WE ARE THE CHOMPIANS!!

Benjamin Chen; Neil Malur; Anastasia Polina; Michael Gerovitch; Ezra Erives; Poonam Sahoo; Hyunjun Ahn; Jeremy Fleming; Richard Chen; Tejas Gopalakrishna

Mentor: Tanya Khovanova

The Games

The Games

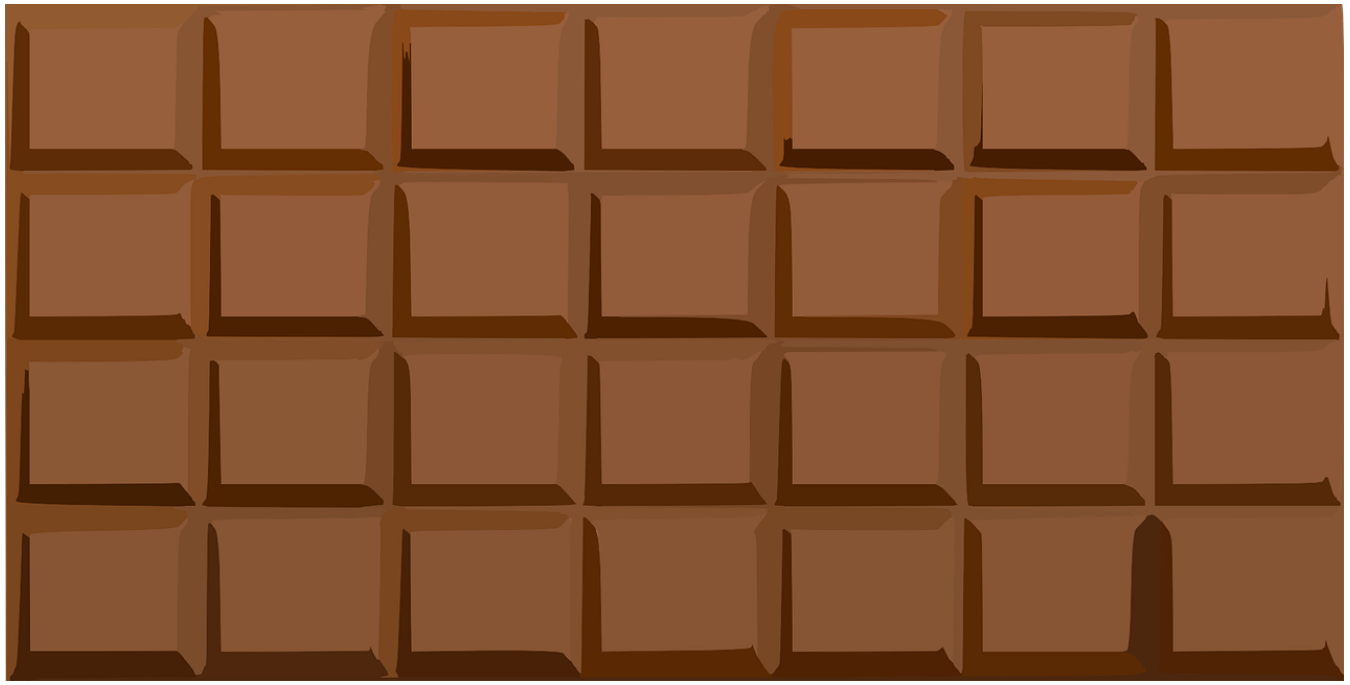
CHOMP

The Games

CHOMP

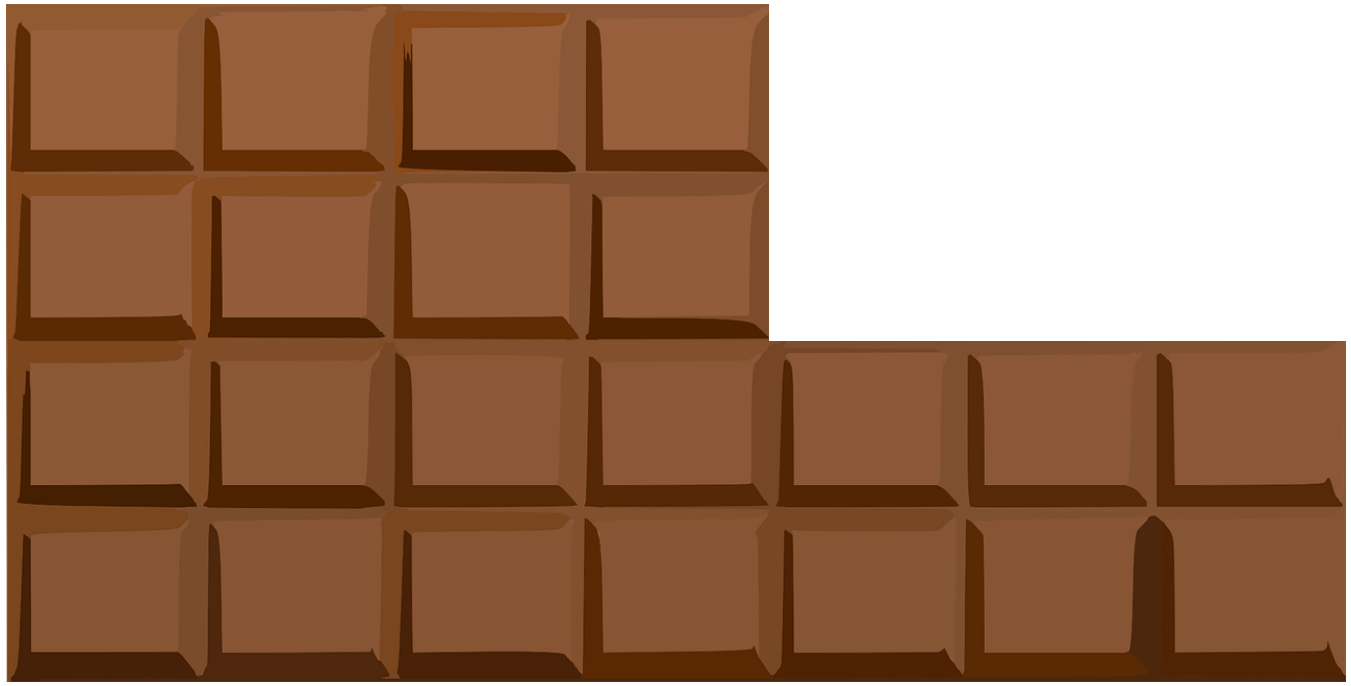
NIM

