

Improved Performance for Private Information Retrieval

By Boyan Litchev

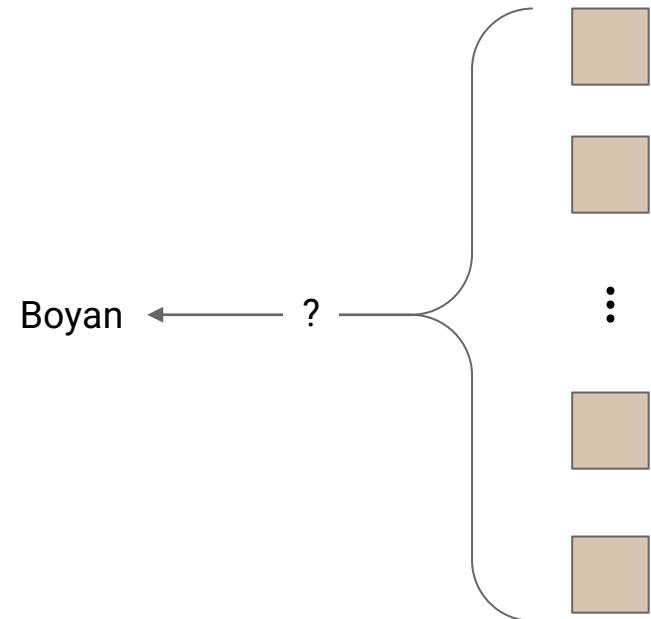
Mentored by Simon Langowski

Private Information Retrieval (PIR)



The Problem

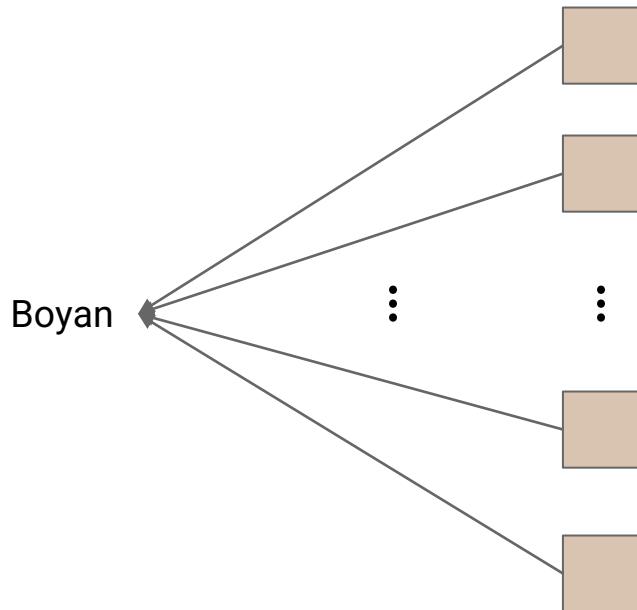
- Retrieve an item without revealing which one



Use Cases

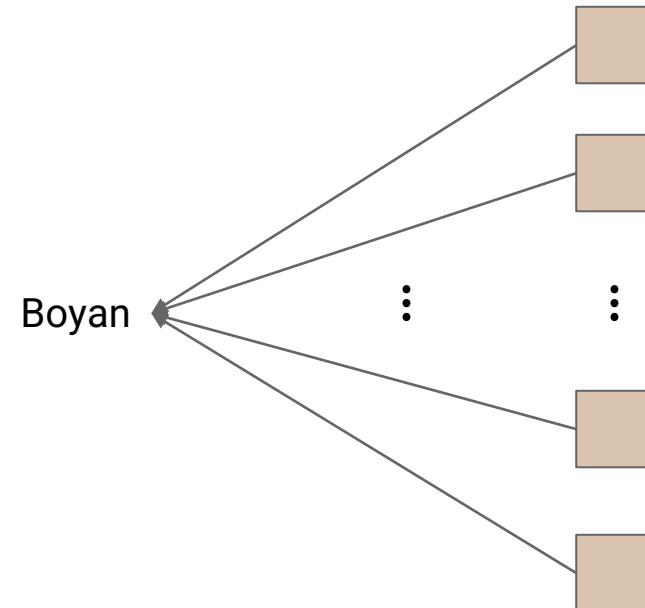
- Private Browsing
- Private Streaming
- Anonymous Messaging

