

# Stable Marriage Problem



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What is the problem?

***\*ahem\* \*attention\_please.wav\****

Why are we here?



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**Thanks For Viewing!**

bye

But actually..?

# Stable Marriage Problem



# Introduction

# Assumptions



Man marries woman



Preferences don't change



No Equal Preferences

# Setting Up the Stable Marriage Problem



Same number of men and women



Rank people



Marry into stable pairs

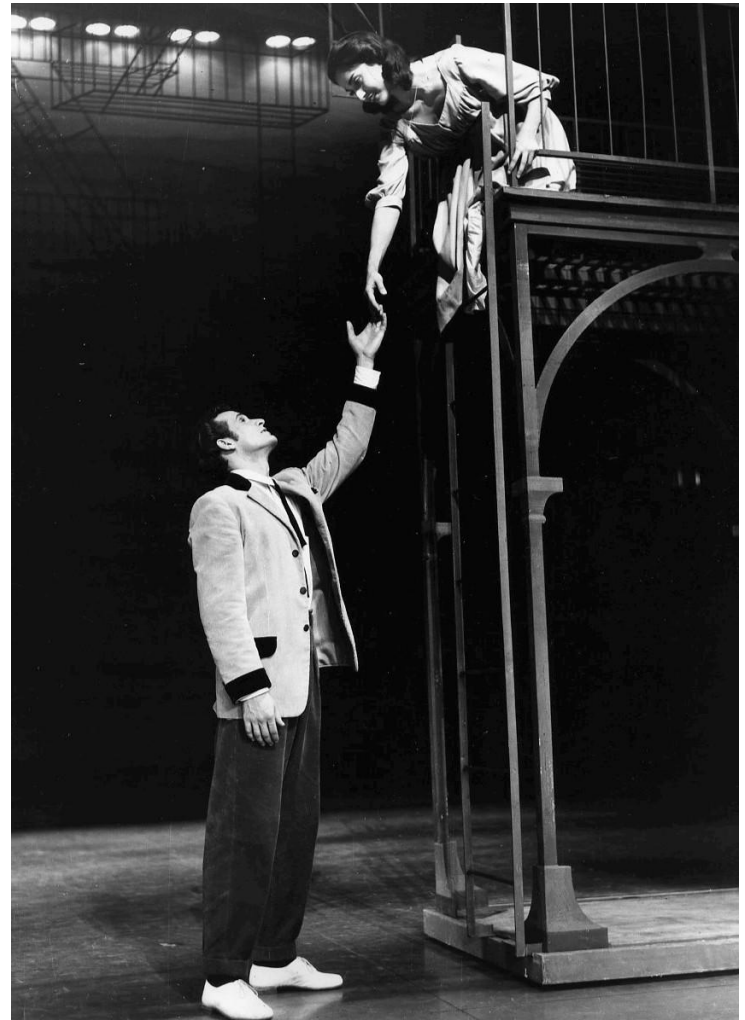


# Rogue Couples: A Forbidden Romance



- Rogue couple = a man and woman are NOT married to each other, but prefer each other over their current partners
- Rogue couples  $\Rightarrow$  unstable matching
- $\Rightarrow$  Stable = no rogue couples

*My only love sprung from my only hate!  
Too early seen unknown, and known too late!  
(Romeo and Juliet, 1.5.138-139)*



