data scarcity and patient variability: retraining models locally lacks power, while borrowing models fails to capture context-specific causes. Current approaches frame these challenges as a tradeoff: transfer learning enhances generalization by leveraging data from other settings but sacrifices patient-specific adaptation, while contextualized learning adapts to specific contexts but struggles with limited data. We introduce Contextualized Transfer Learning (CTL) as a novel approach that reconciles these conflicting goals by modeling the joint distribution of predictors and outcomes, $p(x, y \mid c) \sim f(z(c))$, where $f(z(c))$ represents the latent structure shared across contexts. This enables information sharing across disparate outcomes, patients, and predictors, introducing a new dimension to transfer learning: generalizing across tasks while simultaneously tailoring predictions to individual patient contexts. Data scarcity and patient variability is an especially prominent problem in neurological diseases. We apply CTL to predicting Alzheimer's disease and show that CTL reduces mean square error by 22.9% compared to contextualized regression (CR) and boosts classification accuracy by 8%, outperforming population-based methods by 30%. We also show the interpretibility of CTL, which places heavy emphasis on a select few predictors which is critical for understanding biological insight. These results highlight CTL's potential as a powerful tool for precision diagnostics, particularly in resource-limited settings.

Rajarshi Mandal

Epigenetic Clocks and Aging Biomarkers: A Multi-Omics Exploration of DNA Methylation, SURF1 Mutation, and Regenerative Therapies

Mentor: Dr. Gil Alterovitz, Harvard Medical School

Aging is a complex biological process influenced by various epigenetic and environmental factors. To uncover the underlying biochemical mechanisms, we utilized a comprehensive longitudinal multi-omics dataset containing cytological, bioenergetic, DNA methylation, gene expression, secreted proteins, mitochondrial DNA copy number and mutations, cell-free DNA, telomere length, and whole-genome sequencing data. By examining this high temporal resolution dataset across the replicative lifespans of cultured primary human fibroblasts, we identified biomarkers with strong correlations to aging, providing insights into the biological processes driving cellular senescence. Furthermore, we explored the impact of the SURF1 mitochondrial mutation on the biomarkers to determine mitochondrial metabolism's role in aging. In addition, statistical tests on various experimental treatments (5-azacytidine, betahydroxybutyrate, dexamethasone, oligomycin, etc.) identified potential interventions for developing targeted therapies in dermatology with enhanced skin repair and regeneration. We also validated the aging process in cultured fibroblasts using epigenetic clocks, such as Hannum and PhenoAge. Finally, we compared machine learning models including Huber, SVR, CatBoost, and TabNet regressor for biological age prediction from DNA methylation measurements at various CpG sites.

SESSION 11

Marina Lin

CAACS: A Carbon-Aware Ant Colony System

Mentor: Prof. Laura Schaposnik, University of Illinois at Chicago

In an era where sustainability is becoming increasingly crucial, we introduce a novel *Carbon-Aware Ant Colony System (CAACS) Algorithm* that addresses the Generalized Traveling Salesman Problem (GTSP) while minimizing carbon emissions. This innovative approach harnesses the natural efficiency of ant colony pheromone trails to find optimal routes, balancing both environmental and economic objectives. By integrating sustainability into transportation models, the *CAACS Algorithm* is a powerful tool for real-world applications, including network design, delivery route planning, and commercial aircraft logistics. Our algorithm's unique bi-objective optimization represents a significant advancement in sustainable transportation solutions.

Weian (Andrew) Xie

Worst-Case Error Bounds on Online Learning of Smooth Functions

Mentor: Dr. Jesse Geneson, SJSU

We investigate worst-case error bounds for the online learning of real functions with certain smoothness constraints. Suppose that $\mathcal{F}_q$ is the class of all absolutely continuous functions $f : [0,1] \to \mathbb{R}$ such that $\|f'\|_q \leq 1$, and $\text{opt}_p(\mathcal{F}_q)$ is the best possible upper bound on the worst-case sum of the p^{th} powers of absolute prediction errors for any number of trials guaranteed by any learner. We show that for any $\delta, \epsilon \in (0,1)$, $\text{opt}_{1+\delta}(\mathcal{F}_{1+\epsilon}) = O(\min(\delta,\epsilon)^{-1})$. Combined with the previous results of Kimber and Long (1995) and Geneson and Zhou (2023), we achieve a complete characterization of the values of $p, q \geq 1$ that result in $\text{opt}_p(\mathcal{F}_q)$ being finite, a problem open for nearly 30 years.