A Simple Solution (1/2)



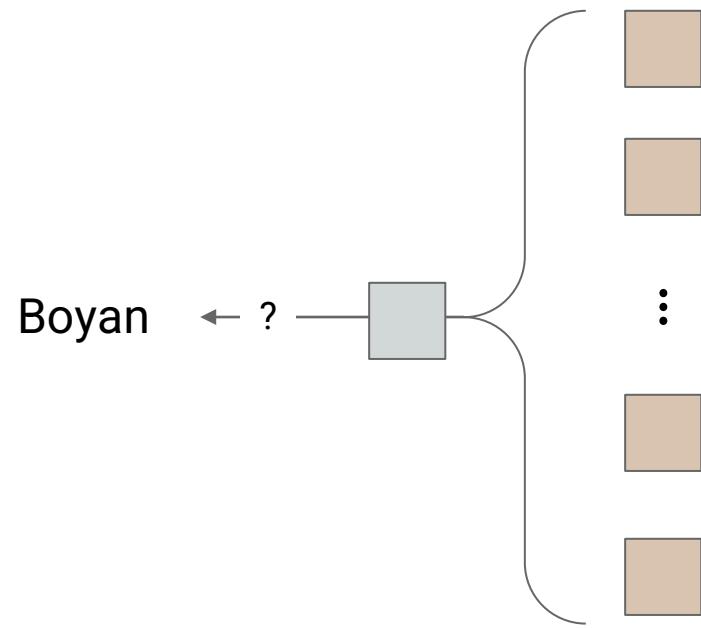
A Simple Solution (2/2)

- Network Costs are the entire database
 - Too high



The Goal

- Compress the database into one element
 - Minimizes network costs



Visible



Encrypted

The Approach

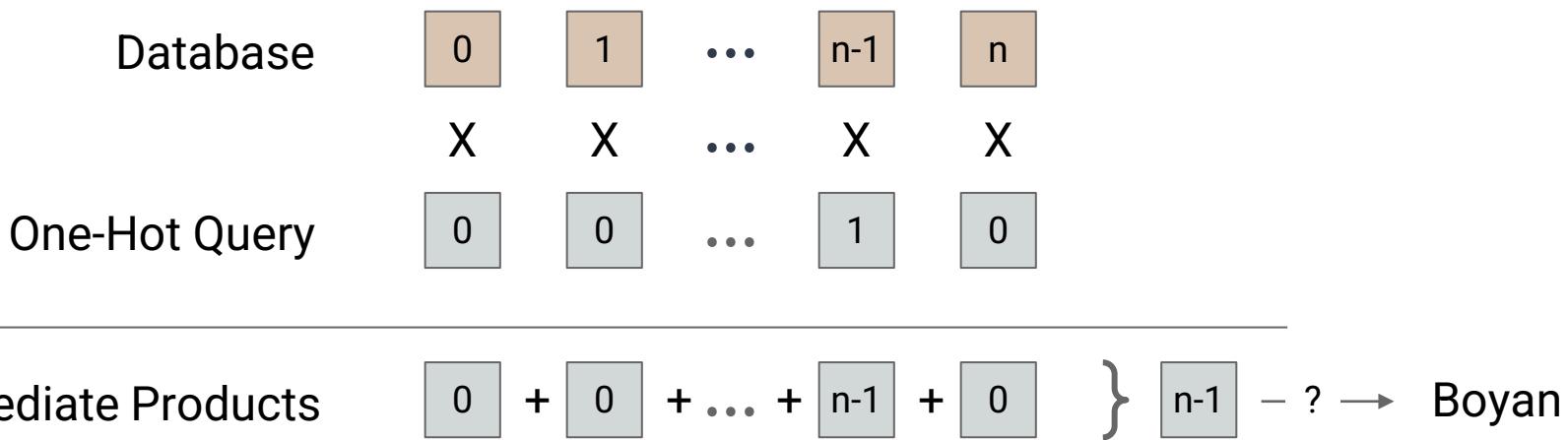
Database	0	1	...	n-1	n
	X	X	...	X	X

One-Hot Query	0	0	...	1	0
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Intermediate Products 0 + 0 + ... + n-1 + 0 } n-1 - ? → Boyan

