Card Tricks and Information

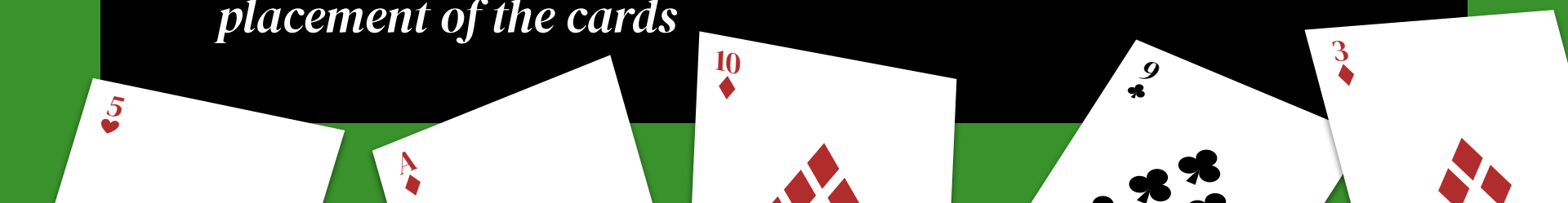
Aria Chen, Tyler Cummins, Rishi De Francesco, Jate Greene, Alexander Meng, Tanish Parida, Anirudh Pulugurtha, Anand Swaroop, and Samuel Tsui

Mentored by: **Tanya Khovanova**

SOME IMPORTANT QUESTIONS

What is a card trick?

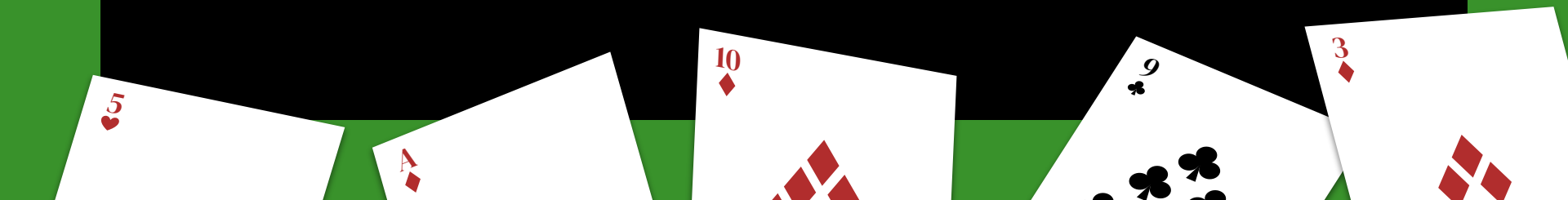
- Uses a deck of cards
- Magician's assistant receives cards and arranges them, hiding one or more
- The magician sees these and guesses the hidden card(s)
- *The only information available to the magician is the placement of the cards*



Fitch Cheney's
5 Card Trick

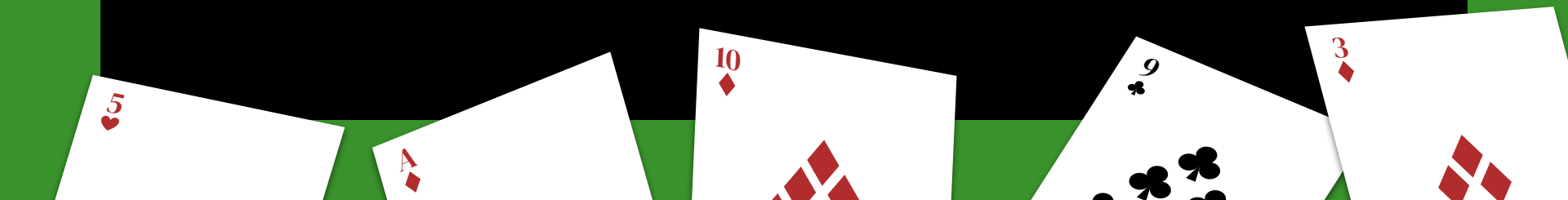
Fitch Cheney's 5 Card Trick

- Assistant receives 5 cards from standard deck
- Assistant chooses one card to hide and arranges the rest face-up in a row
- Magician guesses the hidden card



