

18.702 Problem Set 2

due Wednesday, March 2

1. Chapter 10, Exercise 6.5. (*the standard representation of S_n*)

Hint: This problem is closely related to problem M.1 of Chapter 4, that was assigned in 18.701. I don't recommend using characters.

2. Let ρ and σ be representations of a group G , and suppose that σ has dimension 1. Prove that setting $\rho'_g = \sigma_g \rho_g$ defines a representation of G , and determine its character in terms of the characters of ρ and σ .

3. Chapter 10, Exercise M.9. (*Frobenius Reciprocity*)

Hint: This comes out when one writes what is to be proved carefully. Keep in mind that a isn't in H . So $a^{-1}ha$ and h are conjugate elements of G , but not necessarily conjugate elements of H .

I recommend that you begin by working the example in which $G = S_3$, $H = C_3$ is the subgroup generated by a 3-cycle, and S is a one-dimensional representation of H .

4. Determine the character table of the symmetric group S_5 . You may use any method, including the character table of A_5 that is in the text, except that you are not allowed to look the table up on the web or in a book.

Hint: There is more than one way to proceed. One way is to begin by decomposing the operation of G on the set of five indices, and the operation by conjugation on set of ten pairs of indices. This will determine characters of dimension four and five. Then one can use the result of problem 2. One could also use Frobenius reciprocity and the character table of A_5 .