Photo: Slava Gerovitch

Thirteenth Annual
Spring-Term
PRIMES Conference

May 13-14, 2023

Thirteenth Annual Spring Term PRIMES Conference, May 13-14, 2023

Room 2-190, MIT
Open to the public

Saturday, May 13

1:15 pm: Welcoming Remarks

- Prof. Pavel Etingof, PRIMES Chief Research Advisor
- Dr. Slava Gerovitch, PRIMES Program Director

1:20-1:50 pm: Session 1. Math Reading Groups

- Rishi Gujjar, “Theorems in the Differential Geometry of Curves and Surfaces” (mentor Dr. Jingze Zhu)
- Isha Agarwal & Victoria Li, “Knot Theory” (mentor Julius Baldauf)

2:00-3:10 pm: Session 2. PRIMES Circle

- Marisa Gaetz & Mary Stelow, PRIMES Circle coordinators, Welcoming remarks
- Chahat Kalia & Cynthia Tian, “Stirling Numbers of the Second Kind” (mentor Paige Dote)
- Cathal Stephens & Roonak Thapa, “Euler’s Totient Function” (mentor Ariana Park)
- Elian Martinez & Jorge Sanchez, “Knot Theory” (mentor Luis Modes)

3:20-4:10 pm: Session 3. PRIMES Circle

- Matheus Alves & Gael Medina, “Data Structures and Graph Algorithms (mentor Carlos Alvarado)
- Veronika Moroz, Ada Tsui & Athena Wang, “Combinatorics and Representation Theory” (mentor Katherine Tung)

4:20-4:50 pm: Session 4. PRIMES Circle

- Sophia Heinrich & Lucy O'Brien, “Introductory Combinatorics and Its Applications” (mentor Lily Chen)
- Lillian MacArthur, “Classifying Isometries” (mentor Honglin Zhu)

Sunday, May 14

1:00-1:55 pm: Session 5. Computer Science and Computational Biology

- Anshul Rastogi, “Threshold-Based Inference of Dependencies in Distributed Systems” (mentor Prof. Raja Sambasivan, Tufts University)
- Alicia Li & Mati Yablon, “Adversarial Attacks Against Online Learning Agents in MDPs” (mentor Mayuri Sridhar)
- Steven Tan, “Improved Models for Somatic CAG Repeat Expansion in Huntington’s Disease” (mentors Bob Handsaker and Seva Kashin, Broad Institute)

2:00-2:40 pm: Session 6. PRIMES STEP

- Jasper Caravan, Michael Han, Andrew Kalashnikov, Ella Li, Alexander Meng, Vaibhav Rastogi, Gordon Redwine, Lev Strougov, Angela Zhao (PRIMES STEP Junior group), “SOS” (mentor Dr. Tanya Khovanova)
- Nikhil Byrapuram, Irene Choi, Adam Ge, Selena Ge, Sylvia Zia Lee, Evin Liang, Rajarshi Mandal, Aika Oki, Daniel Wu, and Michael Yang (PRIMES STEP Senior group), “Cards, Magic Squares, and Codes” (mentor Dr. Tanya Khovanova)

3:00-4:10 pm: Session 7. PRIMES Circle

- Adrian Baez & Danielle Harrington, “Types of Normal-Play Games” (mentor Yuyuan Luo)
- Ryan Chao & Yakir Propp, “Theoretical Machines” (mentor Zhao Yu Ma)
- Luba Slesarenko & Chloe Zhong, “Introduction to Cryptography” (mentor Aparna Gupte)

4:30-5:50 pm: Session 8. PRIMES Circle

- Emmanuel Mateo & Jonathan Nguyen, “Graph Theory” (mentor Lexy Zitzmann)
- Niva Sethi, Tzofia Strey & Angela Zhao, “A Brief Survey of Key Topics in Group Theory” (mentor Kaili Liu)
- Iris Liu, Bella Qu & Niyathi Srinivasan - Applications of Probability” (mentor Lexi Spinetta)

2023 PRIMES CONFERENCE ABSTRACTS

Saturday, May 13

Session 1:Math Reading Groups

Rishi Gujjar

Theorems in the Differential Geometry of Curves and Surfaces

Mentor: Dr. Jingze Zhu

In this presentation, we will cover the Differential Geometry of Curves and Surfaces. We will explore differentiable curves in three-dimensional space and study their properties. We will then delve into important theorems, such as the 4 Vertex Theorem, which states that all closed curves must have at least four vertices. Additionally, we will cover the Isoperimetric Inequality, which shows that given a fixed length of rope, a circle maximizes the area it bounds. Finally, we will introduce the Cauchy-Crofton formula, which relates the length of a curve to the number of times it intersects a family of lines. We will also define regular surfaces as the two-dimensional counterparts to regular curves and introduce the first and second fundamental forms, as well as the Gauss map. This presentation serves as an introduction to the basics of differential geometry and its various applications.