5 Card Trick Explanation (1)

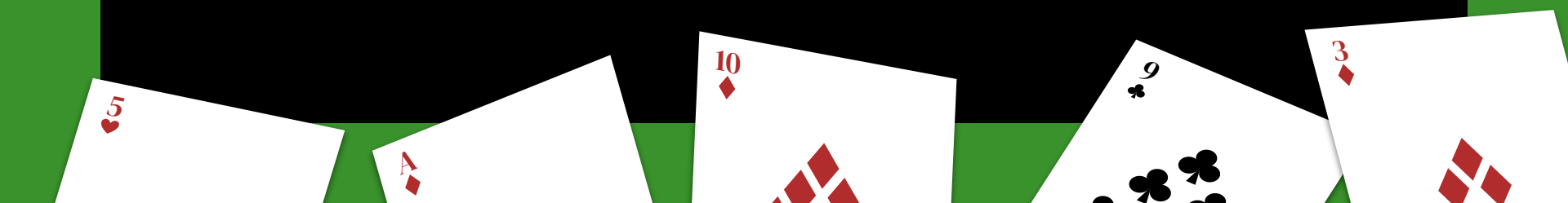
- There will be at least two with same suit (pigeonhole principle)
- One of these cards will be hidden and the other one will be the leftmost card known as the signaling card



5 Card Trick Explanation (2)

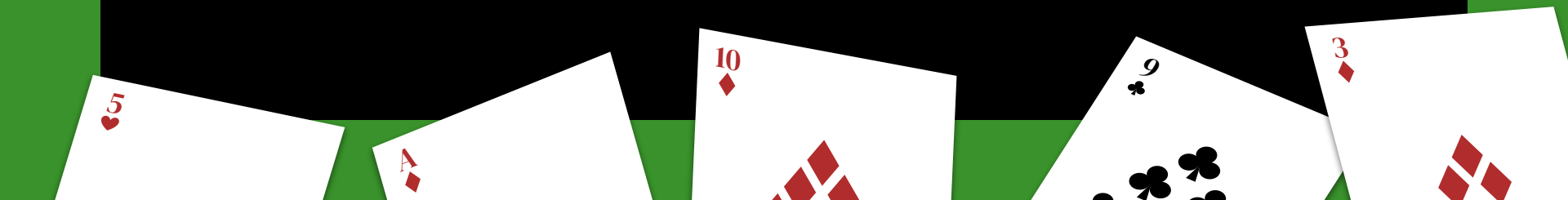
- Magician knows the suit of the hidden card but not the rank/number

- To find the rank, the magician adds a number called the signaling number, which he/she finds from the order of the other cards, to the rank of the signaling card and takes mod 13.



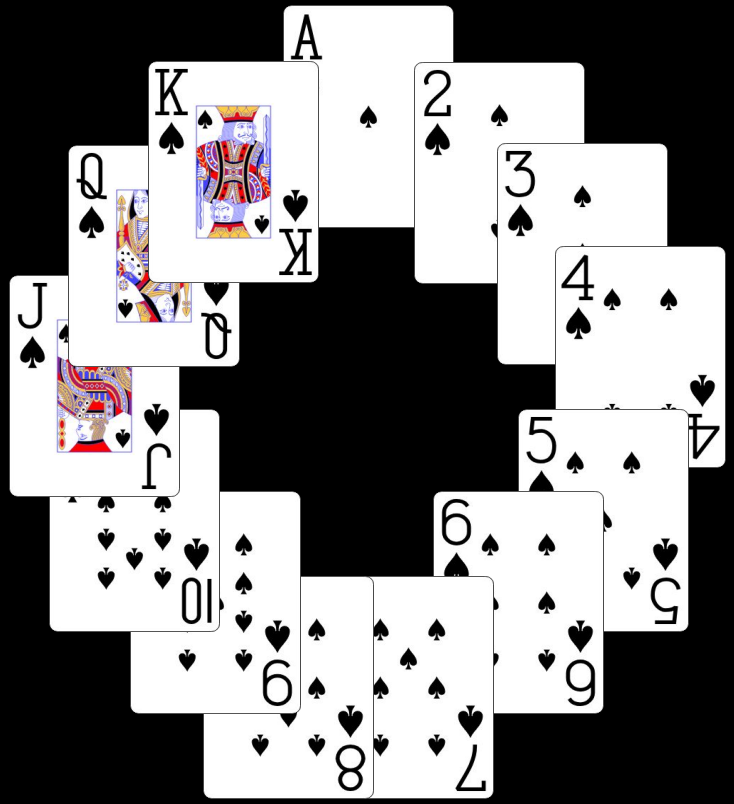
5 Card Trick Explanation (3)

