

# GEOMETRIC ANALYSIS SEMINAR

## “The complexity of a minimal subvariety and the index conjecture”

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Abstract: There are several ways to quantify the "complexity" of a minimal subvariety: on the one hand we have analytic data (like the Morse index, the value of the  $p$ -th eigenvalue of the Jacobi operator etc...), on the other we have geometric invariants (like the Betti numbers, the sigma invariant etc...). But what is the relation between these pieces of information? Are these measures equivalent in some sense? I will give a broad overview of this class of problems and will then focus on recent joint work with L. Ambrozio and B. Sharp: Motivated by a conjecture due to Schoen and recently presented in extended form by F. Marques and A. Neves in their ICM lectures, we study the relation between the Morse index and the first Betti number of minimal hypersurfaces inside positively curved closed Riemannian manifolds. We present a unified framework to address such conjecture and we settle it for a wide class of ambient spaces. More specifically, we give a curvature condition on the ambient manifold which ensures that the Morse index of any minimal hypersurface is bounded from below by a fixed linear function of its first Betti number. As a result, such conclusion is shown to hold true on all compact rank one symmetric spaces, on product of spheres and on suitably pinched submanifolds of the Euclidean space. Differently from earlier works of Ros and Savo, our methods are "flexible" meaning that they also apply to spaces that are neither rigid nor special in any reasonable sense.

**Wednesday, March 16<sup>th</sup>, 2016**  
**MIT, Room 2-105**  
**Time: 4:00 PM**

