

# COMBINATORICS SEMINAR

## THE JORDAN TYPE OF NILPOTENT MATRICES WITH CERTAIN CONSTRAINTS

TERADA ITARU  
University of Tokyo

### ABSTRACT:

We extend Gansner and Saks' result on the Jordan types of nilpotent matrices to those with certain symmetry and skewsymmetry constraints designated by an oriented graph with partial arrow-reversing fixed-point-free involution. Let  $G = (V, E)$  be an oriented graph with no loops, no multiple edges, and no oriented cycles,  $Z = Z^+ \sqcup Z^-$  a subset of  $V$  admitting an involution  $\sigma: Z \rightarrow Z$ ,  $\sigma(Z^\pm) = Z^\mp$  such that  $(\alpha, \beta) \in E$  if and only if  $(\sigma(\beta), \sigma(\alpha)) \in E$  for  $\alpha, \beta \in Z$ . If  $\alpha \in Z$ , write  $\varepsilon(\alpha) = \pm 1$  according to whether  $\alpha \in Z^\pm$ . Let  $N_p(G, \sigma)$  (resp.  $N_k(G, \sigma)$ ) be the complex vector space formed by the complex matrices  $X = (x_{\alpha\beta})$  whose rows and columns are indexed by  $V$  and satisfying the conditions:

1.  $x_{\alpha\beta} = 0$  unless  $(\alpha, \beta) \in E$ , and
2.  $x_{\sigma(\beta)\sigma(\alpha)} = \varepsilon(\alpha)\varepsilon(\beta)x_{\alpha\beta}$  (resp.  $-\varepsilon(\alpha)\varepsilon(\beta)x_{\alpha\beta}$ ) if  $\alpha, \beta \in Z$ .

Then there exists a unique partition  $\lambda = \lambda_*(G, \sigma)$  ( $* = p, k$ )  $= (\lambda_1, \lambda_2, \dots, \lambda_l)$  of  $|V|$  such that almost all elements of  $N_*(G, \sigma)$  have the Jordan type represented by  $\lambda$ . We show that

- (I) If  $* = p$  and  $1k|V|$ , then  $\lambda_1 + \dots + \lambda_k$  equals the largest cardinality of a  $k$ -path of  $G$  satisfying one additional condition.
- (II) If  $* = k$ ,  $1k|V|$  and  $E$  contains all arrows  $(\alpha, \sigma(\alpha))$ ,  $\alpha \in Z^+$ , then  $\lambda_1 + \dots + \lambda_k$  equals the largest cardinality of a  $k$ -path of  $G$ . This is equal to the Jordan type of almost all  $(x_{\alpha\beta})$  satisfying the condition (1) only.

These conditions are motivated by Trapa's bijection for the generalized Steinberg variety for the (complexified) symmetric space  $GL_{2n}/Sp_{2n}$  and another variety related to the same symmetric space considered in the speaker's earlier paper.

Friday, November 3, 2006  
4:15 p.m.

M.I.T. Room 2-136

Refreshments will be served at 3:30 PM in Room 2-349.

<http://www-math.mit.edu/~combin>