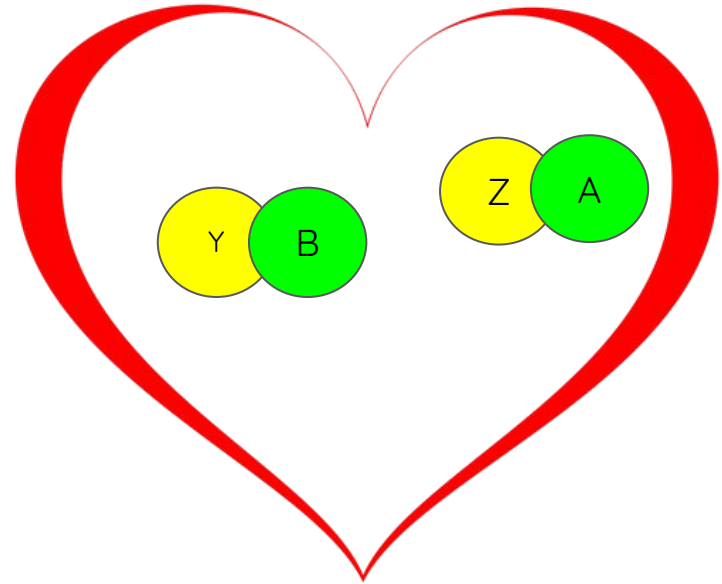
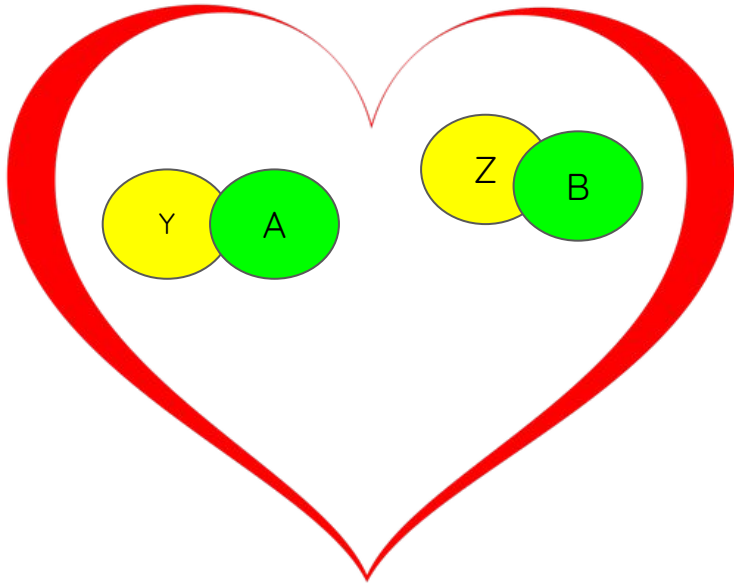
# Example