CHOMP: Misére Play Rules



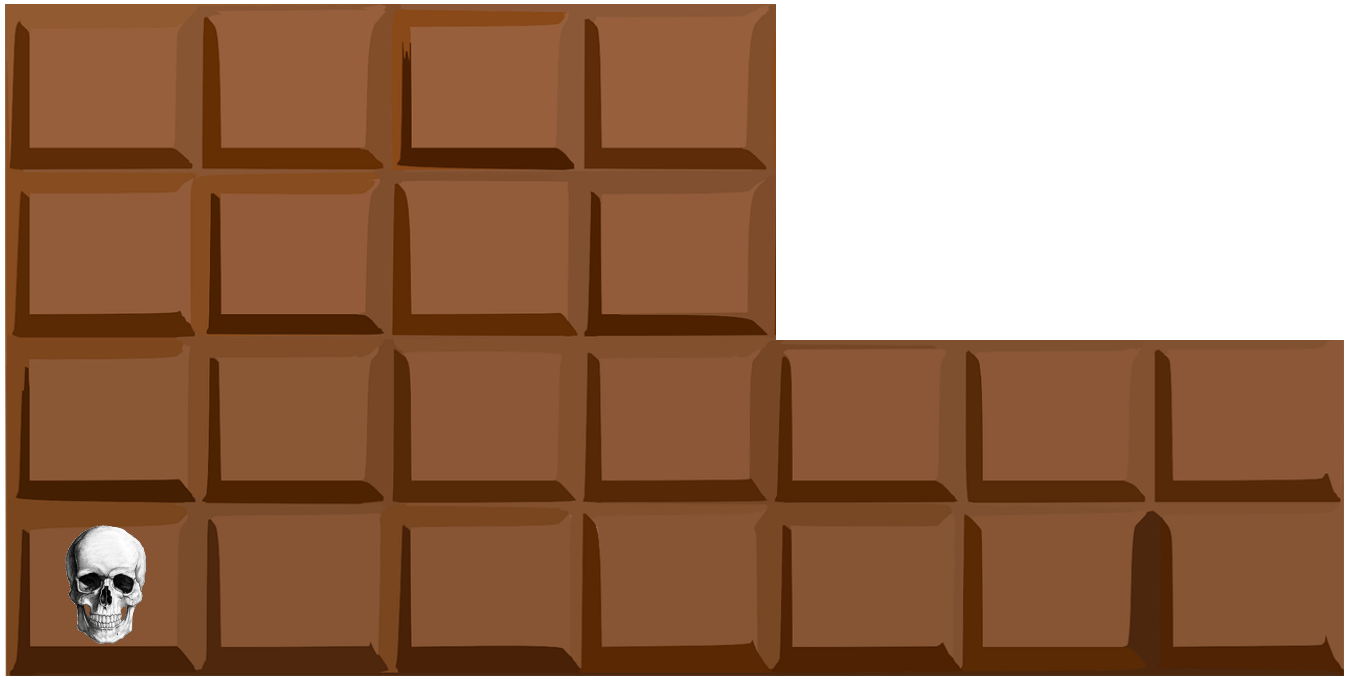
The game of chomp is played on a rectangular m by n chocolate bar with grid lines dividing the bar into mn squares.

CHOMP: Misére Play Rules



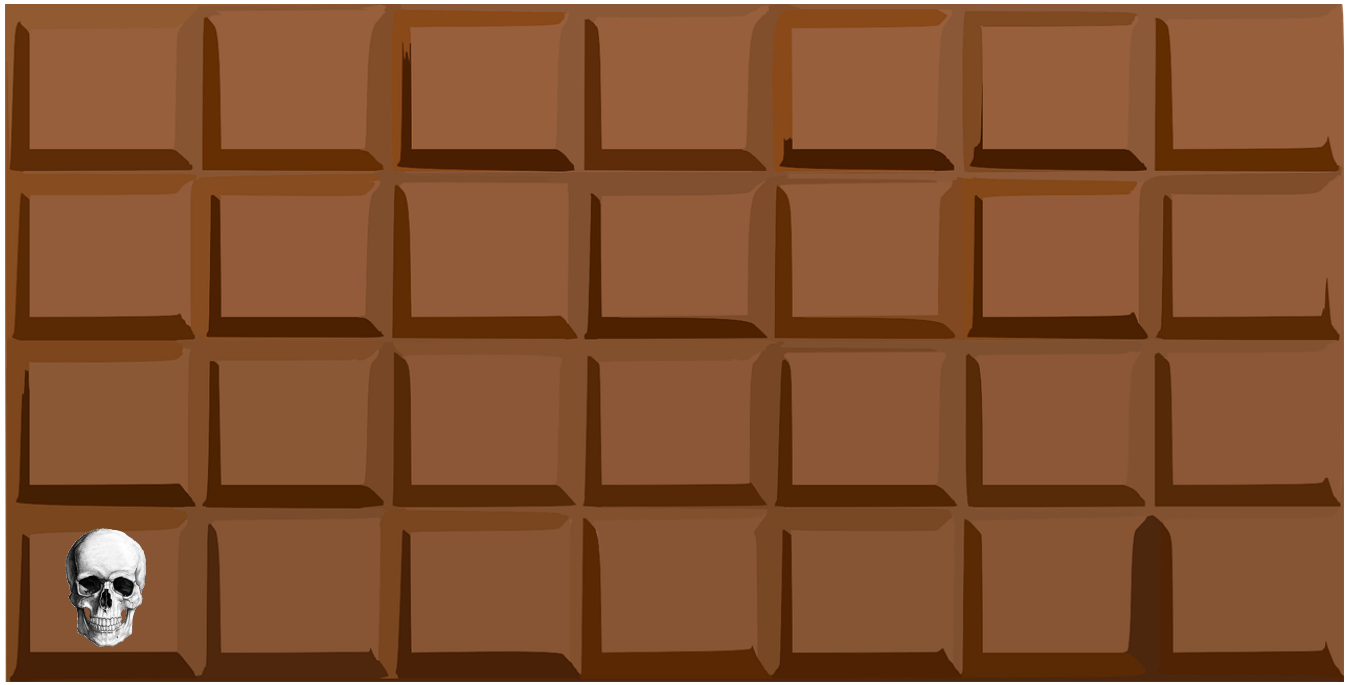
A move consists of chomping a square out of the chocolate bar along with any squares to the right and above.

