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18.702 Comments on Problem Set 8

1. Chapter 14, Problem 2.4. (*ideals that are free modules*)

Two elements a, b of a ring R won't be independent because $ba - ab = 0$. An ideal I is submodule of the module R. So the only way that it can be free is if it is free or rank 1, which means that it is a principal ideal. A principal ideal Ra of R is a free module if a isn't a *zero divisor*, if $ra \neq 0$ for all $r \neq 0$.

A quotient R/I cannot be a free module unless I is the zero ideal. If $a_!, ..., a_r$ are elements of R and b is a nonero element of I, then the residue of $ba_1 + 0a_2 + \cdots$ in R/I will be zero, so the elements $a_1, ..., a_r$ aren't independent.

2. Chapter 14, Problem 7.3(d). (*writing an abelian group as a sum of cyclic groups*)

One diagonalizes the matrix of coefficients. I got the diagonal matrix With diagonal entries $1, 6, 0$. The group is cyclic of order 6.

3. Chapter 14, Problem M.2. (*modules over the ring* $R = \mathbb{Z}/6\mathbb{Z}$)

We have a surjective homomorphism $\mathbb{Z} \to R$. Using this homomorphism, an R-module becomes a $\mathbb{Z}$-module, an abelian group, on which scalar multi;ication by 6 is the zero operation. So the problem becomes to classify abelian group V such that $6V = 0$. They are the direct sums of abelian groups of orders $2, 3$ and 6, with the relation that the sum of cyclic groups of orders 2 and 3 is cyclic of order 6. Or, perhaps better here, V will be a direct sum of a finite number of cyclic groups of orders 2 and 3