MEN	1st ↓	2nd ↓
Y →	A	B
Z →	B	A

WOMEN	1st ↓	2nd ↓
A →	Z	Y
B →	Z	Y

# Examples and Non-Examples

MEN	1st	2nd	WOMEN	1st	2nd
Y	A	B	A	Z	Y
Z	B	A	B	Z	Y

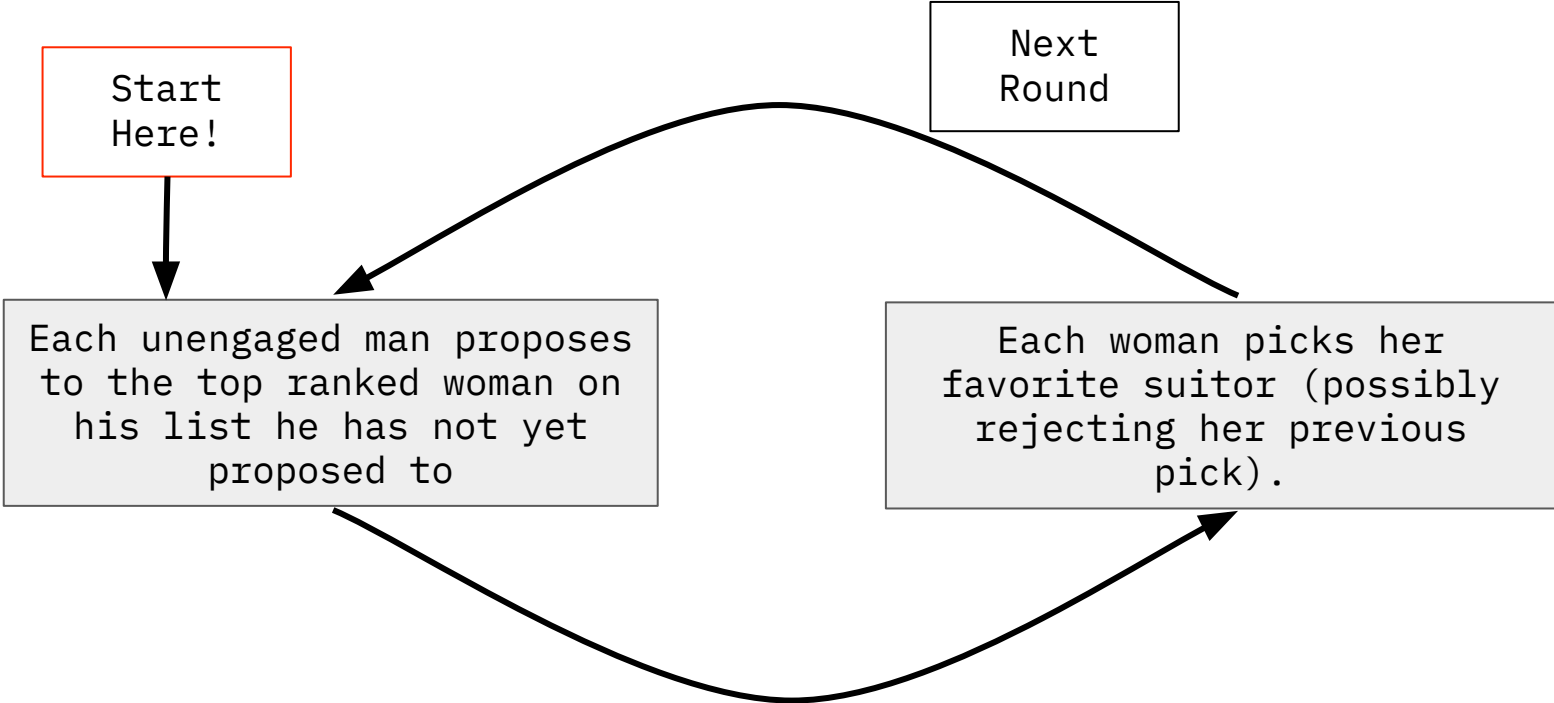




# Gale-Shapley Algorithm

(A systematic method for finding stable marriages)

# How to Create Stable Marriages (Gale-Shapley)



# Example

Men

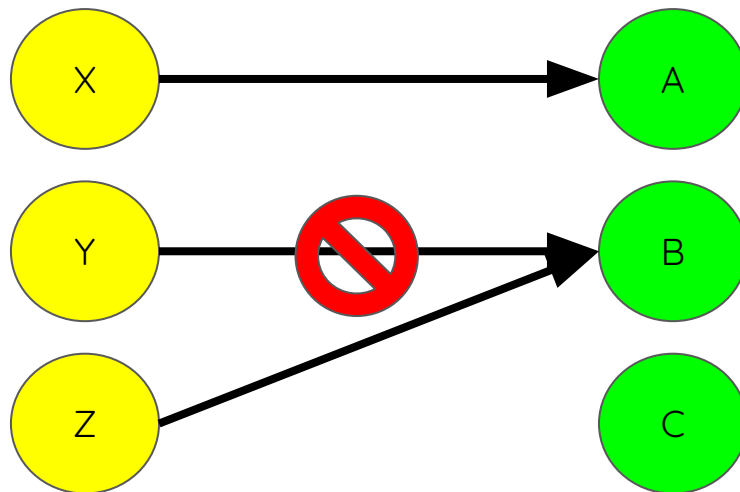
	1	2	3
X	A	B	C
Y	B	C	A
Z	B	A	C

Women

	1	2	3
A	Y	X	Z
B	Z	X	Y
C	X	Z	Y

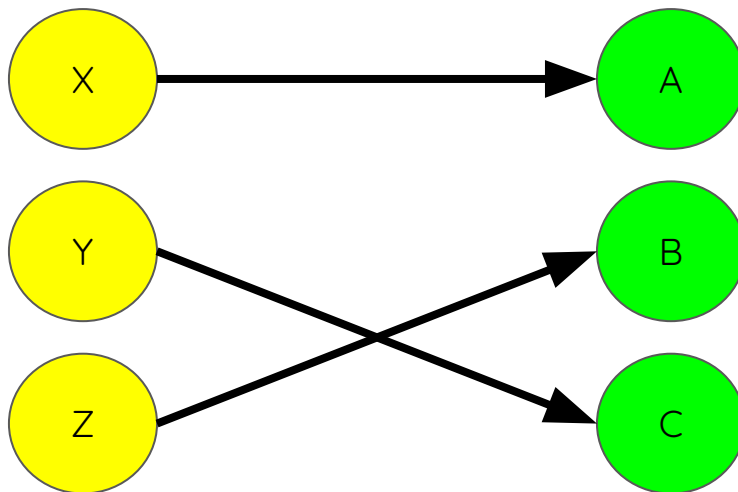
# Round 1

	Men			Women			
X:	A	B	C	A:	Y	X	Z
Y:	B	C	A	B:	Z	X	Y
Z:	B	A	C	C:	X	Z	Y



Round 2

	Men			Women			
X:	A	B	C	A:	Y	X	Z
Y:	B	C	A	B:	Z	X	Y
Z:	B	A	C	C:	X	Z	Y





# Does Gale-Shapley Always End in a Stable Marriage?

Yes!

It was proved 50 years ago, relax and move on.