CHOMP: Misére Play Rules



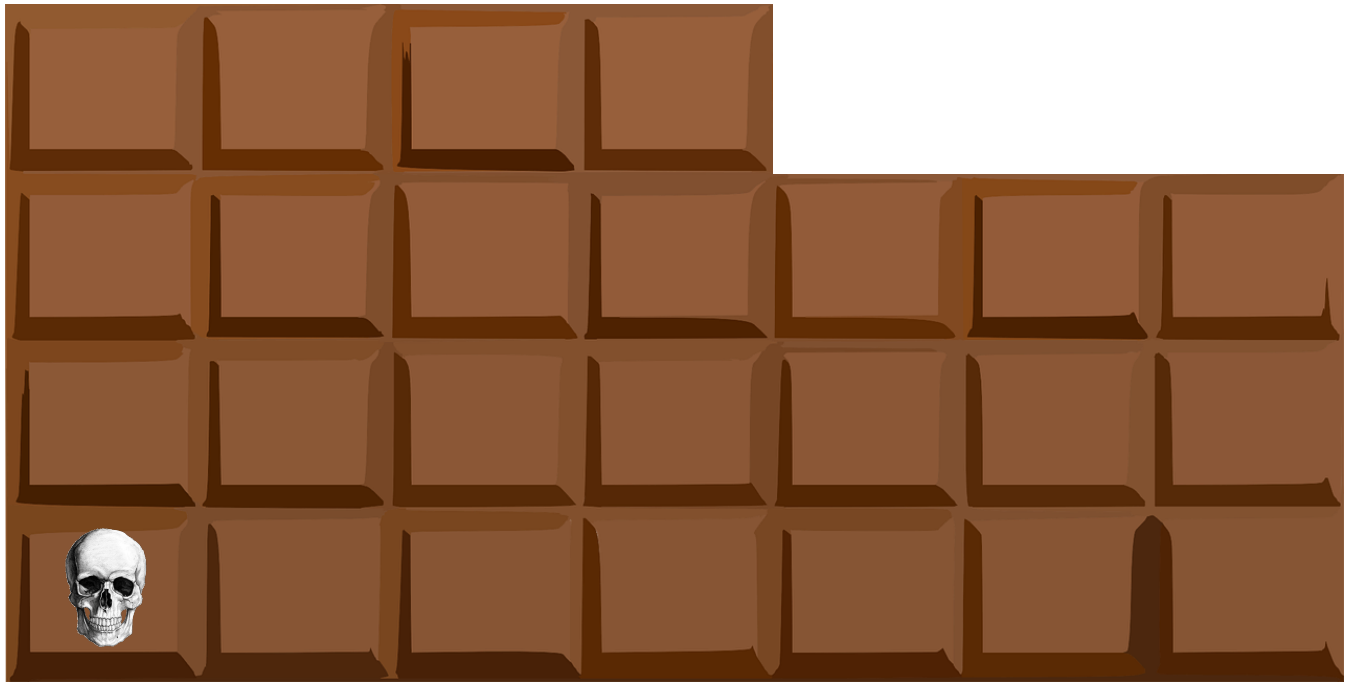
The player eats the chomped squares. Players alternate moves. The lower left square is poisonous and the player forced to eat it dies and loses.

Chomp Misére Play Example Game



Let’s play CHOMP!!!!

Chomp Misére Play Example Game



Player 1 eats three squares.

Chomp Misére Play Example Game

Player 2 Chomps off most of the squares!!!