Isha Agarwal & Victoria Li

Knot Theory

Mentor: Julius Baldauf

This talk gives an introduction to knot theory and the classification of knots. We begin by introducing what it means for two knots to be distinct. Then, we discuss basic invariants, starting with tricolorability, which only allows us to sort knots into two groups. For a more nuanced conception of knots, we then discuss knots in a topological context culminating with Seifert's algorithm and an exploration of the genus of a knot. Finally, we use these results to prove a fundamental result of knot theory: the trivial knot is not a composition of two nontrivial knots.

Session 2: PRIMES Circle

Chahat Kalia & Cynthia Tian

Stirling Numbers of the Second Kind

Mentor: Paige Dote

Suppose we had n kids and k ice cream flavors to give to each kid (each kid only getting one scoop of one flavor). How many different ways are there to give away scoops of ice cream, assuming that we use all k flavors? This question, and more, we will answer during our final presentation on *Stirling numbers of the second kind*. We will define what the Stirling numbers are, and use them to prove the "Stirling Number Dual of the Binomial Theorem."

Cathal Stephens & Roonak Thapa

Euler's Totient Function

Mentor: Ariana Park

We discuss Euler's totient function or Euler's φ-function as well as the surjectivity of the function. The totient function is a fundamental concept in number theory which defines the number of integers a given modulo can have in its reduced residue system. It plays an important role in various branches of mathematics, including cryptography and algebraic number theory. The function is closely related to congruence relations, which are a way of characterizing two integers that have the same remainder when divided by a given Modulus. The study of reduced residue systems, which are the set of integers coprime to a given modulus and modulo that modulus, is closely tied to Euler's totient function and its applications.

Elian Martinez & Jorge Sanchez

Knot Theory

Mentor: Luis Modes

This presentation will be about knot theory. We will focus mainly on tricolorability, Dowker's notation, and knots and sticks. First, we will review the basic definition of knots and give some examples. Later, we will define tricolorability, an invariant for knots, and Dowker's notation, a way to encode the projection of a knot using numbers. Finally, we will talk about some interesting formulas regarding constructing a knot with sticks.

Session 3: PRIMES Circle

Matheus Alves & Gael Medina

Data Structures and Graph Algorithms

Mentor: Carlos Alvarado

Data Structures and Algorithms are integral parts of our day to day lives. From the GPS in our cars to the routers connecting our computers, algorithms are fundamental. This presentation will go over several types of data structures and how they connect to the Dijkstra shortest path algorithm. This presentation will also go over how different data structures are connected to one another and how they compare using time complexity.

Veronika Moroz, Ada Tsui & Athena Wang

Combinatorics and Representation Theory

Mentor: Katherine Tung

The problem of expressing positive integers as sums of squares has been considered since antiquity. It is known that any positive integer is the sum of four squares, but classical proofs of this result are non-constructive. We provide an explicit decomposition of $n! = 1 \cdot 2 \cdots n$ into the sum of relatively few squares, and link this decomposition with ideas from combinatorics and representation theory, including partitions, Young tableau, and irreducible representations of the symmetric group.

Session 4: PRIMES Circle

Sophia Heinrich & Lucy O'Brien

Introductory Combinatorics and Its Applications

Mentor: Lily Chen

Our presentation will focus on foundational concepts of combinatorics, such as permutations, arrangements of lists, and counting subsets. Each of these concepts will be demonstrated through examples of relevant problems and proofs/theorems if applicable. Later, we will explore more advanced topics such as partitions, compositions, and how to represent these topics visually. The second half will focus on the Binomial Theorem and its proof, combinatorial identities associated with the theorem, and how these identities can be found in Pascal's Triangle. Finally, our presentation will conclude with an overview of the applications of combinatorics and the other areas of math that can be connected to combinatorics.