A Note on Costs

- Response is 1 element
- The query can be compressed

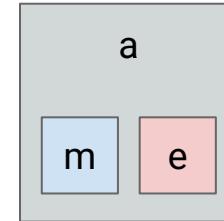


Homomorphic Encryption & Multiplications

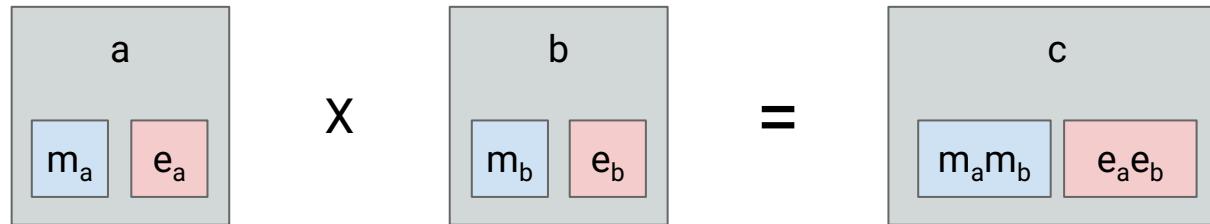


Homomorphic Ciphertext

- A ciphertext a encodes
 - A hidden message m
 - With some error e



Multiplication



Multiplication

- Error scales multiplicatively
- Is very expensive

$$\begin{array}{c} a \\ \boxed{m_a} \quad \boxed{e_a} \end{array} \times \begin{array}{c} b \\ \boxed{m_b} \quad \boxed{e_b} \end{array} = \begin{array}{c} c \\ \boxed{m_a m_b} \quad \boxed{e_a e_b} \end{array}$$

Gadget Inversions



Base-2 Gadget Inversion (1/2)

