

Double affine Hecke algebras and algebraic geometry workshop

May 17 through 21, 2010

Monday, May 17

9:00 am Registration

9:20 am Opening Remarks

9:30 — 10:30 am **Roman Bezrukavnikov**

“Modular representations and geometry”

In lecture 1 I will review a joint project with Ivan Mirkovic devoted to proving Lusztig’s conjectures which generalize Kazhdan-Lusztig theory to representations of Lie algebras in positive characteristic.

10:30 — 11:00 am *Coffee break*

11:00 — 12:00 pm **Roman Bezrukavnikov**

“Symplectic resolutions and canonical bases”

In the second lecture I will describe a conjectural generalization of the story from lecture 1 to other geometrically similar contexts, such as quiver varieties. I will also mention conjectured connections to quantum cohomology — partly known due to works of Braverman, Maulik, Okounkov, Pandharipande et. al — and perhaps hoped for connections to mirror symmetry.

12:00 — 1:30 pm Lunch

Monday, May 17

1:30 — 3:05 pm **Stephen Griffeth**

“Generalized Jack polynomials and the representation theory of rational Cherednik algebras” (joint work with C. Dunkl)

We will explain how to describe the set of parameters c for which the rational Cherednik algebra H_c of type $G(r, p, n)$ is equivalent to its spherical subalgebra (this is an analog of previous work of Gordon-Stafford and Bezrukavnikov-Etingof in the symmetric group case). The technique is based on vector-valued analogs of Jack polynomials and Cherednik-style intertwining operators. Time permitting, we'll describe a couple of examples in low rank where the same method gives more detailed information.

3:05 — 3:25 pm *Break*

3:25 — 5:00 pm **Davesh Maulik**

“Quantum cohomology of the Springer resolution”

We give an overview of some of the algebraic structures associated to the quantum cohomology of equivariant symplectic resolutions and illustrate them in the example of the Springer resolution. Joint with A. Braverman and A. Okounkov.

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Tuesday, May 18

9:30 — 11:00 am **P. Etingof**

“Wreath product symplectic reflection algebras and affine Lie algebras”

I will review known results on representations of wreath product symplectic reflection algebras (in particular, cyclotomic rational Cherednik algebras). Then I will state some conjectures (e.g., those on the number of irreducible finite dimensional representations) that link the representation theory of such a symplectic reflection algebra to the structure of the basic representation over the corresponding affine Lie algebra of type ADE (extended by a Heisenberg algebra) as a module over its subalgebras. These conjectures arise from the idea, due to R. Bezrukavnikov and A. Okounkov, that the representation theory of a wreath product symplectic reflection algebra should be ‘controlled’ by the quantum connection on the equivariant quantum cohomology of the Hilbert scheme of n -tuples of points on the resolution of the corresponding Kleinian singularity.

11:00 — 11:25 am *Coffee Break*

11:25 — 1:00 pm **A. Okounkov**

“The monodromy of the quantum differential equation for the Hilbert scheme of points”

The talk will be about the monodromy of the quantum differential equation for the Hilbert scheme of points in the plane. Joint work with Roma Bezrukavnikov.

Tuesday, May 18

1:00 — 2:30 pm *Lunch*

2:30 — 4:05 pm **A. Braverman**

“From DAHA to double affine Grassmannian”

The purpose of this talk will be to explain certain (rather peculiar) combinatorial results of Macdonald and Cherednik (proved using double affine Hecke algebras) using the geometry of the so called double affine Grassmannian. If time permits, I shall also explain some new way how a close relative of DAHA appears via “double loop groups”. This potentially should lead to “double affine Kazhdan-Lusztig theory”.

Based on joint works with M. Finkelberg, D. Kazhdan and M. Patnaik.

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Wednesday, May 19

9:30 — 11:00am **Michela Varagnolo**

“Canonical bases and affine Hecke algebras of type B ”

In this talk I will explain how to prove some conjectures of Enomoto and Kashiwara concerning canonical bases and branching rules of affine Hecke algebras of type B . The main ingredient will be a new graded Ext-algebra associated to a quiver with involution and Morita equivalent to the affine Hecke algebra of type B . (This is a joint work with E. Vasserot). At the end of the talk I will say some words on an analog construction (given by Shan and Vasserot) for double affine Hecke algebras.

11:00 — 11:25am *Coffee Break*

11:25 — 1:00pm **Eric Vasserot**

“Double affine Hecke algebras and affine Springer fibers”

I’ll review the geometric construction of representations of DAHA’s via the K-theory of affine flag varieties. I’ll insist on the realization of some finite dimensional modules in the cohomology of affine Springer fibers.