- There are 3 cards left that the assistant can use
- This means that the assistant can show 6 different permutations, so the signaling number is (1-6)

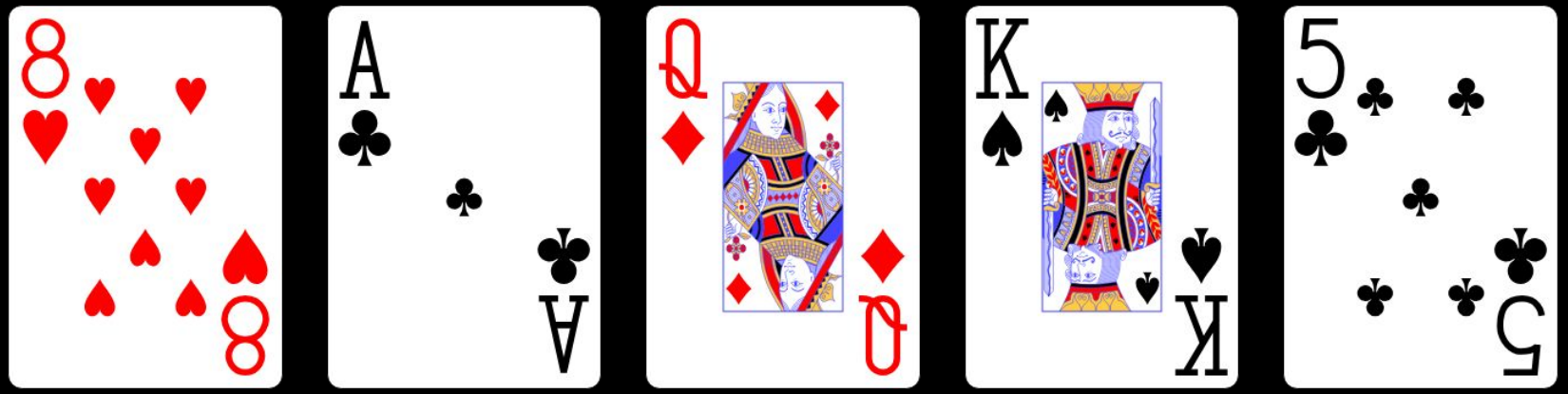


Explanation (4)

- Since the assistant can choose which of the 2 cards of the same suit to hide, he/she can make sure that the difference between the hidden card and the signaling card is between (1-6)



Example
(Assistant's POV)



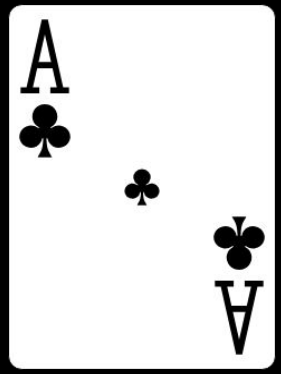
Same Suit

Ace (1) + 4 ≡ 5 (mod 13)

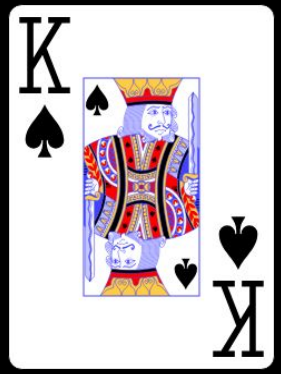
5 + 9 ≡ 1 (mod 13)

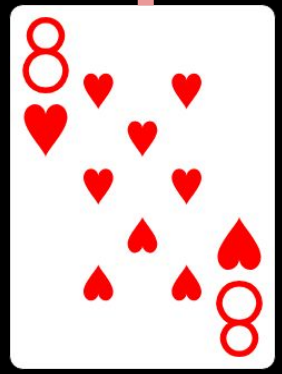
4 < 9, the Ace is the signaling card and the 5 is the hidden card