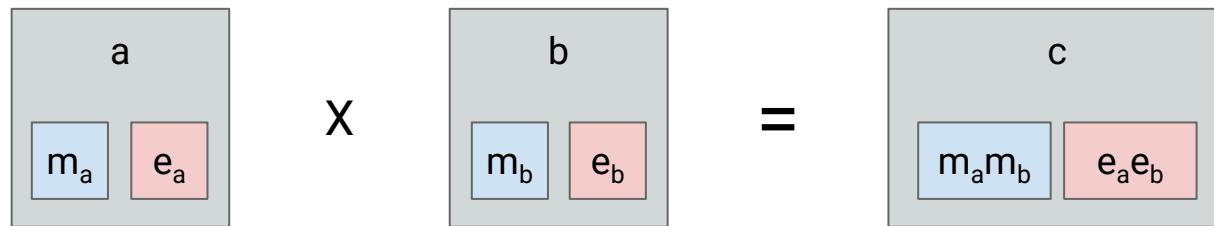
$$97 = 64 + 32 + 1$$

$$97 = \boxed{1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1}$$

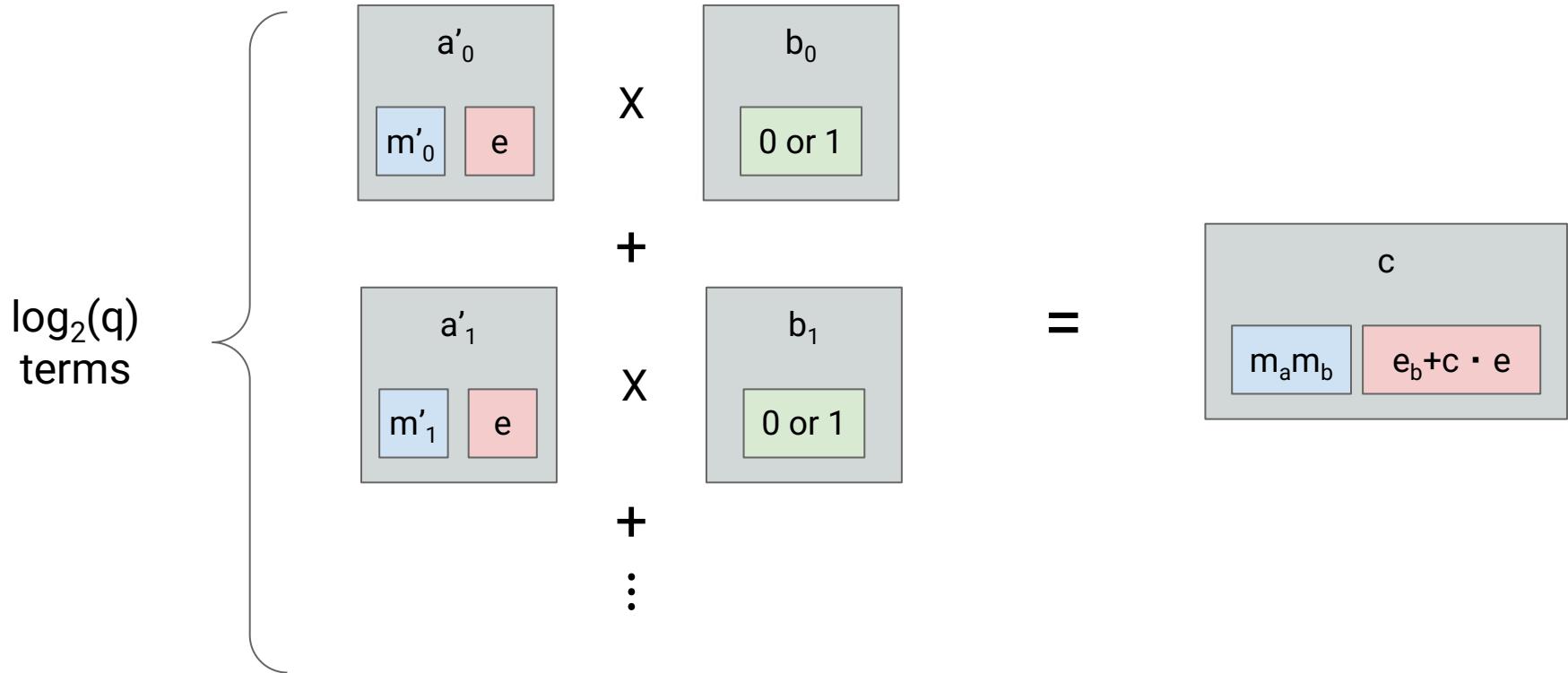
Base-2 Gadget Inversion (2/2)

$$\begin{matrix} b \\ m_b \quad e_b \end{matrix} = \begin{matrix} b_0 \\ 0 \text{ or } 1 \end{matrix} + 2 \cdot \begin{matrix} b_1 \\ 0 \text{ or } 1 \end{matrix} + \dots + 2^{\lceil \log(q) \rceil} \cdot \begin{matrix} b_{\log(q)} \\ 0 \text{ or } 1 \end{matrix}$$

Original Multiplication



Base-2 Gadget Multiplication



Our Work



Base-3 Gadget Inversion (1/2)

