

# Towards Practical Ambiguity Sets

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Benjamin Chen, mentored by Kyle Hogan

October 2020

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# Introduction

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This seems okay, right? After all, Eve gets almost no info about what Alice is actually doing.

However, remember Eve uses Reddit too...







# Alice and Eve

↑  **r/AskReddit** · Posted by [u/btw\\_i\\_use\\_arch](#) 9 hours ago   6  2  2

12.6k  
↓


## Is cereal a soup?

2.6k Comments  Share  Save  Hide  Report 98% Upvoted

Log in or sign up to leave a comment [LOG IN](#) [SIGN UP](#)

SORT BY **Best** ▾

[View discussions in 3 other communities](#)

↑ [PhysicsIsPhun](#) 4.8k points · 4 hours ago 

↓ Yes!

↑ [EpicGamer6612](#) 1.6k points · 4 hours ago

↓ No.

↑ [PhysicsIsPhun](#) 402 points · 50 minutes ago

↓ Yes.

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↓ No. Why would you say that?

↑ [PhysicsIsPhun](#) 125 points · 44 minutes ago

↓ Because it's the truth.

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Alice's traffic

251 min ago

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# Alice and Eve

The screenshot shows a Reddit post from the subreddit r/AskReddit. The post title is "Is cereal a soup?" and it has 12.6k upvotes. The post was made by user u/btw\_i\_use\_arch 9 hours ago. Below the title are options to comment, share, save, hide, and report, along with a "98% Upvoted" indicator. A login/sign-up prompt is visible. The comment section shows a thread of replies:

- PhysicsIsPhun (4.8k points, 4 hours ago) says "Yes!".
- EpicGamer6612 (1.6k points, 4 hours ago) replies "No.".
- PhysicsIsPhun (402 points, <1 hour ago) replies "Yes.".
- EpicGamer6612 (272 points, <1 hour ago) replies "No. Why would you say that?".
- PhysicsIsPhun (125 points, <1 hour ago) replies "Because it's the truth.".

Alice's traffic
240 min ago
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0 min ago

Alice's fake messages are called **dummy messages**.

## Alice, Bob, and Eve

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- Alice can up her dummy traffic to make her look like Bob (lots of overhead)
- Alice can give up on looking like Bob and just post enough dummies to look like PhysicsIsPhun.

We would say EpicGamer6612 (Alice) and PhysicsIsPhun are in an **ambiguity set**, since Eve can't determine which of the two Alice is.

## More Realistic Examples

“Eve” could be...

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Basically any adversary who can see the users' activity, but not the contents of incoming or outgoing traffic (hidden with encryption).

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- How do we group people to look the same in a good way (and what does “good” entail)?
- How do we pick the budget for a group of people?
- Is such a system practical in real life?

## **Our Project**

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# Ambiguity Sets

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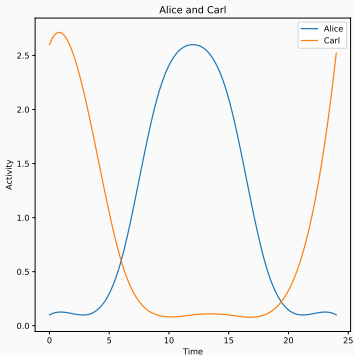
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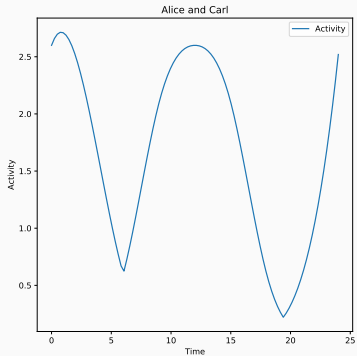
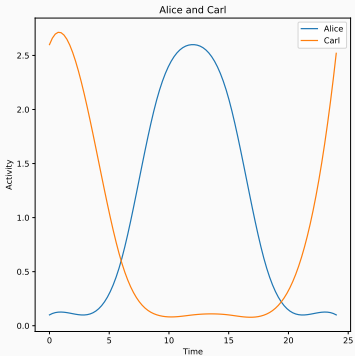
- Users are placed into ambiguity sets of size at least  $k$ , for some integer  $k$ .
- Each user in the set looks identical to every other user in the set from the adversary's point of view.
- We try to create sets to find a balance between performance and privacy.