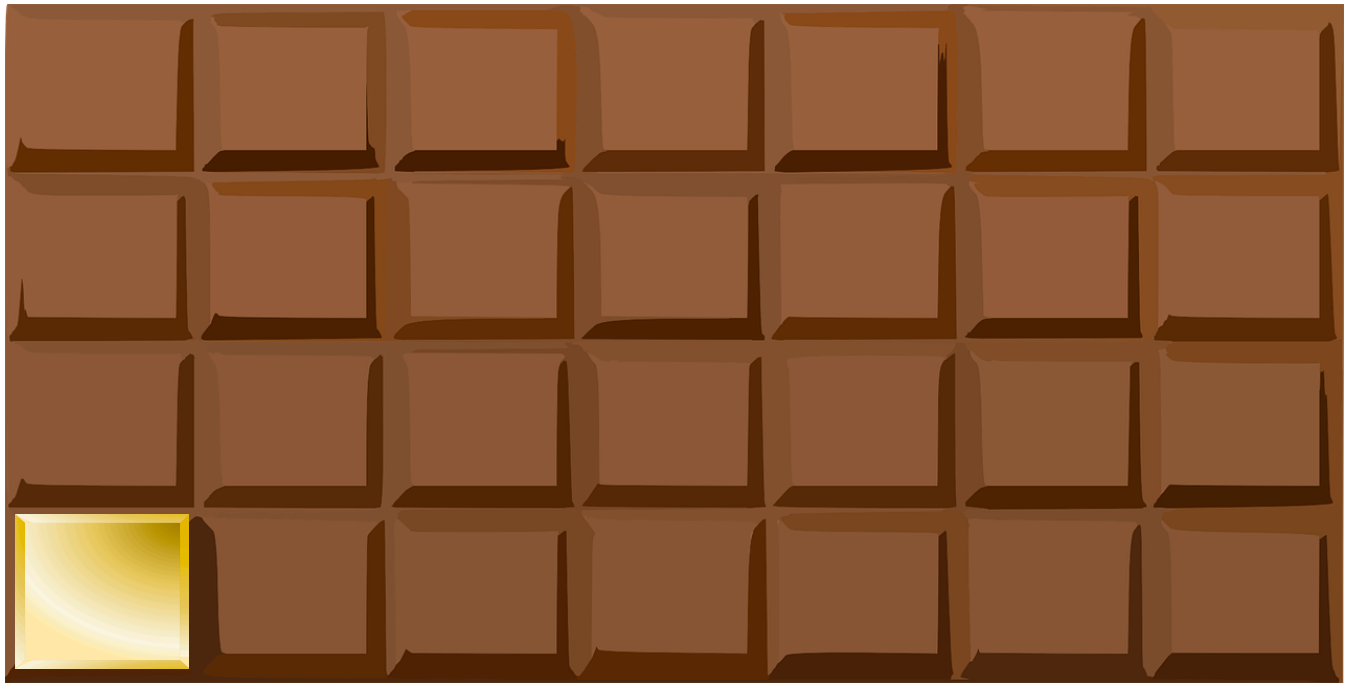
Chomp Misére Play Example Game

Player 1 chomps off three more squares.

Chomp Misére Play Example Game

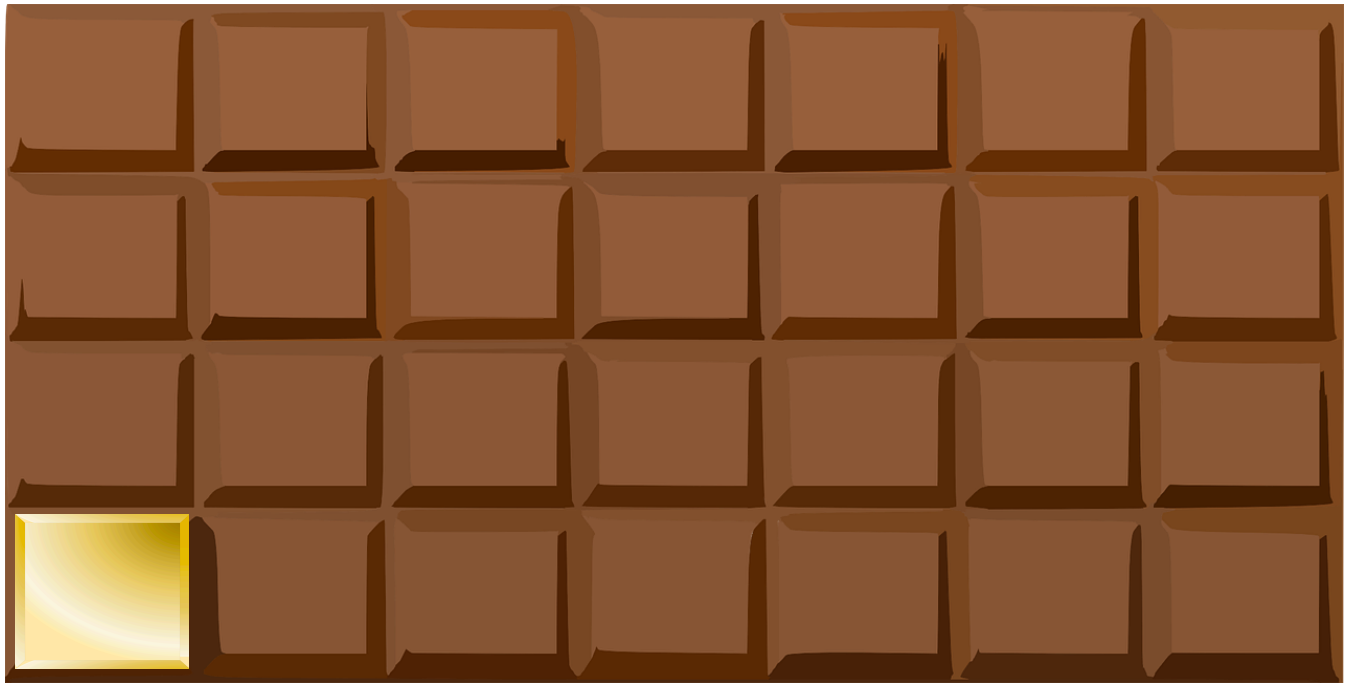
Player 2 leaves Player 1 with nothing but a poisonous square!!!! Player 1 dies and loses.

Normal Play Chomp



Normal Play Chomp is exactly like Misére Play, except that the lower left square is golden, and the player who eats it wins.

Normal Play Chomp

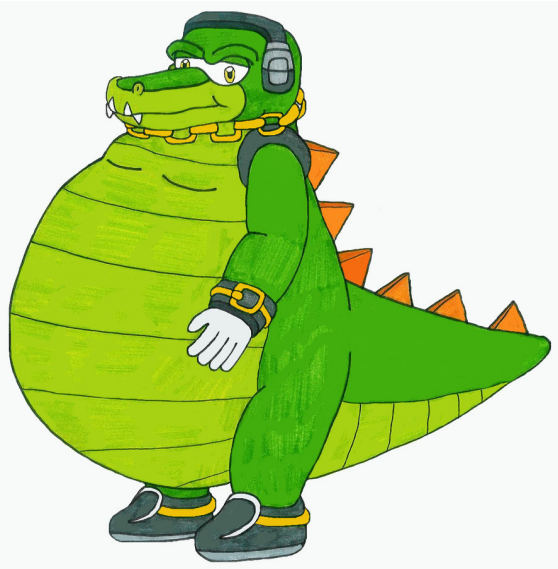


This game is trivial, we can just eat the whole bar.