# Sudoku

# Connection to Sudoku

$$(n!)^{2n}$$

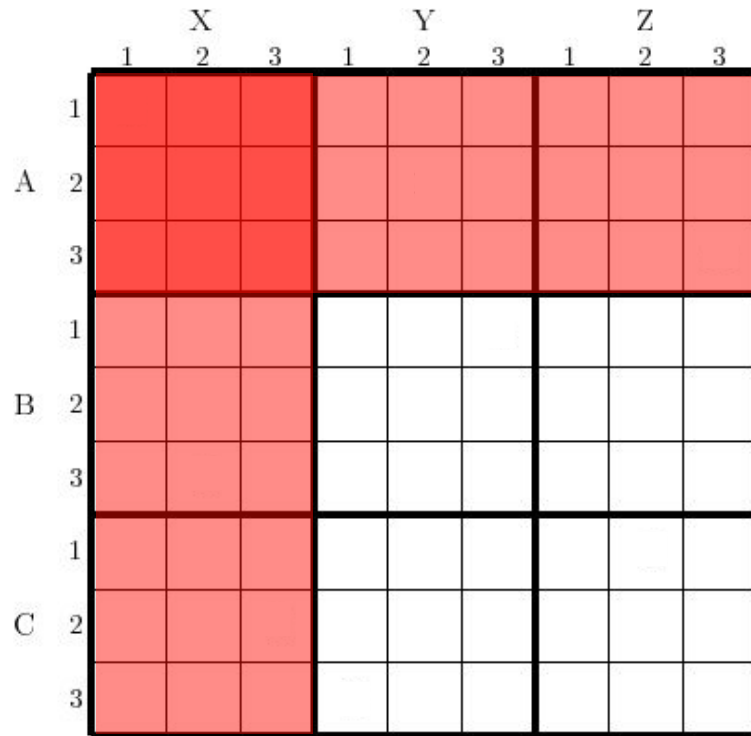
The number of ways of placing a single digit within an  $n^2 \times n^2$  Sudoku so that all rows, columns, and  $n \times n$  blocks have exactly one copy of the digit.

The number of preference profiles with  $n$  men and  $n$  women.

# Setting Up a Sudoku From Preference Profiles

Bands (group of n rows)

	Men			Women			
X:	A	B	C	A:	X	Y	Z
Y:	B	A	B	B:	Y	Z	X
Z:	B	C	A	C:	Z	X	Y





# Interesting Couples (As Seen on Sudoku)

# Rogue Couples

**Definition:** Pair of people who prefer each other to their current partners.

**Step 1:** Highlight the row of the woman's current partner and the column of the man's current partner.

**Step 2:** Find the H in the pair's box.

**Step 3:** To determine if a rogue couple: is the H above the row and to the left of the column?  
 Yes = rogue couple  
 No = not rogue couple

		X			Y			Z		
		1	2	3	1	2	3	1	2	3
A	1						H			
	2	H								
	3								H	
B	1							H		
	2		H							
	3				H					
C	1			H						
	2									H
	3					H				

# Soulmates

**Definition:** Pair of people who rank each other first (always married to each other).

		X			Y			Z		
		1	2	3	1	2	3	1	2	3
A	1						H			
	2	H								
	3								H	

$$\binom{n}{k}^2 \cdot k! \cdot (n-1)!^{2k} \cdot \left( \sum_{i=0}^{n-k} (-1)^i \cdot \binom{n-k}{i}^2 \cdot (n-1)!^{2i} \cdot i! \cdot n!^{2n-2k-2i} \right)$$

C	1			H						
	2									H
	3					H				







# Gale-Shapley in Sudoku

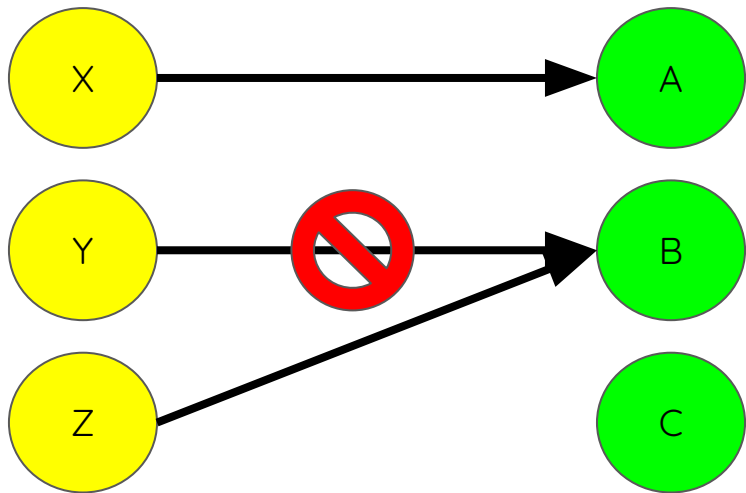
# Gale-Shapley in Sudokus

	Men			Women			
X:	A	B	C	A:	Y	X	Z
Y:	B	C	A	B:	Z	X	Y
Z:	B	A	C	C:	X	Z	Y

		X		Y		Z			
	1	2	3	1	2	3	1	2	3
A	1					H			
	2	H							
	3							H	
B	1						H		
	2		H						
	3				H				
C	1			H					
	2								H
	3					H			