# Alice and Carl

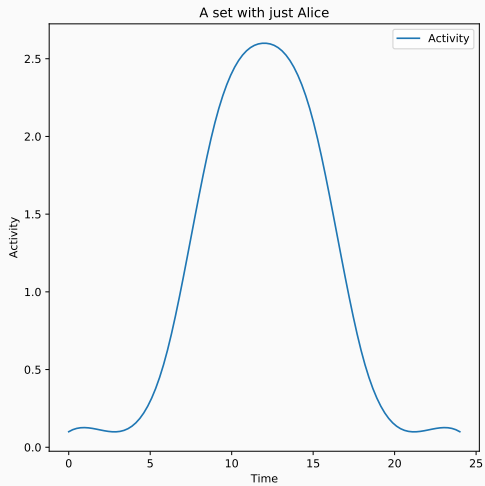




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# The Perfect Ambiguity Sets

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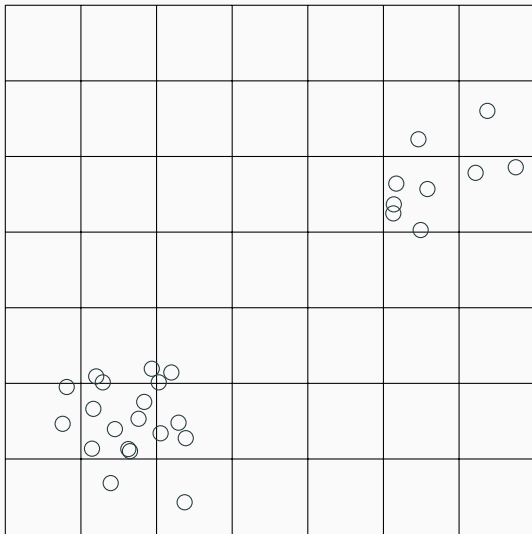
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Achieving both of these at the same time is hard.