The Effects of Chomp on the Crocodile

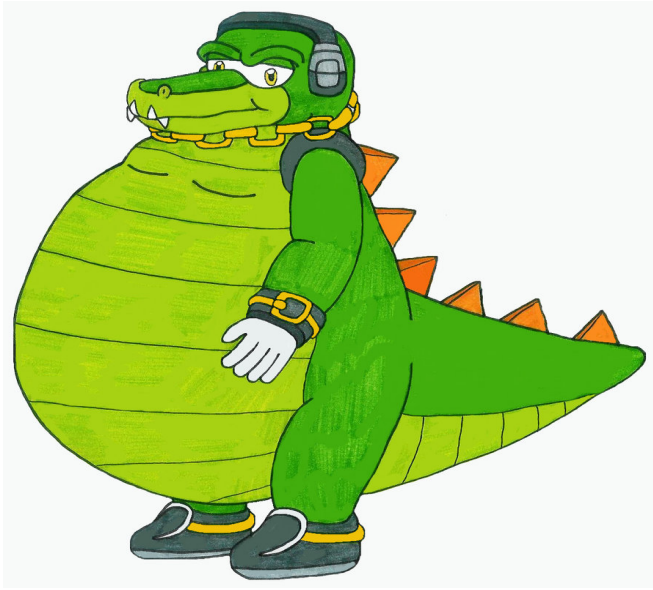
BEFORE GAME

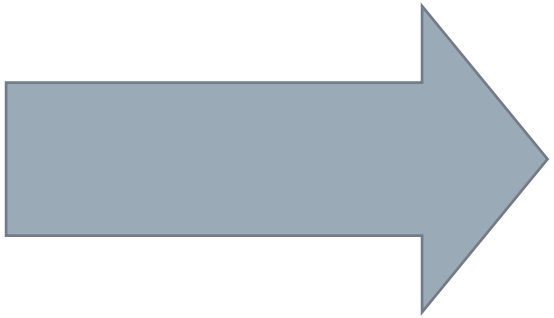
AFTER GAME

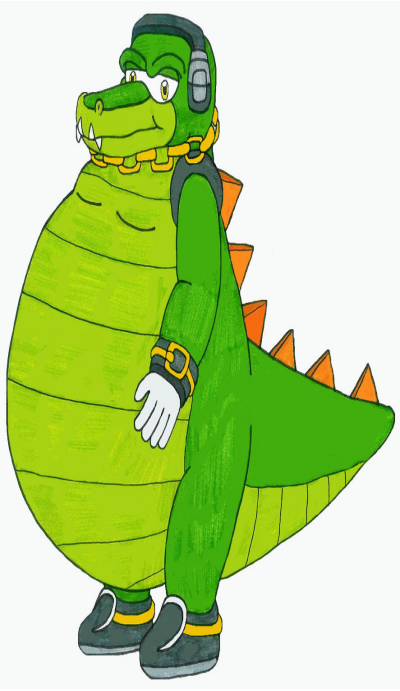


Chomp makes our crocodiles obese, and it makes them VERY unhealthy.

Diet Chomp

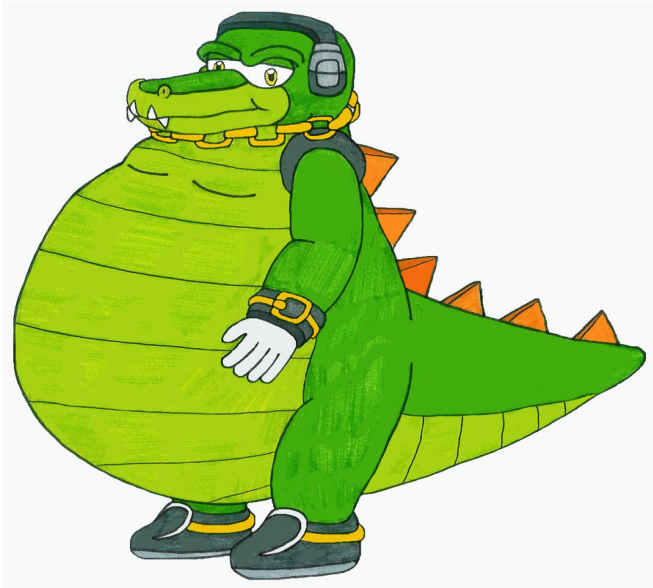


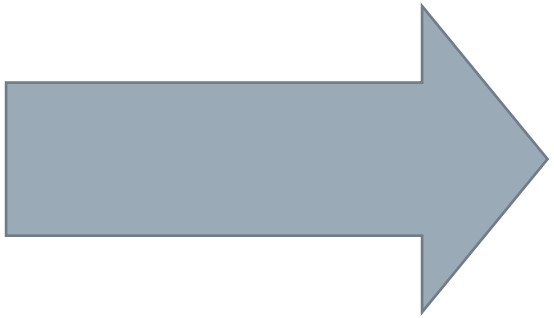


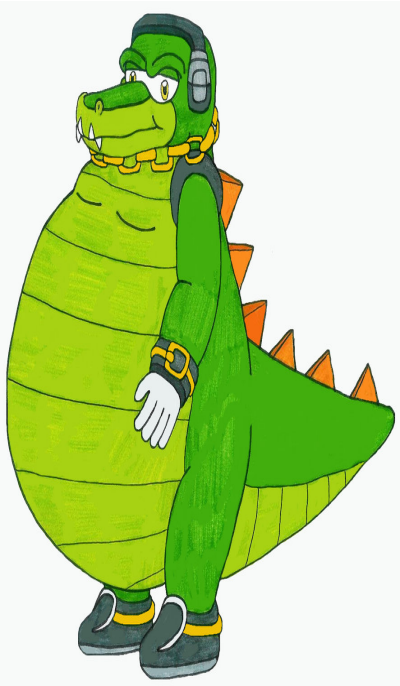


With the revolutionary DIET CHOMP, our crocodiles are getting more fit and healthier than ever before.

Diet Chomp







Here, you are limited to taking either one or two pieces of chocolate.

Types of Positions

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- P-Position: This position is good for the previous player. All moves from a P-position lead to an N-position.

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- P-Position: This position is good for the previous player. All moves from a P-position lead to an N-position.
- N-position: This position is good for the next player. There exists a move from an N-position to a P-position.

Diet Chomp Results – Normal Play

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- In normal play, we have a proof that a position is a P-position if the number of chocolate squares is divisible by 3.

Diet Chomp Results – Normal Play: Proof

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- All positions with the number of squares divisible by three cannot move to another position with the number of squares divisible by three.

Diet Chomp Results – Normal Play: Proof

- All positions with the number of squares divisible by three cannot move to another position with the number of squares divisible by three.

- Therefore, a P-position cannot move to another P-position. This satisfies our rule that a P-position always moves to an N-position.

