18.702 Problem Set 5   (corrected)

due Friday, March 26

1. Chapter 12, Exc. 2.4. (*infinitely many primes in $F[x]$*)

2. In the ring of integers $\mathbb{Z}$, the greatest common divisor d of two positive integers a, b is the positive integer that generates the ideal $a\mathbb{Z} + b\mathbb{Z}$. So $a\mathbb{Z} + b\mathbb{Z} = d\mathbb{Z}$. The intersection $z\mathbb{Z} \cap b\mathbb{Z}$ is also an ideal. It is a principal ideal $m\mathbb{Z}$ for some positive integer m. The integer m is called the *least common multiple* of a and b.
(i) Prove that a and b divide m, and that if an integer n is divisible by a and b, then it is divisible by m.
(ii) Prove that $ab = md$.

3. Chapter 12, Exercise 4.5. (*irreducibility of some polynomials*)

4. Chapter 12, Exc. 4.6. (*factoring $x^5 + 5x + 5$*)

5. Chapter 12, Exc. 4.8. (*factoring certain quartics*)