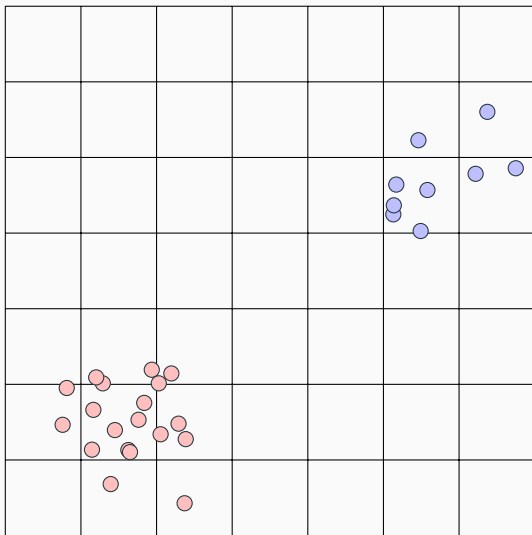
## Creating Ambiguity Sets

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# K-Means

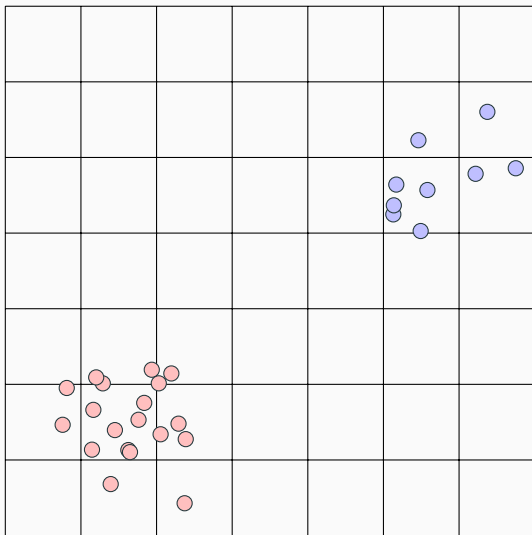


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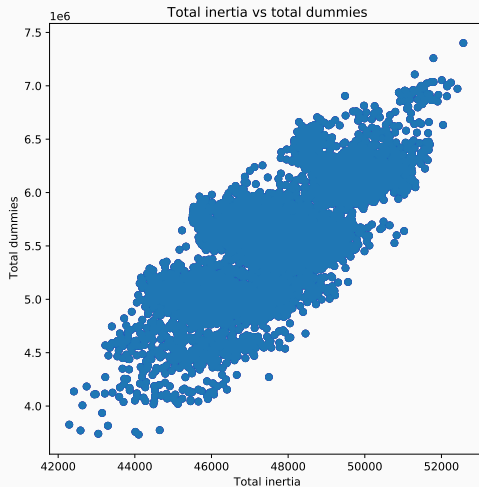


## K-Means



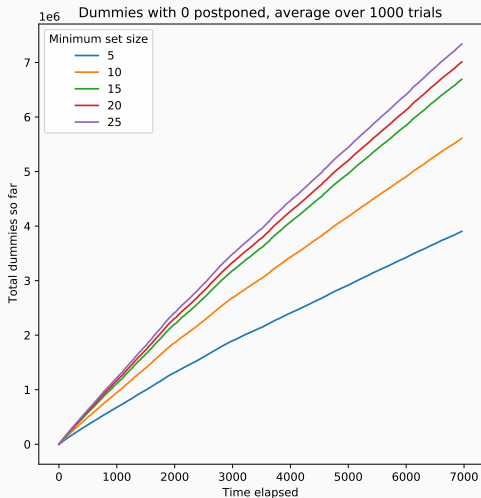
K-means attempts to minimize the **inertia** of each cluster.

# Inertia as an Indicator of Performance



Each dot here represents 1 clustering setup (with a different random seed)

# Cluster Sizes



(On a dataset of 100 users)

## Picking a Budget

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# The Budget

Once the ambiguity sets are created, we define a budget (how much traffic people should send) based on the mean activity over users in the set.

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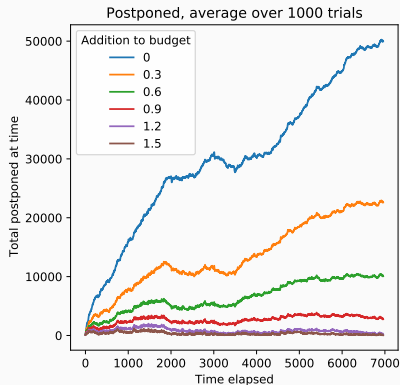
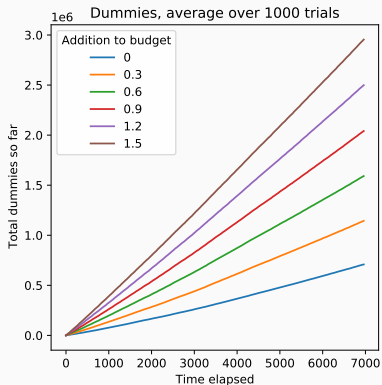
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In general, we care more about reducing postponed messages over reducing dummy messages.



# The Solution

$$\text{budget} = \text{mean} \cdot (1 + \text{addition to budget})$$



- Testing this on bigger datasets

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- Looking more closely at the people who make up the sets here

# Acknowledgements

- Kyle Hogan
- Dr. Gerovitch & Prof. Devadas
- PRIMES Program & MIT

Thanks for listening!