Continuum Modelling of Traffic Systems with Autonomous Vehicles

Kaiying Hou, Brian Rhee
Mentor: Andrew Rzeznik

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Traffic Flow

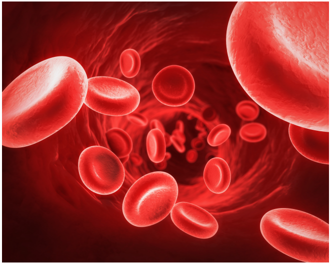
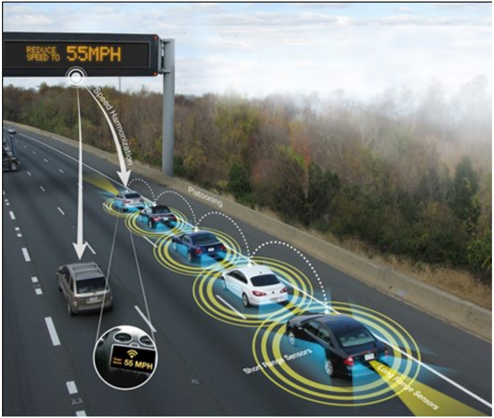


Figure: Red Blood Cells

Figure: Traffic Flow

Autonomous Vehicle



Continuum Variables

$\rho(x, t)$: Density defined as the numbers of cars per unit length.

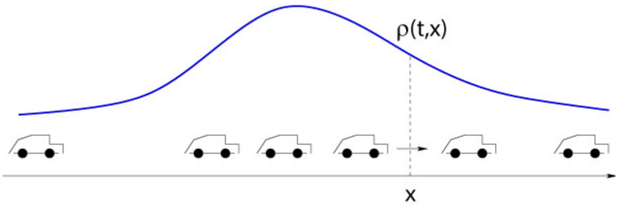
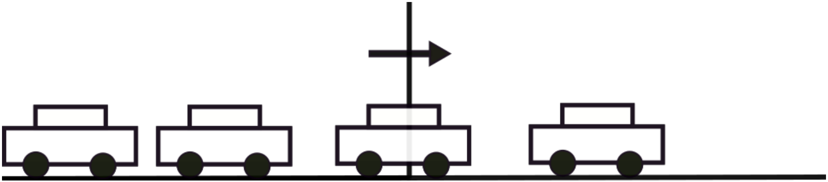


Figure: The density function from a PDE. Here, the ρ function is the density of cars

Flux

$J(x,t)$: Flux defined as the amount of car that pass through x per unit time.

$$J = \frac{cars}{time} = \frac{cars}{length}\frac{length}{time} = \rho v$$

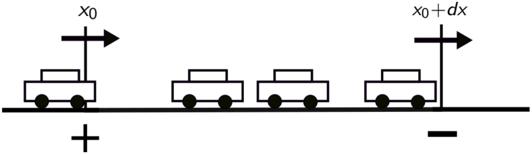


Conservation Equation

$\rho_t + J_x = 0$

Integral Form:

$$\frac{d}{dt}\int_{x_0}^{x_0+dx} \rho(x,t)dx = J(x_0,t) - J(x_0+dx,t)$$

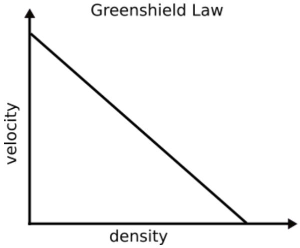


Constitutive Laws

Let $v = v(\rho)$

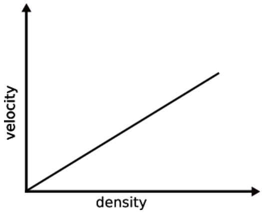
- Greenshield's Law

$$v = v_m(1 - \frac{\rho}{\rho_m})$$
$$J = v_m(1 - \frac{\rho}{\rho_m})\rho$$



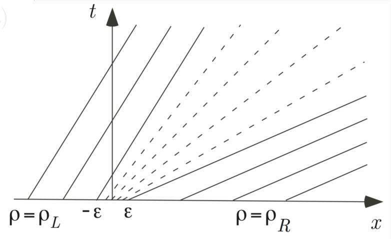
- Burger's Equation

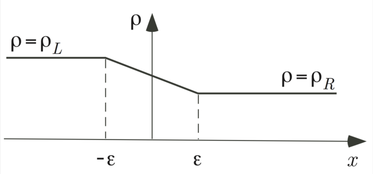
$$v(\rho) = \frac{1}{2}\rho$$
$$J(\rho) = \frac{1}{2}\rho^2$$

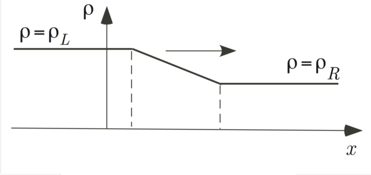


Solving Conservation Equations

$$\rho_t + J'(\rho)\rho_x = 0$$
$$\frac{dx}{dt} = J'(\rho)$$
$$x = x_0 + J'(\rho)t$$