$$97 = 81 + 9 + 2 \cdot 3 + 1$$

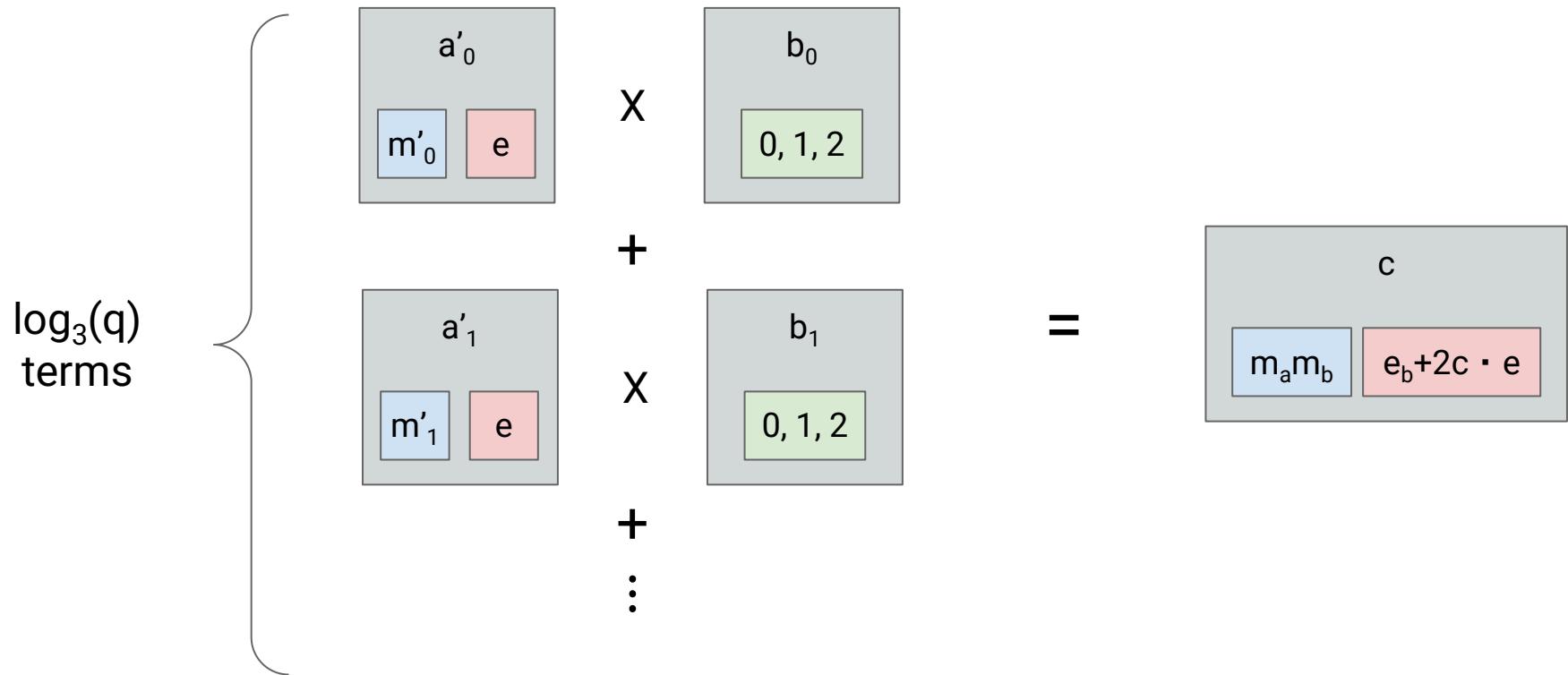
$$97 = \boxed{1 \quad 0 \quad 1 \quad 2 \quad 1}$$

Base-3 Gadget Inversion (2/2)

$$b = b_0 + 3 \cdot b_1 + \dots + 3^{\lceil \log(q) \rceil} \cdot b_{\log(q)}$$

The equation illustrates the decomposition of a base-3 number b into its components. On the left, a gray box labeled b contains two smaller boxes: a blue one labeled m_b and a red one labeled e_b . An equals sign follows, followed by a sum of terms. Each term consists of a gray box with a label b_i above it and a green box with the values $0, 1, 2$ below it. The first term is b_0 , the second is $3 \cdot b_1$, and so on, up to $3^{\lceil \log(q) \rceil} \cdot b_{\log(q)}$.

Base-3 Gadget Multiplication



Balanced Base-3 Gadget Inversion (1/2)