Lillian MacArthur

Classifying Isometries

Mentor: Honglin Zhu

An isometry is a geometric transformation that preserves distances between pairs of points. We present methods to classify isometries in the Euclidean plane, and extend these methods to spherical, single elliptical, and hyperbolic geometry. We then explore isometries of the plane with different metrics such as the taxicab metric and the l_p metric for $p > 1$ and $p = \infty$.

Sunday, May 14

Session 5: Computer Science and Computational Biology

Anshul Rastogi

Threshold-Based Inference of Dependencies in Distributed Systems

Mentor: Prof. Raja Sambasivan, Tufts University

Many current online services rely on the interaction between different components that form a distributed system. Analyzing distributed systems is important in performance analysis (e.g. critical path analysis), debugging, and testing new features. However, the analysis of these systems can be difficult due to limited knowledge of how components work and the variety of services and applications that are usually instrumented. *The Mystery Machine*, introduced by Chow et al. in 2014, has a "big data" approach, using logged events across many traces to generate and refine a causal model. We introduce *Scooby Systems*, our extension of *The Mystery Machine*'s algorithm to address particular limitations of *The Mystery Machine*. Specifically, we introduce thresholds to increase the tolerance to violations in the formation of causal relationships. In the future, we hope to implement backtrace information into the identification of logged events to address event repetitions.

Alicia Li & Mati Yablon

Adversarial Attacks Against Online Learning Agents in MDPs

Mentor: Mayuri Sridhar

Consider a typical streaming problem, where an agent dynamically interacts with its environment to learn an optimal behavior. Such methods are used in a variety of applications, including playing Atari games and robotic hand manipulation. We analyze an agent that learns the rewards of each path in its environment, which can be modeled as determining the edge weights of a graph. We study an agent that follows an ϵ-greedy sampling strategy because this model is widely used and has been successfully applied to many problems. However, in recent years, numerous attacks have been devised against graph learning algorithms, with some methods exploiting graph structure and node features. To ultimately create a robust graph streaming algorithm based on ϵ-annealing, we first construct, implement, and analyze worst-case attacks against random-sampling and ϵ-greedy victim models. Our adversarial strategy exploits path overlaps and stalls the victim to effectively increase the corruption budget.

Steven Tan

Improved Models for Somatic CAG Repeat Expansion in Huntington's Disease

Mentosr: Bob Handsaker and Seva Kashin, Broad Institute

Huntington's Disease (HD) is an inherited neurodegenerative disease caused by alleles with 36 or more repeats of the trinucleotide sequence CAG in the huntingtin (HTT) gene. A person with HD inherits an allele with a certain CAG length (>35) at birth, but somatic expansion within the brain is known to occur throughout their lifetime, resulting in a situation in which individual cells have longer and highly variable numbers of CAG repeats. As somatic expansion is thought to be a driver of disease onset, a better understanding of the mechanisms behind CAG repeat expansion is important for developing therapeutics. In this study, we applied statistical models to simulate CAG repeat expansion. We used a new kind of biological data, in which CAG length has been measured precisely in individual neurons of specific cell types from post mortem brain samples of HD patients. Through applying these models to simulate CAG expansion in different cell types from the caudate and cortex, we found that effectively fitting the data required models consisting of two phases of expansion. These phases appear to have differing rates and CAG length thresholds: one at roughly 36 CAGs — a known threshold for somatic instability — and another at roughly 70 CAGs, which we hypothesize is a threshold for accelerated expansion. The models suggest that somatic expansion is a slow process for a large period of time, but may increase to extremely fast rates in a span of just a couple years. The models deepen our understanding of somatic expansion, and suggest that therapeutics targeting the somatic expansion process may be highly effective. These insights may also inform the design of clinical trials in terms of the types of patients to include and the disease stage which may provide the most leverage for the therapeutic.

Session 6: PRIMES STEP

Jasper Caravan, Michael Han, Andrew Kalashnikov, Ella Li, Alexander Meng, Vaibhav Rastogi, Gordon Redwine, Lev Strougov, Angela Zhao (PRIMES STEP Junior group)

SOS

Mentor: Dr. Tanya Khovanova

SOS! EMERGENCY! The game of SOS is a classic pen-and-paper game similar to tic-tac-toe. This game involves two players who go toe-to-toe and alternate placing letters with the goal of creating SOS. In our talk, we explore this game and its many variations including SOO, SSS, and SOSO. Aren't you SOO ready to learn about this? Come to our talk, where we discuSSS the game of SOS! When trapped in a mathematician's dream, how will you call for help if you can't even win a game of SOS? Did you get the joke? It's just SOSO.