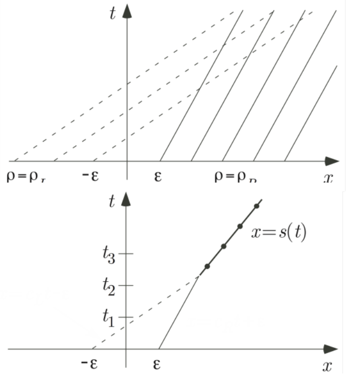


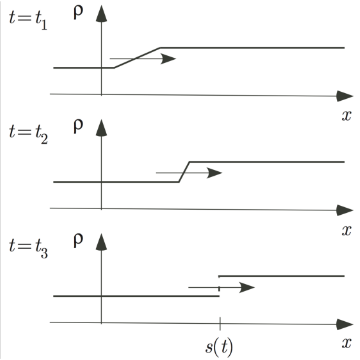




Shockwave

$$s'(t) = \frac{J(\rho_R) - J(\rho_L)}{\rho_R - \rho_L}$$

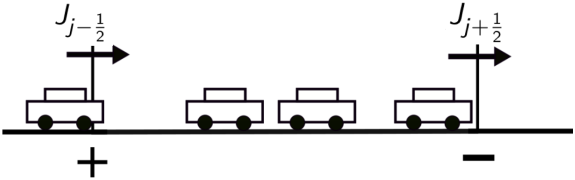




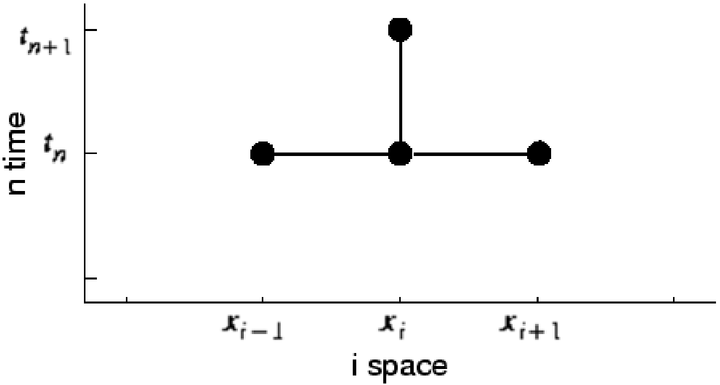
Finite Volume Method

Let function $f(x,t)$ at $x = x_0$ and $t = t_0$ be written as $f_{x_0}^{t_0}$

$$\frac{d}{dt}\int_{x_0}^{x_0+dx}\rho(x,t)dx = J(x_0,t) - J(x_0+dx,t)$$
$$\frac{\Delta x}{\Delta t}(\overline{\rho}_x^{t+1} - \overline{\rho}_x^{t}) = J_{x-\frac{1}{2}} - J_{x+\frac{1}{2}}$$
$$\overline{\rho}_x^{t+1} = \overline{\rho}_x^{t} + \frac{\Delta t}{\Delta x}(J_{x-\frac{1}{2}} - J_{x+\frac{1}{2}})$$



Upwinding

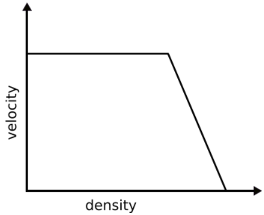


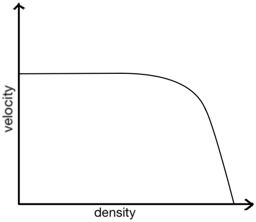
Constitutive Laws for Autonomous Vehicles

Linear Piecewise: $v(\rho) = \begin{cases} v_m & (\rho \leq \rho_c) \\ -\frac{v_m}{\rho_m - \rho_c}\rho + \frac{v_m \rho_m}{\rho_m - \rho_c} & (\rho_c < \rho \leq \rho_m) \end{cases}$

Arctan: $y = tan^{-1}(x)$
$\therefore v(\rho) = A tan^{-1}(\mathrm{C}\rho + \mathrm{D}) + \mathrm{B}$

ERF: $erf(x) = \int_{-\infty}^{x} e^{-t^2} dt$
$\therefore v(\rho) = A erf(\mathrm{C}\rho + \mathrm{D}) + \mathrm{B}$





System of Conservation Equation

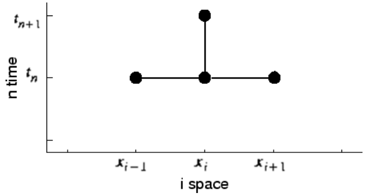
$\rho(x,t)$: the density of regular cars
$\sigma(x,t)$: the density of autonomous cars

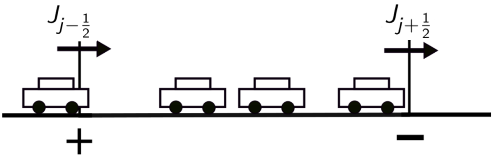
$$\rho_t + J_1(\rho,\sigma)_x = 0$$
$$\sigma_t + J_2(\rho,\sigma)_x = 0$$

Lax-Friedrichs Method

$$J_{x-\frac{1}{2}} = \frac{1}{2}(J_{x-1} + J_x) - \frac{\Delta x}{2\Delta t}(\overline{\rho}_x - \overline{\rho}_{x-1})$$

$$\overline{\rho}_x^{t+1} = \overline{\rho}_x^t + \frac{\Delta t}{\Delta x}(J_{x-\frac{1}{2}} - J_{x+\frac{1}{2}})$$





Future Goals

- Different Methods
- Solving the Coupled Conservation Equations
- Multiple Lanes
- Adding Diffusive Terms

Acknowledgements

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- Parents
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