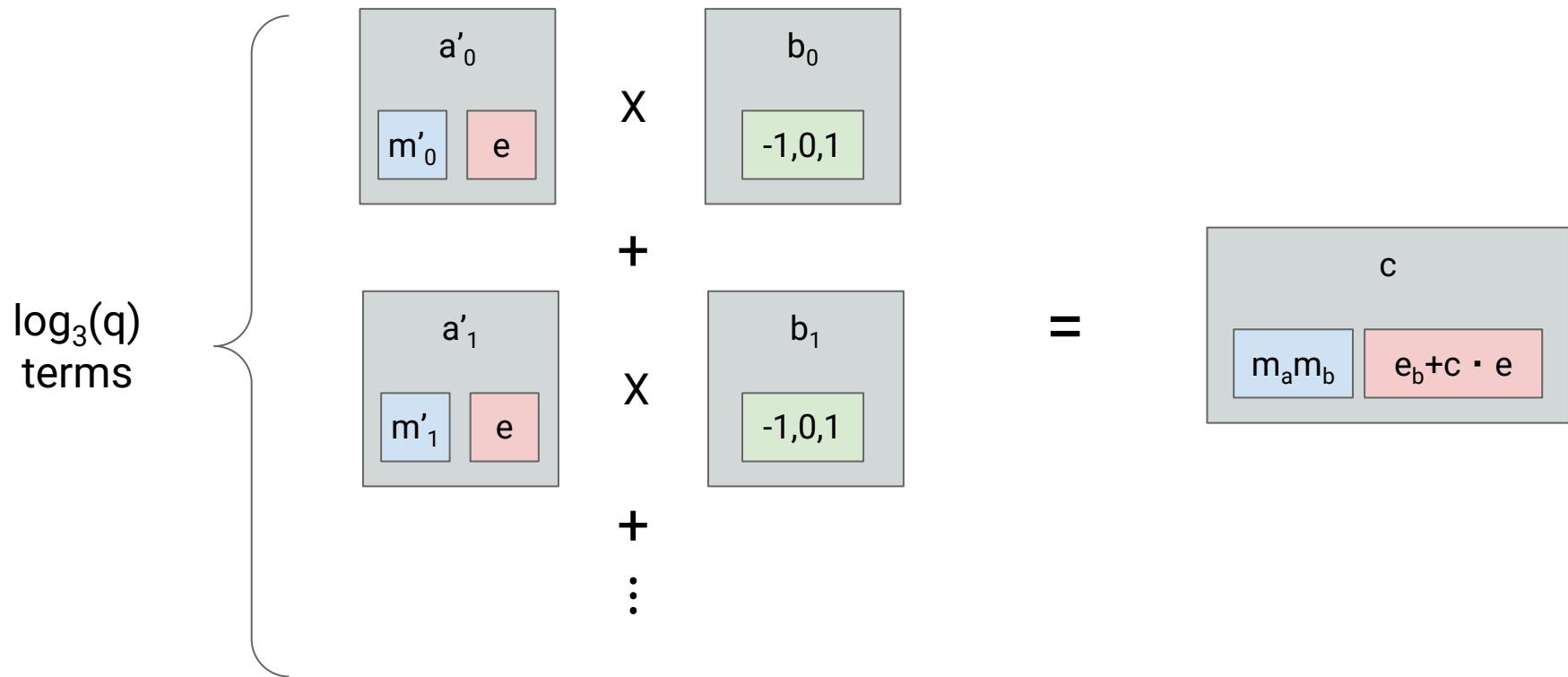
$$97 = 81 + 27 - 9 - 3 + 1$$

$$97 = \boxed{1 \quad 1 \quad -1 \quad -1 \quad 1}$$

Balanced Base-3 Gadget Inversion (2/2)

$$\begin{matrix} b \\ m_b \quad e_b \end{matrix} = \begin{matrix} b_0 \\ -1,0,1 \end{matrix} + 3 \cdot \begin{matrix} b_1 \\ -1,0,1 \end{matrix} + \dots + 3^{\lceil \log(q) \rceil} \cdot \begin{matrix} b_{\log(q)} \\ -1,0,1 \end{matrix}$$

Balanced Base-3 Gadget Multiplication



Preliminary Results



Per-Multiplication Costs

	Total Time (ms)	Gadget Inversion (ms)	Other Costs (ms)
Optimized Base 3	2.86		
Base 3	3.84	1.25	2.59
Base 2	5.29	0.6	4.62
Improvement	27%	-87%	44%

Future Work

- Testing Larger Databases
- Modified Parameter Sets
- Alternate Decompositions

Acknowledgments!

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this project possible!