Metaplectic representations of Hecke algebras, Weyl group actions and associated polynomials

Joint work with Jasper Stokman and Vidya Venkateswaran

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- Related to Eisenstein series for "metaplectic" covers of adelic groups.

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- This construction also yields a new family of "metaplectic" polynomials, which generalize Macdonald polynomials.

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- Affine Weyl group $= W \ltimes P$, where $P = \{\lambda\}$ is the weight lattice.
- W also acts on $\mathbb{C}_x[P] = \langle x^\lambda \rangle$ group algebra, $\mathbb{C}_x(P)$ fraction field.

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- For $f \in \mathbb{C}_x(P^n)$ and $\lambda \in P$ define

$$\sigma_i\left(fx^\lambda\right) := s_i\left(fx^\lambda\right) \times \left(1 - vx^{m(\alpha_i)\alpha_i}\right)^{-1} \times$$
$$\left[x^{-\mathbf{r}_{m(\alpha_i)}\left[-\frac{\mathbf{B}(\lambda,\alpha_i)}{\mathbf{Q}(\alpha_i)}\right]\alpha_i}[1-v] - vg_{\mathbf{Q}(\alpha_i)-\mathbf{B}(\lambda,\alpha_i)}x^{[1-m(\alpha_i)]\alpha_i}\left[1 - x^{m(\alpha_i)\alpha_i}\right]\right]$$

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- Theorem (C-G). *This defines an action of W on $C_x(P)$.*

Finite Hecke algebra

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$$T_i u_\mu = \begin{cases} u_{s_i\mu} & \text{if } (\mu, \alpha_i^\vee) > 0 \\ k u_\mu & \text{if } (\mu, \alpha_i^\vee) = 0 \\ (k - k^{-1}) u_\mu + u_{s_i\mu} & \text{if } (\mu, \alpha_i^\vee) < 0 \end{cases}$$

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- Induced representation: $\pi = Ind_H^{\mathcal{H}}(U_C)$ realized on $N_C = U_C \otimes \mathbb{C}_Y$

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The identity on $\mathbb{C}_Y (P^n)$ extends to an algebra isomorphism $\mathcal{A}_{loc} \approx \mathcal{H}_{loc}$, with $s_i \mapsto c_i(Y)(T_i - k) + 1$ where $c_i(Y) = \left(1 - Y^{\alpha_i^n}\right) / \left(k^{-1} - kY^{\alpha_i^n}\right)$.

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Theorem (S.-Stokman-Venkateswaran)

The metaplectic representation ϖ of $\mathcal{H}$ on $\mathbb{C}_x [P]$ extends (uniquely) to a representation ϖ_{loc} of $\mathcal{H}_{loc}$ on $\mathbb{C}_x (P)$. The action of W induced by the isomorphism $\mathcal{A}_{loc} \approx \mathcal{H}_{loc}$ coincides with the CG action.

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- For general n the metaplectic polynomials bear an analogous relationship with p-parts of WMDS.

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Theorem (S.-Stokman-Venkateswaran)

ϖ can be extended to the double affine Hecke algebra.

- This leads to Cherednik operators and "metaplectic" polynomials depending on additional parameters $g_1, \ldots, g_{n-1}$.
- For $n = 1$, these are the "usual" nonsymmetric Macdonald polynomials
- The p-parts of WMDS are Whittaker functions for p-adic groups
- For $n = 1$, by the Casselman-Shalika formula they are specializations of Macdonald polynomials
- For general n the metaplectic polynomials bear an analogous relationship with p-parts of WMDS.
- One can similarly obtain symmetric metaplectic polynomials, which generalize symmetric Macdonald polynomials

Some Metaplectic polynomials for GL(3)

Formulas for $E_\lambda^{(n)}(x)$, $1 \le n \le 5$ and $\lambda \in \mathbb{Z}^3$ of weight at most 2.