Signaling Number : 4



Signaling Card

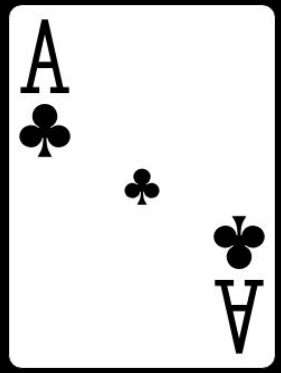


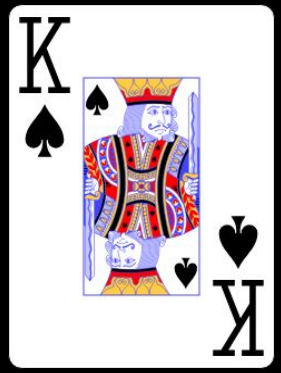


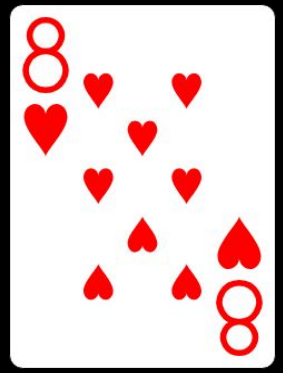
Hidden Card
(5 of Clubs)

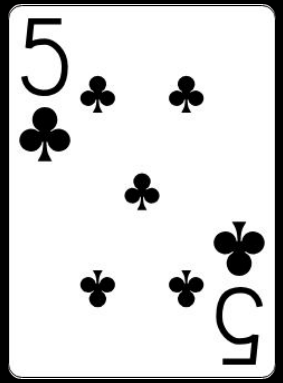
Example
(Magician's POV)

Suppose the magician gets these cards:









Signaling Card:

Hidden card is a club

Signaling number: 4

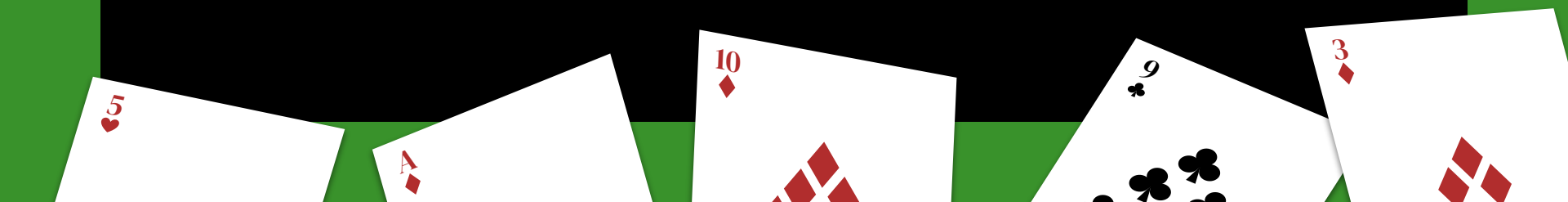
Value of hidden card: 1 + 4 = 5

Hidden Card

Kleber and Vakil's
5 Card Trick

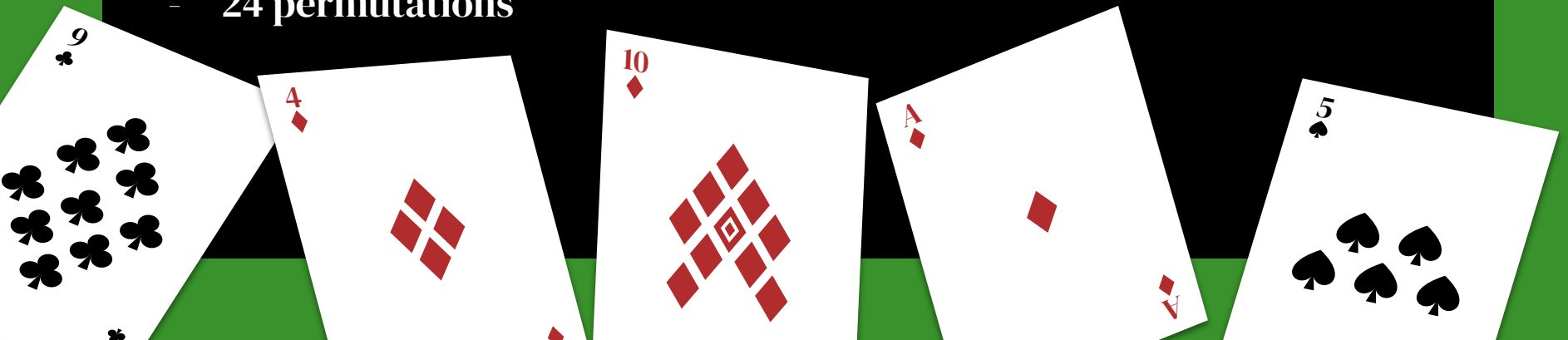
Kleber and Vakil's 5 Card Trick

- Discovered the "Best Trick"
- Proved that 124 card deck is the largest possible if the assistant is given 5 cards
- With K cards, they showed the largest deck size $N = K! + K - 1$
- They also showed a way to perform it



Kleber and Vakil Trick Explanation

- Magician excludes the 4 cards showing as a possible hidden card, 120 possible hidden cards remaining
- Assistant can choose the hidden card such that the magician can figure out value mod 5
- 24 permutations



Bounding the deck size

- Each possible ordering of cards the assistant can show is called a **message**
- Each hand the assistant gets must correspond to at least one message
- The total number of hands cannot exceed the total number of messages.
- Bounding in this way gives us $N \leq K! + K - 1$ where N is the number of cards in the deck and K is the number of cards the assistant gets

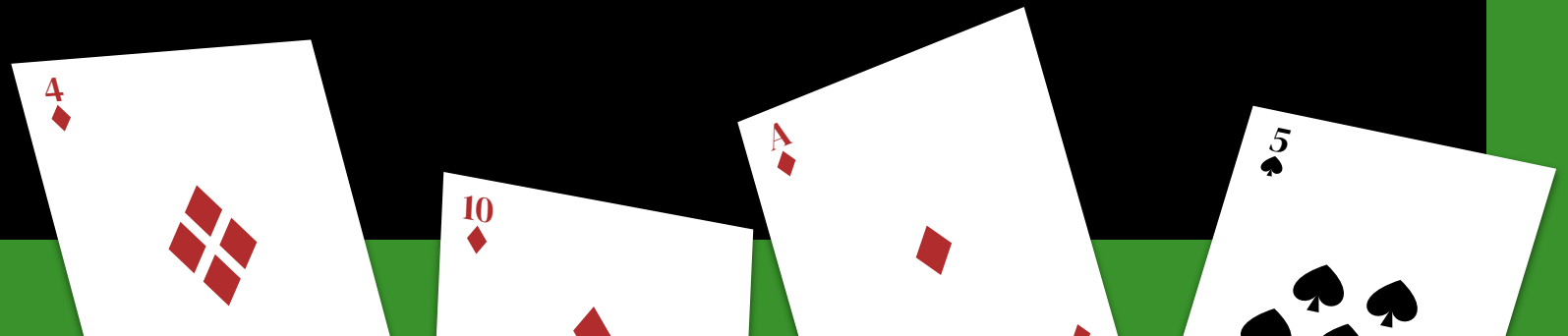
The 4 Card Trick

Mathematical card magic:
fifty-two new effects

The 4 Card Trick

- The assistant is given 4 cards instead of 5

- Similar to the 5 card trick but the assistant is allowed to place cards face down



Our Research Topics:

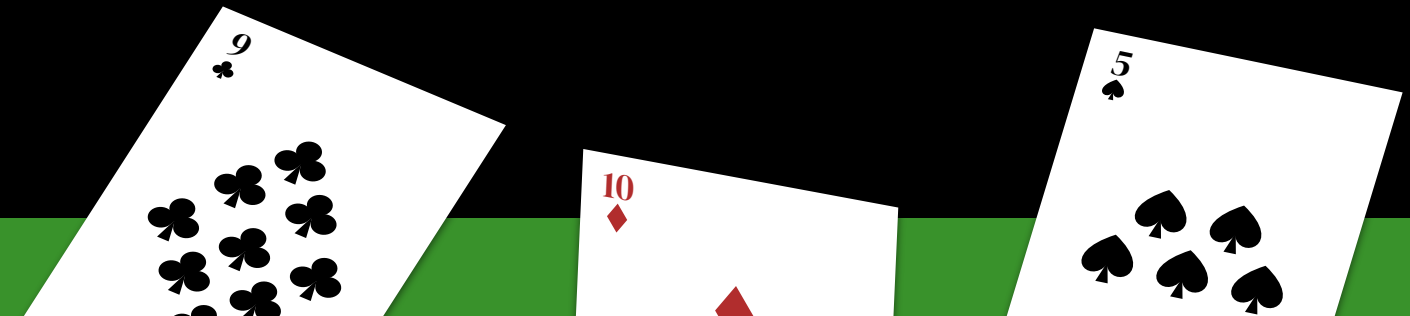
- What if the audience chooses the hidden card?
- What if there are multiple cards hidden?
- What if cards can be rotated?
- What if the cards are in a circle?
- What if the cards in the deck are duplicated?
- In each case, what is the largest possible deck size?

*We also looked out for tricks that could be performable with the standard deck of cards

Our 3 Card Trick

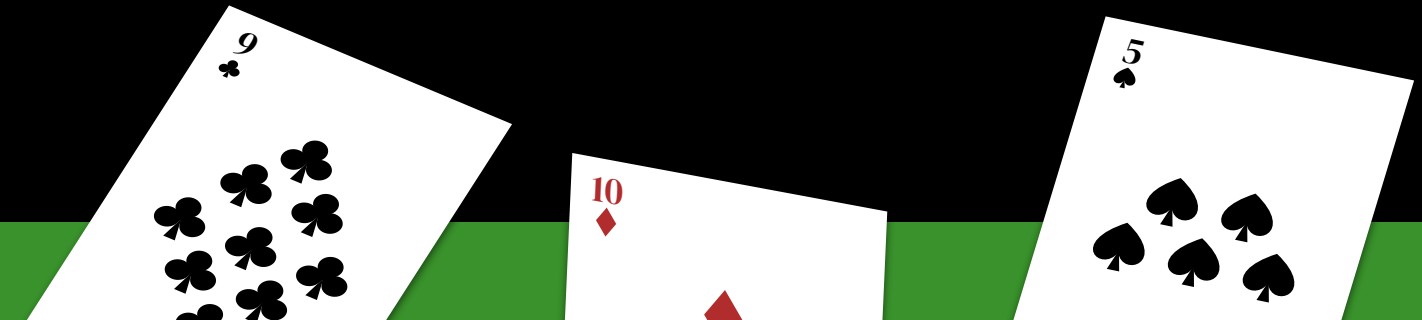
The Trick:

- The assistant is given 3 cards instead of 4 or 5
- Cards can be placed horizontally or vertically
- Cards can be face up or face down
- Our method of performing works for a deck size of up to 54, so jokers could be included



Performing This Trick: The Assistant (1)

- Special “Ace Case” exists
- If assistant received Ace, places all cards face down
- Rotations signal suit (4 possibilities, 4 suits)

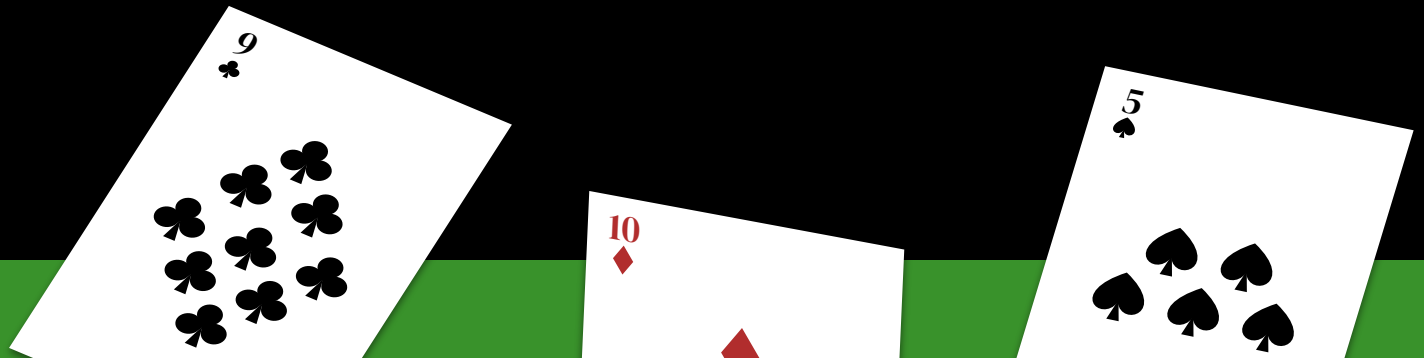


Performing This Trick: The Assistant (2)