Nikhil Byrapuram, Irene Choi, Adam Ge, Selena Ge, Sylvia Zia Lee, Evin Liang, Rajarshi Mandal, Aika Oki, Daniel Wu, Michael Yang (PRIMES STEP Senior group)

Cards, Magic Squares, and Codes

Mentor: Dr. Tanya Khovanova

In this presentation, we discuss various card games and how they can help teach linear algebra. The games discussed include SET, Socks, and a new game called EvenQuads. For people who like to multipurpose their cards, we explain how to play EvenQuads on the Socks deck and Socks on the EvenQuads deck. We also discuss magic squares using the EvenQuads and SET decks, as well as a related class of squares we call strongly magic squares. Finally, we discuss how to create error-correcting codes from EvenQuads cards and vice versa.

Session 7: PRIMES Circle

Adrian Baez & Danielle Harrington

Types of Normal-Play Games

Mentor: Yuyuan Luo

Game theory can be described as a series of methods which are used in order to determine the outcome of a game decisively without needing to complete said game. Within game theory, the genre of games which we discuss are those classified under normal-play games. Normal-play games are a subgenre of combinatorial games, and they have an outcome of either a win or a loss, and with this comes classifications for these results known as types, which specifies the player with the winning strategy. In our presentation, we provide tools and formalism which allow us to determine the types of various games.

Ryan Chao & Yakir Propp

Theoretical Machines

Mentor: Zhao Yu Ma

We will be discussing three classes of automata: Finite Automata, CFG & PDAs, and Turing Machines. We will be providing the definition of all these automata, as well as their properties and uses. We will go into depth on the equivalence theorems between different subtypes of the same class.

Luba Slesarenko & Chloe Zhong

Introduction to Cryptography

Mentor: Aparna Gupte

In this talk, we will describe a foundational algorithm in cryptography: the Diffie-Hellman key exchange protocol. This algorithm enables public-key encryption, which means two parties can communicate privately over a channel without ever having to meet to share a secret key. We will build up to this cryptosystem by first talking about secret-key encryption, and introducing the necessary tools in number theory, group theory, and complexity theory.

Session 8: PRIMES Circle

Emmanuel Mateo & Jonathan Nguyen

Graph Theory

Mentor: Lexy Zitzmann

In this presentation, we will introduce the history, fundamentals, and real-life applications of graph theory. Planning efficient routes and understanding network flows is essential for navigation, telephone networks, the internet, and many other real-world situations. After beginning with an introduction of the history of the field through the Königsberg Bridge Problem, we will first cover the traversability of graphs–namely Eulerian and Hamiltonian graphs–and introduce important algorithms with real-life applications. We then shift away from traversability to focus on a specific type of algorithm that finds the shortest path between two points–Dijkstra's algorithm–and discuss how this algorithm can be used for cost efficiency and route optimization. Finally, we narrow our focus to tree graphs to tackle the Minimum Spanning Tree problem and present the two main algorithms used to solve it.

Niva Sethi, Tzofia Strey & Angela Zhao

A Brief Survey of Key Topics in Group Theory

Mentor: Kaili Liu

In this talk we will provide a brief survey of key topics in group theory, with the goal of showing some proofs and the geometric intuition behind it. We begin with an overview of the basics of group theory and lead into a discussion of notable mathematical events driving its development in the 19th century. We then provide some basic groups, subgroups and cosets. We also show the proof of the 5th quadratic and the Cayley theorem. Finally, we close with some interesting applications.

Iris Liu, Bella Qu & Niyathi Srinivasan

Applications of Probability

Mentor: Lexi Spinetta

The applications of probability, which stems from the study of the likeliness of certain events, determine much of the workings of our world, whether we realize it or not. In this presentation, we will be exploring several ideas and concepts in probability theory, starting from the fundamental axioms of probability, and then discussing conditional probability, and independence and dependence of events. We will apply these fundamental theories to more complicated distributions of random events, discussing discrete random variables (the Binomial, Poisson, and Geometric distributions especially), and continuous random variables. Further, we will discuss the properties of expected value and variance of a data set, all the while solving challenging, real-world problems to support our conjectures.