# Round 1

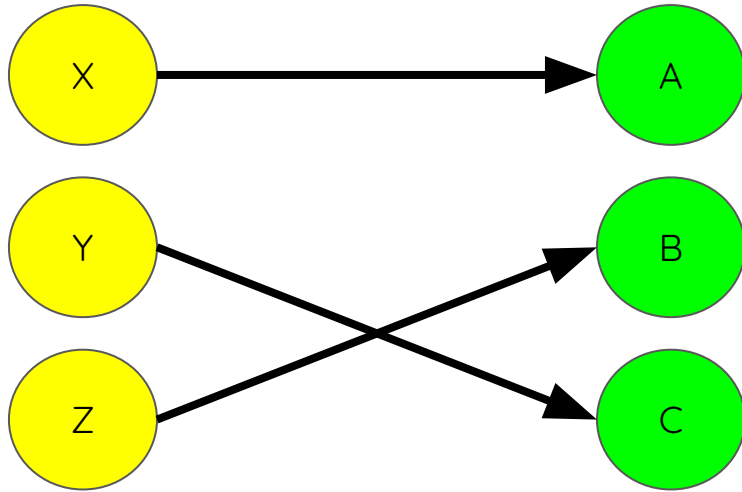
	Men			Women			
X:	A	B	C	A:	Y	X	Z
Y:	B	C	A	B:	Z	X	Y
Z:	B	A	C	C:	X	Z	Y



	X			Y			Z		
	1	2	3	1	2	3	1	2	3
1									
A 2	H								
3								H	
1							H		
B 2		H							
3				H					
1			H						
C 2									H
3					H				

# Round 2

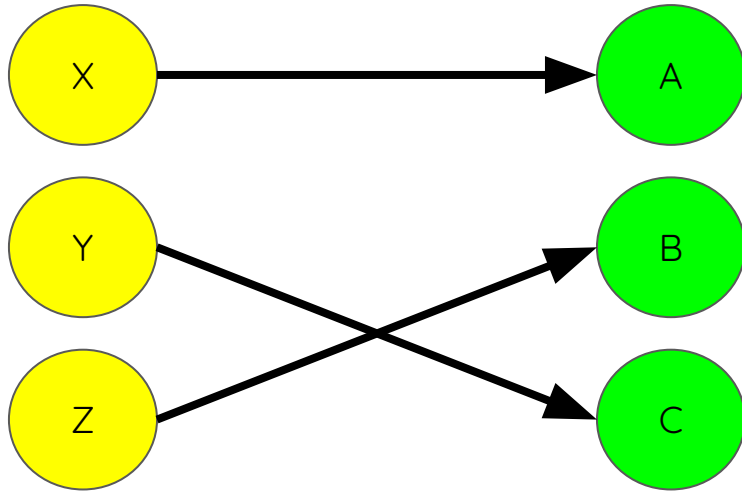
	Men			Women			
X:	A	B	C	A:	Y	X	Z
Y:	B	C	A	B:	Z	X	Y
Z:	B	A	C	C:	X	Z	Y



	X			Y			Z		
	1	2	3	1	2	3	1	2	3
A	1					H			
	2	H							
	3							H	
B	1						H		
	2		H						
	3			H					
C	1		H						
	2								H
	3				H				

# Final Form

	Men			Women			
X:	A	B	C	A:	Y	X	Z
Y:	B	C	A	B:	Z	X	Y
Z:	B	A	C	C:	X	Z	Y



		X			Y			Z		
		1	2	3	1	2	3	1	2	3
A	1						H			
	2	M								
	3								H	
B	1							M		
	2		H							
	3				H					
C	1			H						
	2									H
	3					M				

**IN CONCLUSION**

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RIP Arkady Yurievich Zorkin-Breloshkin who was  
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unpronounceable name  
Shout-out to Yavor who replaced him at the last minute



# References

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