$E^{(1)}_{(0,0,0)}(x) = 1$
$E^{(2)}_{(0,0,0)}(x) = 1$
$E^{(3)}_{(0,0,0)}(x) = 1$
$E^{(4)}_{(0,0,0)}(x) = 1$
$E^{(5)}_{(0,0,0)}(x) = 1$

$E^{(1)}_{(1,0,0)}(x) = x_1$
$E^{(2)}_{(1,0,0)}(x) = x_1$
$E^{(3)}_{(1,0,0)}(x) = x_1$
$E^{(4)}_{(1,0,0)}(x) = x_1$
$E^{(5)}_{(1,0,0)}(x) = x_1$

Some Metaplectic polynomials for GL(3)

$$E^{(1)}_{(0,1,0)}(x) = \frac{(k-1)(k+1)}{k^4 q-1} x_1 + x_2$$
$$E^{(2)}_{(0,1,0)}(x) = \frac{(k-1)(k+1)}{k(kq^2+\epsilon)} x_1 + x_2$$
$$E^{(3)}_{(0,1,0)}(x) = \frac{(k-1)(k+1)g_1}{k^4 g_1^3 q^3+1} x_1 + x_2$$
$$E^{(4)}_{(0,1,0)}(x) = \frac{(k-1)(k+1)g_1}{k^4 g_1^3 q^4+1} x_1 + x_2$$
$$E^{(5)}_{(0,1,0)}(x) = \frac{(k-1)(k+1)g_1}{k^4 g_1^3 q^5+1} x_1 + x_2$$

$$E^{(1)}_{(0,0,1)}(x) = \frac{(k-1)(k+1)}{qk^2-1} x_1 + \frac{(k-1)(k+1)}{qk^2-1} x_2 + x_3$$
$$E^{(2)}_{(0,0,1)}(x) = -\frac{(k-1)(k+1)}{k(k+\epsilon q^2)} x_1 + \frac{(k-1)(k+1)}{q^2+\epsilon k} x_2 + x_3$$
$$E^{(3)}_{(0,0,1)}(x) = -\frac{(k-1)(k+1)g_1^2}{k^2 g_1^3 q^3+1} x_1 + \frac{(k-1)(k+1)g_1}{k^2 g_1^3 q^3+1} x_2 + x_3$$
$$E^{(4)}_{(0,0,1)}(x) = -\frac{(k-1)(k+1)g_1^2}{k^2 g_1^3 q^4+1} x_1 + \frac{(k-1)(k+1)g_1}{k^2 g_1^3 q^4+1} x_2 + x_3$$
$$E^{(5)}_{(0,0,1)}(x) = -\frac{(k-1)(k+1)g_1^2}{k^2 g_1^3 q^5+1} x_1 + \frac{(k-1)(k+1)g_1}{k^2 g_1^3 q^5+1} x_2 + x_3$$

Some Metaplectic polynomials for GL(3)

$$E^{(1)}_{(0,1,1)}(x) = \frac{(k-1)(k+1)}{qk^2-1}x_1x_2 + \frac{(k-1)(k+1)}{qk^2-1}x_3x_1 + x_3x_2$$
$$E^{(2)}_{(0,1,1)}(x) = -\frac{(k-1)(k+1)}{k(k+\epsilon q^2)}x_1x_2 + \frac{(k-1)(k+1)}{q^2+\epsilon k}x_3x_1 + x_3x_2$$
$$E^{(3)}_{(0,1,1)}(x) = -\frac{(k-1)(k+1)g_1^2}{k^2g_1^3q^3+1}x_1x_2 + \frac{(k-1)(k+1)g_1}{k^2g_1^3q^3+1}x_3x_1 + x_3x_2$$
$$E^{(4)}_{(0,1,1)}(x) = -\frac{(k-1)(k+1)g_1^2}{k^2g_1^3q^4+1}x_1x_2 + \frac{(k-1)(k+1)g_1}{k^2g_1^3q^4+1}x_3x_1 + x_3x_2$$
$$E^{(5)}_{(0,1,1)}(x) = -\frac{(k-1)(k+1)g_1^2}{k^2g_1^3q^5+1}x_1x_2 + \frac{(k-1)(k+1)g_1}{k^2g_1^3q^5+1}x_3x_1 + x_3x_2$$

$$E^{(1)}_{(1,0,1)}(x) = \frac{(k-1)(k+1)}{k^4q-1}x_1x_2 + x_3x_1$$
$$E^{(2)}_{(1,0,1)}(x) = \frac{(k-1)(k+1)}{k(kq^2+\epsilon)}x_1x_2 + x_3x_1$$
$$E^{(3)}_{(1,0,1)}(x) = \frac{(k-1)(k+1)g_1}{k^4g_1^3q^3+1}x_1x_2 + x_3x_1$$
$$E^{(4)}_{(1,0,1)}(x) = \frac{(k-1)(k+1)g_1}{k^4g_1^3q^4+1}x_1x_2 + x_3x_1$$
$$E^{(5)}_{(1,0,1)}(x) = \frac{(k-1)(k+1)g_1}{k^4g_1^3q^5+1}x_1x_2 + x_3x_1$$