1:00 — 2:30pm *Lunch*

2:30 — 4:05pm **Olivier Schiffmann**

“DAHA, elliptic Hall algebra and the equivariant K-theory of Hilbert schemes”

We will describe an action of the Hall algebra of an elliptic curve on the equivariant K-theory of the Hilbert schemes of points in the plane. This elliptic Hall algebra, which is isomorphic to the spherical DAHA of $GL(\infty)$ admits several presentations which we will describe : a ‘Drinfeld realization’, a realization as a shuffle algebra of Feigin-Odesskii type, and a combinatorial realization based on paths in Z^2 . We will also interpret our construction in the framework of the geometric Langlands duality (for $GL(n)$ and for elliptic curves) and explain its generalization to arbitrary curves and reductive groups.

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Thursday, May 20

9:30 — 11:00 am **Valerio Toledano Laredo**

“The trigonometric Casimir connection of a simple Lie algebra”

Let \mathfrak{g} be a finite dimensional complex, simple Lie algebra, G the corresponding simply connected Lie group and H a maximal torus in G . I will describe a flat connection on H with logarithmic singularities on the root hypertori in H and values in the Yangian of \mathfrak{G} . Conjecturally, its monodromy is described by the quantum Weyl group operators of the quantum loop algebra $U_h(L\mathfrak{g})$.

11:00 — 11:25 am *Coffee Break*

11:25 — 1:00 pm **Victor Ginzburg**

“Isospectral commuting variety and Harish-Chandra D-module”

Let \mathfrak{g} be a complex reductive Lie algebra with Cartan algebra \mathfrak{h} . Hotta and Kashiwara defined a holonomic \mathcal{D} -module \mathcal{M} , on $\mathfrak{g} \times \mathfrak{h}$, called Harish-Chandra module. We give an explicit description of $\text{gr } \mathcal{M}$, the associated graded module with respect to a canonical *Hodge filtration* on \mathcal{M} . The description involves the *isospectral commuting variety*, a subvariety $\mathfrak{X} \subset \mathfrak{g} \times \mathfrak{g} \times \mathfrak{h} \times \mathfrak{h}$ which is a finite extension of the variety of pairs of commuting elements of \mathfrak{g} . Our main result establishes an isomorphism of $\text{gr } \mathcal{M}$ with the structure sheaf of the normalization of \mathfrak{X} . It follows, thanks to Saito’s theory of polarized Hodge modules, that the normalization of the isospectral commuting variety is Cohen-Macaulay and Gorenstein. This confirms a conjecture of M. Haiman.

In the special case where $\mathfrak{g} = \mathfrak{gl}_n$, there is an open subset of the isospectral commuting variety that is closely related to the Hilbert scheme of n points in \mathbb{C}^2 . The sheaf $\text{gr } \mathcal{M}$ gives rise to a locally free sheaf on the Hilbert scheme. We show that the corresponding vector bundle is isomorphic to the *Procesi bundle*. This yields a new proof of the positivity result for Macdonald polynomials, established earlier by Haiman.

1:00 — 2:30 pm *Lunch*

2:30 — 4:05 pm **Ivan Losev**

“Symplectic reflection algebras via quantum Hamiltonian reduction, revisited”

Consider a symplectic reflection algebra for a wreath product of a Kleinian group with a symmetric group. It is known that the spherical subalgebra in this algebra. It is known (due to Holland, Etingof-Ginzburg, Oblomkov, Gordon, Etingof-Gan-Ginzburg-Oblomkov) that the spherical subalgebra is isomorphic to a quantum Hamiltonian reduction of the algebra of differential operators on the space of representations of an appropriate quiver. The proof in the most difficult case (due to EGGO) uses several beautiful auxiliary constructions and involves many interesting computations. We will sketch a more boring proof (covering everything but the Holland original proof for the Kleinian group itself). Our proof involves deformation quantization of some symplectic resolutions.

****6:30pm Conference dinner at The Royal East Restaurant**

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Friday, May 21

9:30 — 11:00 am **Peng Shan**

“Crystals of Fock spaces and cyclotomic rational double affine Hecke algebras”

We will explain the construction of a crystal structure on the set of simple modules in the category \mathcal{O} of cyclotomic rational double affine Hecke algebras and its relation to Fock spaces.

11:00 — 11:25 am *Coffee Break*

11:25 — 1:00 pm **Aaron Lauda**

“Categorifying quantum groups”

I’ll explain joint work with Mikhail Khovanov on a categorification of one-half of the quantum universal enveloping algebra associated to a Kac-Moody algebra. This categorification is obtained from the graded representation category of certain graded algebra that can be defined using a graphical calculus. Certain finite-dimensional quotients of these graded algebras give categorifications of irreducible representations of the quantum enveloping algebra.

1:00 — 2:30 pm *Lunch*

2:30 — 4:05 pm **TBA**