GEOMETRIC ANALYSIS SEMINAR

"Higher Willmore invariants for hypersufaces, and energies for these via a Q-curvature"

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Abstract: The Willmore energy of a surface is a conformal measure of its failure to be conformally spherical. In physics the energy is variously called the bending energy, or rigid string action. In both geometric analysis and physics it has been the subject of great recent interest. It turns out that its Euler-Lagrange equation is an extremely interesting equation in conformal geometry: the energy gradient is a fundamental curvature that is a scalar-valued hypersurface analogue of the Bach tensor (of dimension 4) of intrinsic conformal geometry.

We show that that these surface conformal invariants, i.e. the Willmore energy and it gradient (the Willmore invariant), are the lowest dimensional examples in a family of similar invariants in higher dimensions. In particular we prove that a generalising analogue of the Willmore invariant arises directly in the asymptotics associated with a singular Yamabe problem on conformally compact manifolds. Then a result of Graham shows that an energy giving this invariant (as gradient with respect to variation of hypersurface embedding) arises as a so-called "anomaly term" in a related renormalised volume expansion. We show that this anomaly term is, in turn, the integral of a local Q-curvature quantity for hypersurfaces that generalises Branson's Q-curvature by including coupling to the (extrinsic curvature) data of the embedding. This is associated to conformally invariant Laplacian power operators related to the celebrated GJMS operators, but which are coupled to the extrinsic curvature data of the embedding. In connection with these results an interesting new Obata type problem arises and this will be described.

This is joint work with Andrew Waldron.

Wednesday, May 23rd, 2018
MIT, Room 2-131
Time: 4:00 PM

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