- Normal case, if no ace:
- By pigeonhole, assistant will receive 2 cards of same color (red/black)
- One of these cards is hidden card, other is signaling card

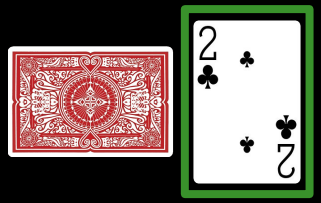
Performing This Trick: The Assistant (3)

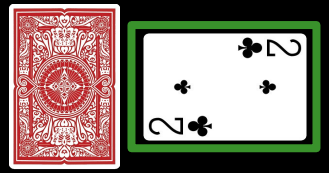
- Number cards 2-13 (no ace). Let one card be A and other be B, $A > B$
- If cards are the same suit: signaling number = $A - B$, A is hidden
- If cards are not the same suit: signaling number = $12 - (A - B)$, B is hidden

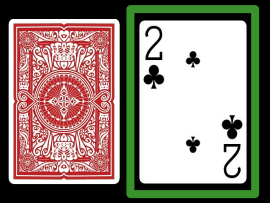


Performing This Trick: The Assistant (4)

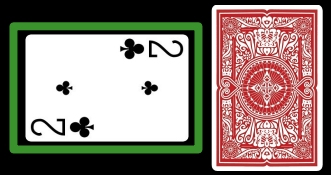

1

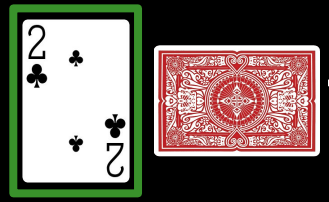

2

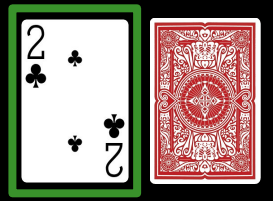

3


4

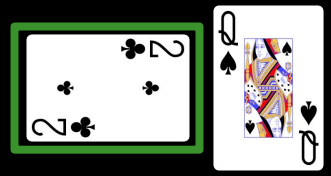

5

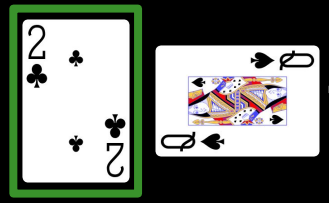

6

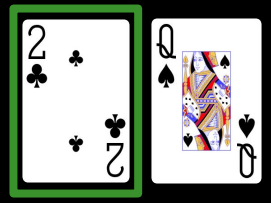

7


8


9

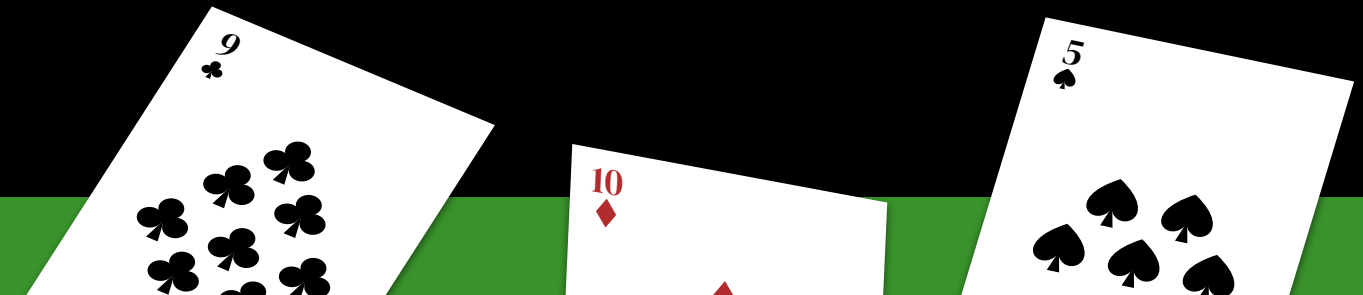

10


11


12

Performing This Trick: The Magician

- Find the signaling number ***S***
- Let the first face-up card (signaling card) be *A*
- If *A* + *S* < *14*: *B* = *S* + *A*, same suit as *A*
- Otherwise, *B* = *S* + *A* - *12*, different suit from *A*