Diet Chomp Results – Normal Play: Proof

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- If the number of squares is not divisible by three, and if we can remove up to two squares, we can move to a position with the number of squares divisible by three.

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- If the number of squares is not divisible by three, and if we can remove up to two squares, we can move to a position with the number of squares divisible by three.
- So, such N-positions have a move to a P-position.

Diet Chomp Results – Normal Play: Proof

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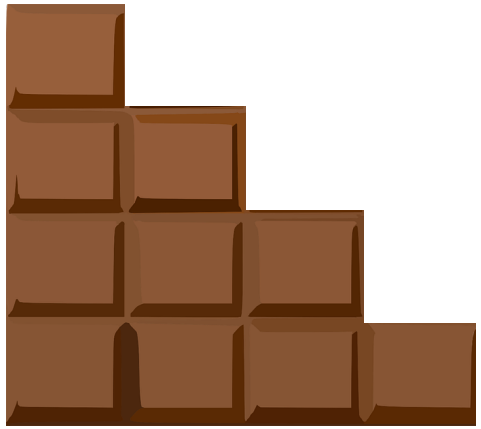
- Sometimes, we can't remove two squares. This occurs in "Perfect stair" positions, like this one:



Diet Chomp Results – Normal Play: Proof

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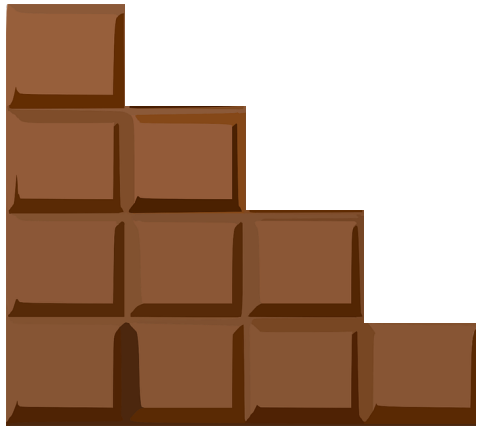

- However, here the number of squares is a triangular number. It is widely known that triangular numbers have a remainder of 0 or 1 when divided by three. So, we can remove one square to get a P-position!

Diet Chomp Results – Normal Play: Proof

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- The base case is the empty position. It is divisible by three, and it is a P-position.

Diet Chomp Results – Normal Play: Proof

- The base case is the empty position. It is divisible by three, and it is a P-position.
- This completes the proof that the positions with the number of squares divisible by three are P-positions.

Three-Row Diet Chomp Misére Play Pictures

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- Orange is a P-Position, blue is an N-position.

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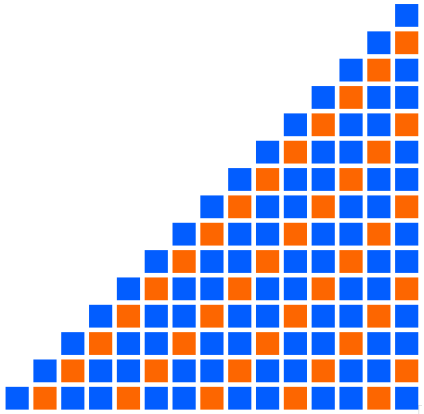
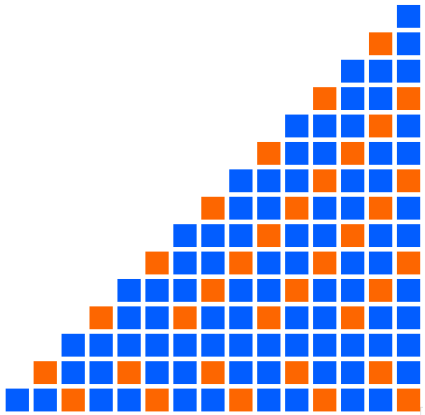
- The x and y coordinates are adjusted, the bottom left corner is (a,a,a) in the a^{th} picture.
- Orange is a P-Position, blue is an N-position.
- The pictures have period 12.

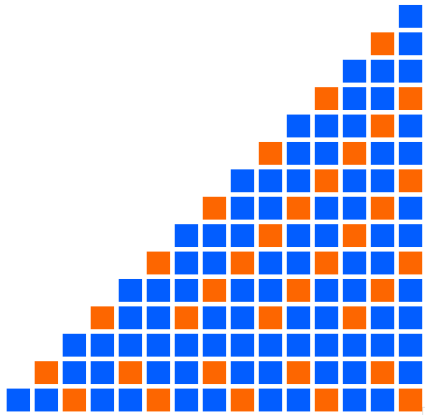
(0,a,b) - (5, a, b)



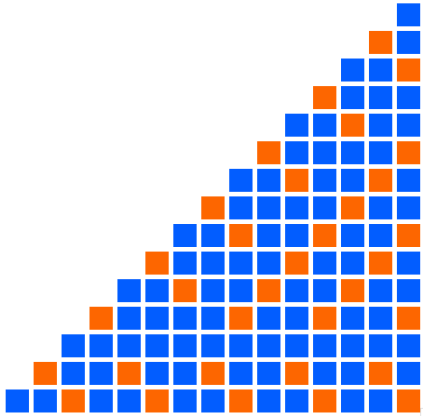
(0, a, b)



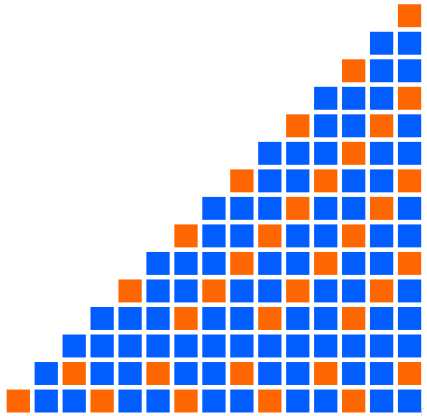
(1, a, b)



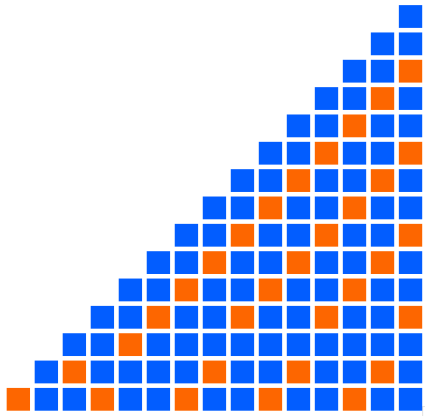
(2, a, b)