We study the learning scenarios of smooth functions that also belong to certain special families of functions, such as polynomials. We prove a conjecture by Geneson and Zhou (2023) that it is not any easier to learn a polynomial in $\mathcal{F}_q$ than it is to learn any general function in $\mathcal{F}_q$. We also define an agnostic model of the online learning of smooth functions, where the learner may receive erroneous feedback up η times, for some positive integer η. We establish inequalities relating the worst-case error in the agnostic setting to the worst-case error in the standard setting.

Aidan Gao

Spatial Clustering and Classification with Graph Neural Networks

Mentor: Junhong Lin

With the rise of machine learning, graph neural networks have been developed to conduct both node-level and graph-level tasks like classification. While typically limited to being used on graphs, through techniques like k-nearest neighbors, graphs can be constructed from different mediums, allowing graph neural networks to be used in other areas. This talk explores the use of graph neural networks in spatial clustering, achieving notable success on benchmark datasets such as MNIST, CIFAR 10, and Imagenet in supervised classification and unsupervised clustering. Using a novel CNN-GNN framework, supervised GNN models surpass 99% accuracy, ranking #1 on current image classification state of the art for MNIST and CIFAR 10 and #2 on Imagenet. In unsupervised learning, using unsupervised pretrained embeddings, models reach over 98% accuracy and 92% NMI on MNIST and CIFAR 10, ranking in the top 5 and 3, respectively.

Rohan Das

New Multiplicative Structures on Frobenius Algebras

Mentors: Prof. Julia Plavnik, Indiana University Bloomington, and Dr. Pablo Ocal, UCLA

In recent years, Frobenius algebras have been studied due to their equivalence in the commutative case to Topological Quantum Field Theories. The tensor product adds a multiplicative structure to them; we want to form new such structures. To do so, we modify the twisted tensor product, which is defined on algebras and has been important recently in introducing noncommutativity to algebras. We define our analog, the warped tensor product, and classify exactly when the warped tensor product of two Frobenius algebras is Frobenius. We define the warped tensor product in a way that preserves commutativity, so our results are exactly the new multiplicative structures on commutative Frobenius algebras.

Enmei (Emma) Yang

Examples of the Reflective Algebra for Various Hopf Algebras

Mentors: Prof. Julia Plavnik and Dr. Héctor Peña Pollastri, Indiana University Bloomington

The Drinfeld double $D(H)$ is a key classical object used to construct certain Hopf algebras whose modules form monoidal categories equipped with a braiding. In analogy to this, define the reflective algebra $R_H(A)$ for a quasitriangular Hopf algebra H and an H-comodule algebra A. It can be shown that the category of modules of $R_H(A)$ has the structure of a braided module category. The goal of our project is to study the structure of $R_H(A)$. We will focus on computing them with the goal of making generalizations about its structure.

Sargam Mondal

Exact Factorizations of G-crossed Braided Fusion Categories

Mentors: Prof. Julia Plavnik, Indiana University Bloomington, and Prof. Monique Müller, Universidade Federal de São João del-Rei, Brazil

Fusion categories are rich algebraic structures with diverse applications in mathematics and physics. For example, they are important in constructing knot invariants and in providing a framework for axiomatizing quantum field theory. As in the study of topological quantum field theories (TQFTs) and related quantum systems, where creating new examples from existing ones and breaking down complex objects into simpler components is key, we are motivated to explore factorization structures in fusion categories. In this work, we investigate the interplay between two structures: exact factorizations and G-crossed braided fusion categories. The tensor product in a fusion category is not necessarily commutative, and G-crossed braided fusion categories feature a group action that governs the non-commutativity of the fusion rules. Exact factorizations, an analog of the Zappa–Szép product in group theory, give insight into how these categories are built. We seek to understand the implications of exact factorizations on G-crossed braided structures and the role of fusion subcategories within these contexts. In particular, we focus on the case where the group G is the universal grading group of the fusion category. We prove that when a G-crossed braided fusion category, with G as its universal grading group, admits an exact factorization into fusion subcategories, these subcategories necessarily inherit the crossed braided structure.