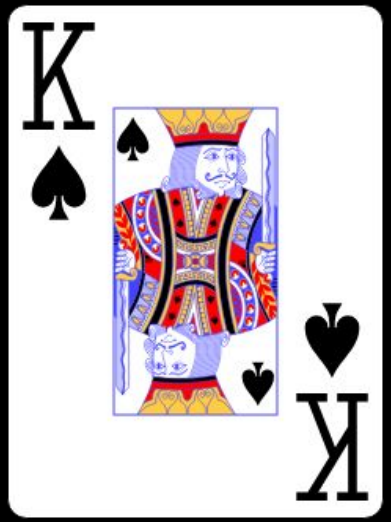


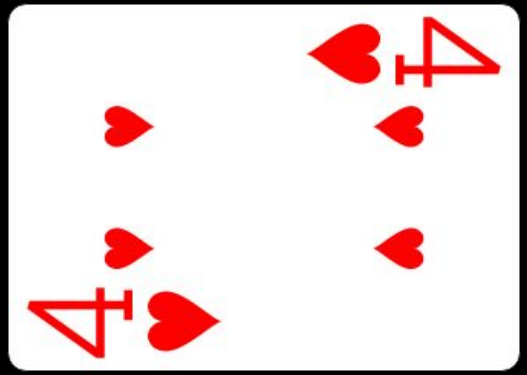
Example
(Assistant's POV)

Same Color

King and Queen are different suits; signaling number is 12-(13-12) = 11

Queen is hidden card, King is signaling card

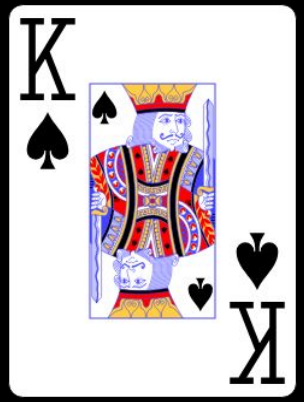


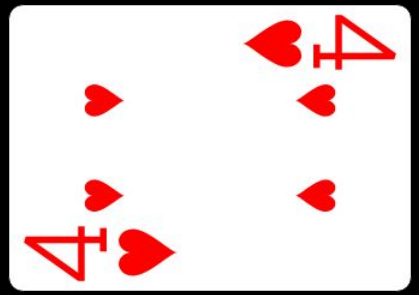


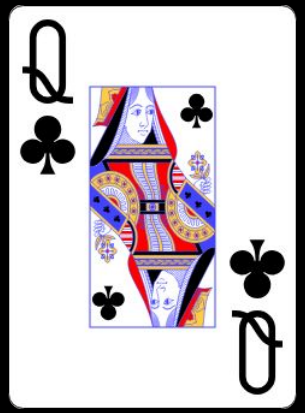
Hidden Card
(Queen of Spades)

Example
(Magician's POV)

Suppose the magician sees these cards:







Signaling Card:

Hidden card is a club or spade

Hidden Card

Signaling Number: 11

13 + 11 > 13, hidden card is a club; value is 13 + 11 - 12 = 12

Other Things We Did:

Calculated upper bounds for:

- Duplicates with all cards face up
- Cards can be face down audience chooses the hidden card
- Assistant chooses multiple cards
- etc.

Found general strategies close to upper bound for:

- Duplicates with all cards face up
- Assistant chooses multiple cards

Found bound-reaching strategies for small K in some cases

Found general bound-reaching strategies for some cases

Acknowledgements

Many thanks to:

- Our mentor, Tanya Khovanova
- The MIT PRIMES STEP Program
- Our parents for supporting our goals and driving us every week

Thank you!

Any questions?