(3, a, b)

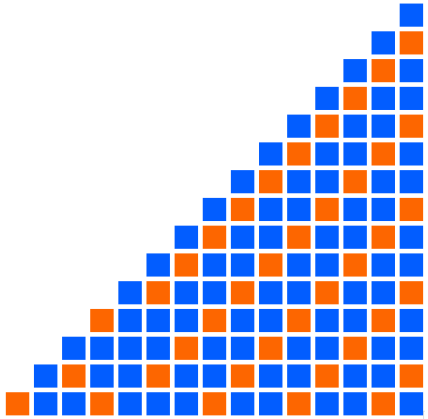


(4, a, b)

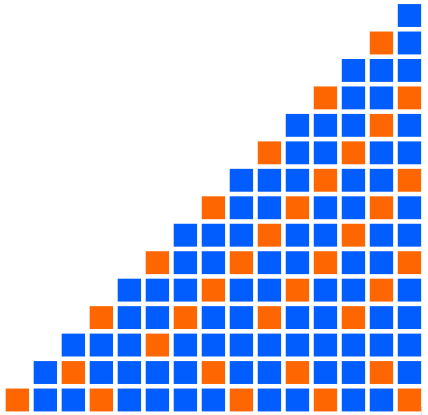


(5, a, b)

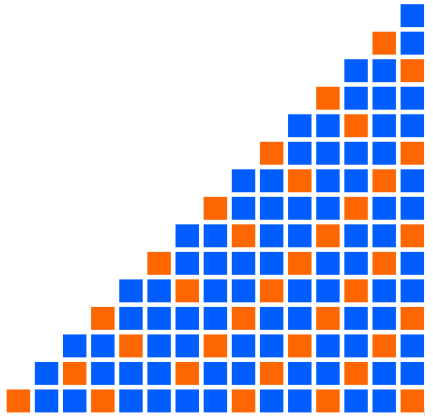
(6,a,b) - (11, a, b)



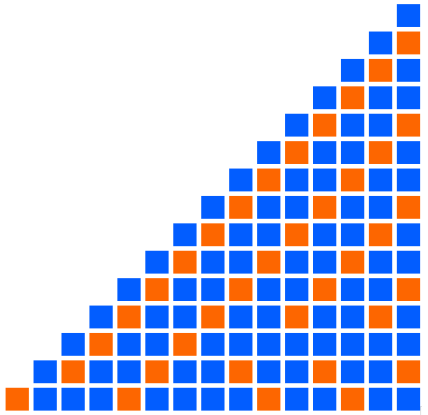
(6, a, b)



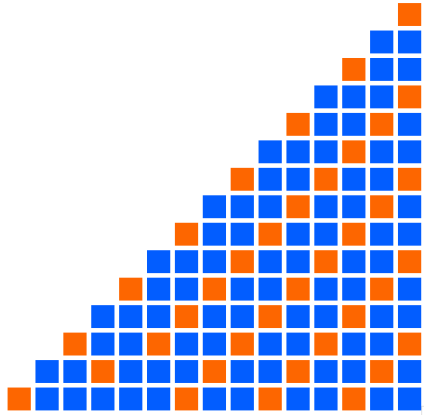
(7, a, b)



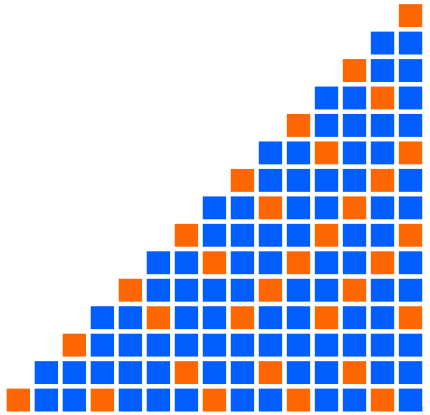
(8, a, b)



(9, a, b)

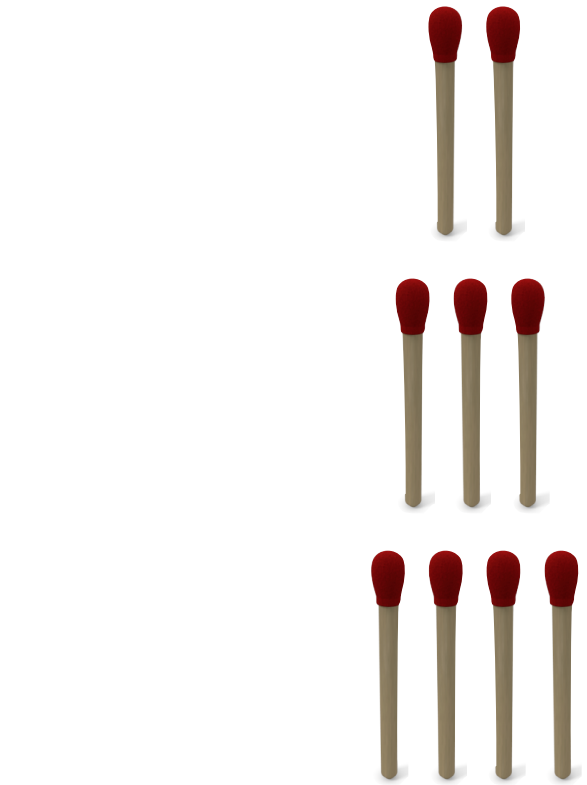


(10, a, b)



(11, a, b)

NIM: Rules



NORMAL PLAY: In a turn, you can take any amount of matches from one row. Players alternate turns. The player without a turn loses.

NIM: Rules

Misere play rules are the same, except that now the player without a turn wins.

SLOW NIM: Rules

Slow Nim is the same as Nim, except that you can only take one or two matches.

EXTENDED NIM: Rules

Extended Nim is the same as Nim, but you can put back matches. To prevent it from becoming an infinite game, we put a limit on the number of matches that can be put back.