Some Metaplectic polynomials for GL(3)

$$E^{(1)}_{(1,1,0)}(x) = x_1x_2$$
$$E^{(2)}_{(1,1,0)}(x) = x_1x_2$$
$$E^{(3)}_{(1,1,0)}(x) = x_1x_2$$
$$E^{(4)}_{(1,1,0)}(x) = x_1x_2$$
$$E^{(5)}_{(1,1,0)}(x) = x_1x_2$$

$$E^{(1)}_{(2,0,0)}(x) = x_1^2 + \frac{q(k-1)(k+1)}{qk^2-1}x_1x_2 + \frac{q(k-1)(k+1)}{qk^2-1}x_3x_1$$
$$E^{(2)}_{(2,0,0)}(x) = x_1^2$$
$$E^{(3)}_{(2,0,0)}(x) = x_1^2$$
$$E^{(4)}_{(2,0,0)}(x) = x_1^2$$
$$E^{(5)}_{(2,0,0)}(x) = x_1^2$$

Some Metaplectic polynomials for GL(3)

$$E^{(1)}_{(0,2,0)}(x) = \frac{(k-1)(k+1)}{(qk^2-1)(qk^2+1)}x_1^2 + x_2^2 + \frac{(k-1)(k+1)\left(k^4q^2+qk^2-q-1\right)}{(qk^2+1)(qk^2-1)^2}x_1x_2 + \frac{(k-1)^2(k+1)^2q}{(qk^2+1)(qk^2-1)^2}x_3x_1 + \frac{q(k-1)(k+1)}{qk^2-1}x_3x_2$$

$$E^{(2)}_{(0,2,0)}(x) = \frac{(k-1)(k+1)}{(q^2k^2-1)(q^2k^2+1)}x_1^2 + x_2^2$$

$$E^{(3)}_{(0,2,0)}(x) = \frac{(k-1)(k+1)g_1^2}{k^2g_1^3+q^6}x_1^2 + x_2^2$$

$$E^{(4)}_{(0,2,0)}(x) = \frac{(k-1)(k+1)}{k(q^8k+\epsilon)}x_1^2 + x_2^2$$

$$E^{(5)}_{(0,2,0)}(x) = \frac{(k-1)(k+1)g_2}{k^4g_2^3q^{10}+1}x_1^2 + x_2^2$$

Some Metaplectic polynomials for GL(3)

$$E^{(1)}_{(0,0,2)}(x) = \frac{(k-1)(k+1)}{(kq-1)(kq+1)}x_1^2 + \frac{(k-1)(k+1)}{(kq-1)(kq+1)}x_2^2 + x_3^2 + \frac{(q+1)(k-1)^2(k+1)^2}{(kq-1)(kq+1)(qk^2-1)}x_1x_2 + \frac{(q+1)(k-1)(k+1)}{(kq-1)(kq+1)}x_3x_1 + \frac{(q+1)(k-1)(k+1)}{(kq-1)(kq+1)}x_3x_2$$

$$E^{(2)}_{(0,0,2)}(x) = \frac{(k-1)(k+1)}{(kq^2-1)(kq^2+1)}x_1^2 + \frac{(k-1)(k+1)}{(kq^2-1)(kq^2+1)}x_2^2 + x_3^2$$

$$E^{(3)}_{(0,0,2)}(x) = -\frac{(k-1)(k+1)g_1}{k^4g_1^3+q^6}x_1^2 + \frac{(k-1)(k+1)k^2g_1^2}{k^4g_1^3+q^6}x_2^2 + x_3^2$$

$$E^{(4)}_{(0,0,2)}(x) = -\frac{(k-1)(k+1)}{k(\epsilon q^8+k)}x_1^2 + \frac{(k-1)(k+1)}{q^8+\epsilon k}x_2^2 + x_3^2$$

$$E^{(5)}_{(0,0,2)}(x) = -\frac{(k-1)(k+1)g_2^2}{k^2g_2^3q^{10}+1}x_1^2 + \frac{(k-1)(k+1)g_2}{k^2g_2^3q^{10}+1}x_2^2 + x_3^2$$