Jiwu Jang

Vertex Functions of Type D Nakajima Quiver Varieties

Mentor: Dr. Hunter Dinkins

Let $\mathcal{M}$ be a Nakajima quiver variety. A vertex function $V(\boldsymbol{z})$ of $\mathcal{M}$ is a certain formal series with coefficients in the localized K-theory of $\mathcal{M}$. We discover and prove a product identity for vertex functions of type D Nakajima quiver varieties, which verifies the conjecture of Smirnov and Dinkins (2020) in the type D case with framing at one vertex. We provide an explicit combinatorial description of zero-dimensional type D Nakajima quiver varieties using the data of symmetric Young diagrams, and use properties of Macdonald polynomials to express the type D vertex function as a product of infinite q-Pochhammer symbols in the language of the type D root system. Our main result, Theorem 5.1, can be seen as a type D generalization of the q-binomial theorem. We prove this theorem for both extremal cases: $\mathsf{w} = (0^{n-1}, 1)$ and $\mathsf{w} = (1, 0^{n-1})$. The proof involves a careful analysis of the combinatorics of type D root systems and the application of skew-Macdonald polynomial theory. This work extends previous results on type A quivers to the type D case, bringing us closer to a complete understanding of vertex functions for all finite type quivers.

Session 13

Hwisoo (Harry) Kim

Subregular Affine Kazhdan-Lusztig Polynomials in Type D

Mentor: Kenta Suzuki

Weyl groups are groups of symmetries of vector spaces, classified into types A G. Kazhdan-Lusztig polynomials are certain polynomials attached to Weyl groups that have connections to representation theory and physics. We focus on a certain subset of the Weyl group known as the subregular cell. Bezrukavnikov, Kac, and Krylov compute part of the subregular Kazhdan-Lusztig polynomials of Weyl groups in types A, D, and E. In types D4 and D5, we extend their work and compute all subregular Kazhdan-Lusztig polynomials.

Sophia Liao

Transitivity of Bender–Knuth Moves on Standard and Semistandard Young Tableaux

Mentor: Prof. Leonid Rybnikov, Université de Montréal

A standard Young tableaux is a Young diagram with unique entries from 1 to n so that each row and column are strictly increasing. A semistandard Young tableaux generalizes this notion so that entries are not required to be unique and rows are only required to be non-decreasing. A Bender–Knuth move t_i acts on a standard Young tableaux by swapping i and $i+1$ if the resulting tableaux is a valid standard Young tableaux. This action can also be generalized in a more complicated manner to semistandard Young tableaux. We find that standard Young tableaux and general pairs of standard Young tableaux of a fixed shape are transitive under Bender–Knuth moves, and we conjecture that general n-tuples of standard Young tableaux are also transitive. However, semistandard Young tableaux are not even singly transitive, and we present a few invariants for their orbits.

Eric Yee

Hilbert Series of Quasi-Invariant Polynomials in Characteristics $p \leq n$

Mentor: Frank Wang

In this talk, we compute the Hilbert series of the space of $n = 3$ variable m-quasi-invariant polynomials in characteristic 2 and 3 and explicitly describe the generators in the characteristic 2 case. In doing so we extend the work of the first author [*New York J. Math.* **29** (2023), 613–634] who worked on characteristics greater than n and prove Ren and Xu's conjecture [*SIGMA* **16** (2020), 107, 13 pages] on when the Hilbert series differs between characteristic 0 and 3 for $n = 3$. This is the first description of m-quasi-invariants in the case where the space forms a modular representation over the symmetric group, bringing us closer to describing the quasi-invariants in all characteristics and number of variables.

Sidarth Erat and Shihan Kanungo

Mixed Tensor Products for Lie Superalgebras

Mentor: Arun Kannan

We study mixed tensor products for the Lie superalgebra $\mathfrak{gl}(m|n)$. Lie superalgebras arise in physics where they are used to describe the mathematics of symmetries between fundamental particles. This is done using representation theory, which uses the tools of linear algebra to understand symmetry. Sometimes, representations can be quite complicated. Mixed tensor products enable us to understand complex representations by relating them to simpler ones.

The theory of mixed tensor products for the Lie algebra $\mathfrak{gl}(m)$, has been developed recently. This was achieved via a homomorphism that lifts representations of smaller superalgebras to representations of larger superalgebras, namely, from the universal enveloping algebra $U(\mathfrak{gl}(m+1))$ to a tensor product of $U(\mathfrak{gl}(m))$ with an algebra of differential operators. In order to develop the corresponding theory for Lie superalgebras, we needed to extend this map to the superalgebra setting. This was quite non-trivial, but in the end we were able to construct the desired homomorphism φ_R with the properties we need, and which also restricts to the original map in the case $n = 0$, where the superalgebra $\mathfrak{gl}(m|n)$ reduces to the algebra $\mathfrak{gl}(m)$.

The map φ_R has many interesting properties related to the representation theory of $U := U(\mathfrak{gl}(m+1|n))$. For instance, we use φ_R to describe a special class of representations of U. We were also able to find an explicit computation of the image under φ_R of the center of U, which is important because it determines much of the representation theoretic structure of the Lie superalgebra.