EXTENDED NIM: Rules

Slow Extended Nim is the same, but it has an additional rule: You can only take/put back up to two matches.

MONOTONIC NIM: Rules

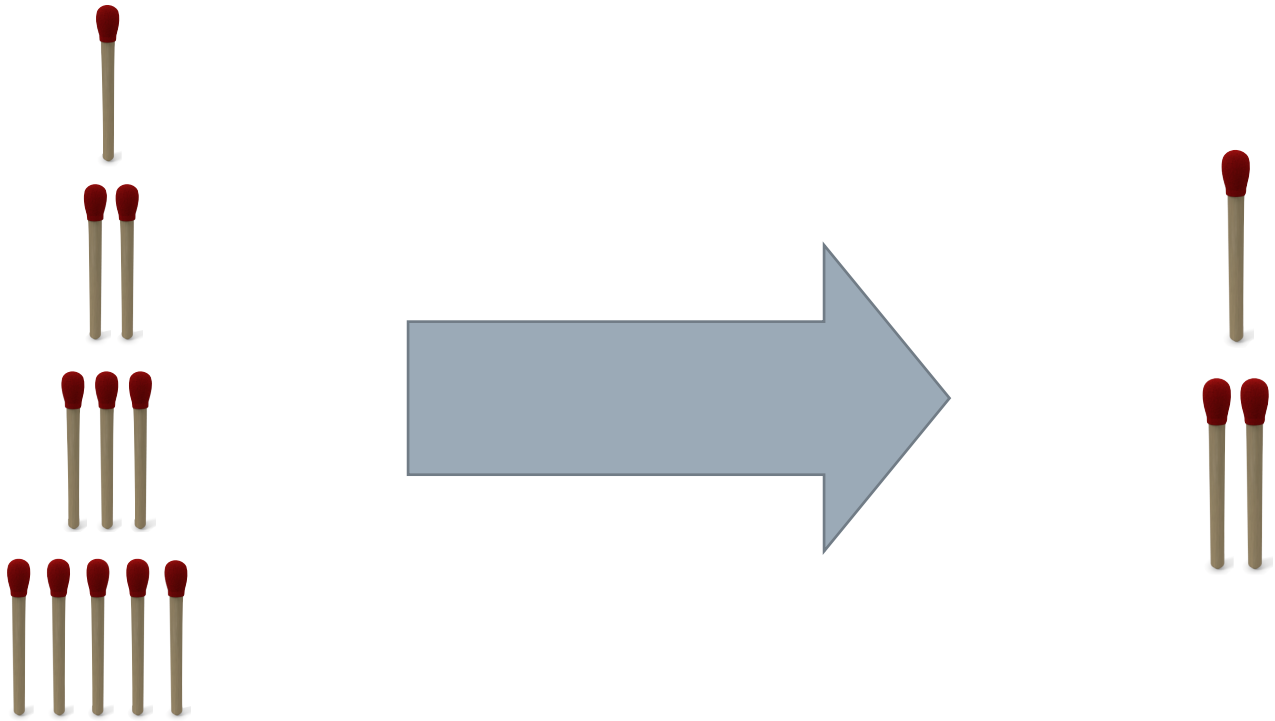
In Monotonic Nim, we can play just like regular Nim, except that the position has to be “Monotonic”. This means that the rows have to be non-decreasing.

MONOTONIC NIM: Rules

Slow Monotonic Nim is the same, but it has an additional rule: You can only take up to two matches.

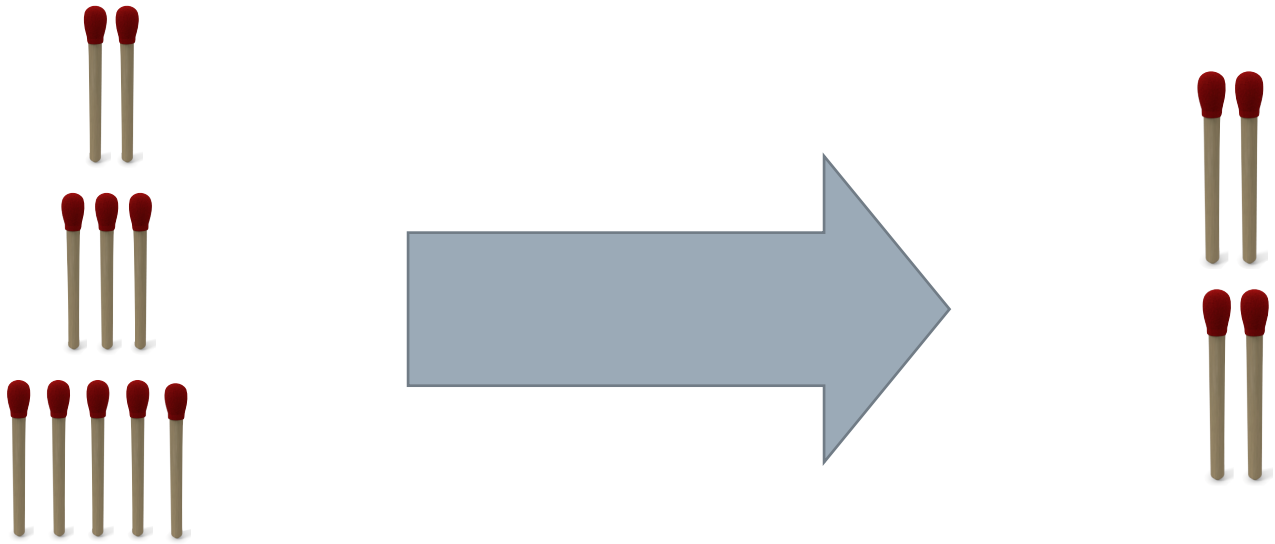
Difference Positions

A difference position is the differences of the rows of a Nim Position.



Difference Positions

If there is an odd amount of rows, add an empty row on the top to find the difference position.



Nim Results

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- The Extended games have the same P-positions as their non-extended counterparts.

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- A Monotonic Nim position is a P-position if and only if the difference position is a P-position in Nim.

Acknowledgements

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Special Thanks to:

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THANK YOU FOR WATCHING

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Mentor: Tanya Khovanova